

# Developing methods for two-digit subtraction by means of the thieving game

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*Multi-digit subtraction is a central part of Grade 1–3 mathematics that many students struggle with and is challenging to teach. We used design research to investigate how and to what extent game-based activities could support Grade 1 students' development of methods for two-digit subtraction. Our results show that a short sequence of activities based in the game led to 85 % of students being able to document at least one method for two-digit subtraction, and 42 % being able to document at least two different methods. However, the way the teacher picked up students' methods for discussion and how the teacher documented them affected what methods students developed.*

Multi-digit addition and subtraction is the only area for which Swedish Grade 3 students are required to know written methods (Skolverket, 2022). Since 2016, one subtest of the national mathematics test for Grade 3 has been devoted to such methods. During these years, this subtest has had the lowest student attainment—about 80 % compared to 86–95 % on the other subtests (Skolverket, 2025). Research studies have also found that students struggle with, particularly, methods for multi-digit subtraction (Heinze et al., 2009; Skolverket, 2025). For example, Heinze et al. (2009) and Kullberg et al. (2024b) both report that the students in their samples solved less than half of the tasks correctly. However, the success rate depends on the methods students use, and what methods students use may depend on the design of teaching.

## **Methods for multi-digit subtraction**

In Sweden, and internationally, students are usually taught the *standard algorithm* for subtraction, writing the minuend above the subtrahend and subtracting from right to left digit by digit. If a subtraction cannot be performed due to the minuend digit being smaller than the subtrahend digit, one is taken from the next digit and exchanged for a ten in the current digit. While understanding how and why this algorithm works require making use of properties of our number system, it is possible to *use* the algorithm without considering the place values of the digits, seeing them as separate one-digit numbers. Therefore, this method can be seen as digit-based (Hickendorff et al., 2019). While seldom explicitly taught, it is known that some students use *One-by-one* counting methods (Hopkins et al., 2022). This can be done in several ways—e.g., counting down from the minuend—but all involve double counting that heavily taxes working memory. When terms and differences get larger, this makes the method prone to mistakes.

In contrast to the standard algorithm and one-by-one counting, most methods for multi-digit subtraction make use of the base-10 structure of our number system (Fuson

et al., 1997) and can thus be seen as number-based (Hickendorff et al., 2019). Some of these methods are based on a “taking away” view of subtraction (Selter et al., 2012). The *Split* method splits both terms into ones, tens and hundreds and entails one subtraction for each number, as in the standard algorithm. Several studies have reported low success rates for using this method for subtractions bridging over ten (Heinze et al., 2009; Kullberg et al., 2024b). A common mistake for when using this method is to subtract the minuend from the subtrahend when the minuend is smaller. The *Stepwise* method splits only the subtrahend, usually into parts that make each sub-subtraction simpler, such as  $32 - 18 = 32 - 2 - 10 - 6 = 30 - 10 - 6 = 20 - 6 = 14$ . This method has been reported to be substantially more successful when used by students (Heinze et al., 2009; Kullberg et al., 2024b; Torbeyns et al., 2009). The *Compensation* method first takes away more than the subtrahend, normally the nearest greater ten, and then adds the difference:  $32 - 18 = 32 - 20 + 2 = 12 + 2 = 14$ . Other methods are based on a “determining the difference” view of subtraction (Selter et al., 2012). *Indirect addition* involves adding parts to the subtrahend up to the minuend using tens and hundreds as benchmarks:  $18 + \underline{2} = 20$ ,  $20 + \underline{10} = 30$ ,  $30 + \underline{2} = 32$ ,  $2 + 10 + 2 = 14$ . *Simplification* involves adding or subtracting the same number to both terms, which can be explained as moving the difference along the number line to a place where it is more easily “seen”:  $32 - 18 = (32+2) - (18+2) = 34 - 20 = 14$ .

### **Teaching designs for methods for multi-digit subtraction**

To some extent, students may develop their own methods independent of teaching. However, research has shown that when students are introduced to the standard algorithm, they abandon other methods, favouring the algorithm even if they cannot use it correctly and when other methods would be more efficient (Hickendorff et al., 2019). More recent studies have shown that this phenomenon can be partly avoided if teaching focusses students’ own development and comparison of methods before introducing the standard algorithm (Heinze et al., 2018). Designing teaching that supports students’ own development of methods may, however, be more difficult since the teaching activities must both require students to develop more advanced methods than One-by-one and support them in doing so. For example, the design used in Heinze et al. (2018) failed to support students’ development of indirect addition.

Kullberg et al. (2024a) has shown that strengthening students’ knowledge of part-whole number relations can increase their ability to solve subtractions bridging over ten. However, the solution frequencies for tasks with multi-digit subtrahends was still below 50 %, and 55 % of students still used error-prone methods (Kullberg et al., 2024b), indicating that additional teaching is needed to support the development of multi-digit subtraction methods.

### **Aim and research question**

The aim of this study is to further understanding of how activities can be designed to support students’ development of multi-digit subtraction methods, by answering the question: How and to what extent can the thieving game support Grade 1 students’ development of two-digit subtraction methods by means of their own reasoning?

## Method

This study is part of a design research project aiming to develop design principles for teaching Grades 1–3 students about numbers and operations. The study took place at the end of Grade 1 in two classes that had been following a teaching design during Grade 1, based on alternating between student independent work, alone or in pairs, and whole-class discussions about students’ methods and reasoning. During Grade 1, teaching had covered additive relationships for numbers 0–20, additive situations (part-part-whole, change, and comparisons), double and half, and two-digit numbers, including adding and subtracting even tens to/from two-digit numbers. The students had worked with several representations of numbers, including the number line, the place-value system and wooden base-10 blocks developed within the project consisting of one- (1B), five- (5B) and ten- (10B) blocks with no visible units (Figure 1A).

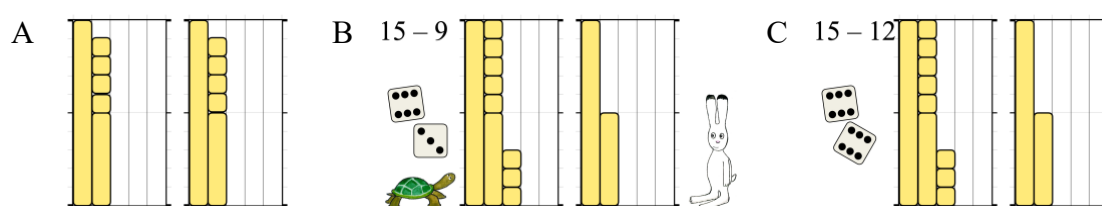


Figure 1. Initial setting for the thieving game and two tricky situations

The study design included developing an initial teaching design for three lessons, which was tested in each of the teachers’ classes in two iterations, with revisions of the design in between. The teaching design included instructions and materials for the students’ independent work, and suggested examples and questions for the whole-class discussion. The initial idea for the design was to support students’ development of methods in the context of *tricky situations* emerging in the thieving game using base-10 blocks played in pairs. In the thieving game, each player starts with an equal amount, in this case 19 represented by one 10B, one 5B and four 1B (Figure 1A). The players take turns in rolling two dice. In each turn, the player rolling the dice is allowed to “thieve” blocks of the same value as the throw of the dice from the other player. A player loses when they have no blocks left. The game forces students to translate from dice to block representations of numbers and thereby practicing number relations such as  $3 + 3 = 5 + 1$ . It also gives rise to tricky situations where the correct number to thief cannot be directly taken from the other player (Figure 1B). The game element was hypothesized to make students less likely to solve such situations by taking too much or too little and playing in pairs would make it impossible to avoid tricky situations by choosing to thief from another player. It was seen as an advantage that tricky situations only start to occur after a few turns, as this would allow all students to get started.

Each lesson started with a whole-class discussion followed by student independent work. The aim of the lesson sequence was to allow students to develop methods in the context of the game and then transfer these methods to subtraction tasks with numbers. The aim of the first lesson was to give students experience of the game and tricky situations. The aim of the second lesson was to develop and practice ways of documenting handling tricky situations with symbolic notation. The aim of the third lesson was to name and practice these ways as methods for subtraction with two-digit

numbers. Through simulated gameplay, we identified three possible methods for handling tricky situations: *Compensate*, and swapping blocks (e.g., one 10B for 1 5B and five 1B) followed by *Stepwise on tens* or *Take from ten* (Table 1).

Code	Description
No documented method	Only an answer or the expression to be evaluated is given.
Not determinable	A method is described or documented, but which one cannot be determined.
One by one	Counting up or down one at a time.
Compensate	Subtracting the nearest larger ten, then adding the difference.
Stepwise on tens	Subtracting stepwise so that the sub-results are even tens.
Stepwise other	Subtracting stepwise so that the sub-results are <u>not</u> even tens.
Split	Subtracting the ones and tens separately
Take from ten	Subtract the subtrahend from one or all tens in the minuend, then adding the ones.

Table 1. Codes for subtraction methods

### Data collection and method of analysis

In each class, the teaching design was planned to be implemented over three lessons, each with 40 min devoted to the mathematical activities. During the planned lessons, one researcher observed the lesson and took notes, and audio was recorded by a device worn by the teacher. Student worksheets were also collected. As the study aims to investigate how and to what extent the thieving game can support Grade 1 students' development of two-digit subtraction methods, the analysis focused on identifying students' use of different methods. The first step of the analysis was therefore to identify segments of observation notes, audio recordings and students' written solutions that concerned one solution to one task. Each such segment that concerned a subtraction task was then coded for which method (if any) was described, when and by who. The coding system started out from methods described by previous research and was then developed by adding codes for methods that did not fit any existing code and removing codes that were not present in our data. The final codes are described in Table 1. The codes were then used to create a narrative of the lesson sequence in each iteration, describing when, how and by whom methods emerged.

## Results

In this section, we present the results of the two iterations including the revised design.

### Iteration 1

The plan for Iteration 1 was: In Lesson 1, introduce the game without discussing tricky situations and let students play in pairs, recording results after each turn, while the teacher identifies and overviews students' handling of tricky situations. In Lesson 2, discuss one non-tricky and one tricky situation, described as occurring when Tortoise and Hare (two figures used since start of Grade 1) are playing. When students suggest ways of handling the tricky situation, the teacher shows ways of documenting these ways using symbolic notation. Each turn in the game includes both a subtraction and an addition, but the teacher was asked to start with and focus on the subtraction. Then, play in pairs, recording how each turn is handled, as done in the discussion. In Lesson 3,

discuss selected examples from the students' documentation, constructing posters for methods used, then individual student work documenting methods for subtractions.

### ***Lesson 1***

The teacher started by introducing the game by playing two turns with a student, visibly to the whole class. This seemed to be enough for the students to understand the rules of the game and get started playing in pairs. All pairs seemed engaged and enjoying playing, and most students did not need any support. However, identifying and overiewing students' handling of tricky situations proved difficult for both the observer and the teacher. The teacher was mostly occupied by organising the turn-taking in one group consisting of three students, who also needed support in handling the tricky situations. In this case, the teacher instructed the students how to swap blocks (one 10B for two 5Bs) when they needed to thief 9 from 14 (one 10B and four 1B). The observer had a hard time capturing the tricky situations, as they were sparse and randomly occurring, but at least one pair was observed to intentionally "forget" what the dice showed to avoid a tricky situation.

### ***Lesson 2***

The class first discussed a non-tricky situation, then the tricky situation showed in Figure 1B. This situation necessitates either using Compensate or swapping blocks (one 10B for one 5B and five 1B) followed by either Take from ten or Stepwise on tens. When asked for suggestions, one student demonstrated Compensate without any assistance from the teacher and, when asked, described what he had done in the following way:

- Student: I swapped a 1 for another.  
Teacher: You swapped a 10 for a 1. And why can one do like that?  
Student: You should say that (quietly).  
Teacher: You can say it—why can one do like that?  
Student: Because then you get 1 minus.  
Teacher: So, 10 minus 1?  
Student: Is 9 (emphatically).  
Teacher: Okay, so if we take the 10 and swap it back for a 1, then you take 9 and give back a 1, or take 10 and give back a 1. That means you take 9.

The teacher documented the actions by writing whole sentences on the board, which took considerable time. When documenting the actions using symbols, the teacher first described the addition as Compensate, followed by a student describing an alternative way of adding. The teacher then documented the subtraction as:  $15 - 9 = 10 - 9 + 5 = 1 + 5 = 6$ , marking that  $10 - 9 = 1$ , and used the blocks to illustrate the block swapping. This aligns with Take from ten rather than Compensate, which the student described.

The teacher then asked for alternative methods for solving the task  $15 - 9$ . One student tried swapping a 5B for five 1B, which proves ineffective in this context. With further assistance with swapping from another student, the student was eventually able to thief 9. The teacher documented this on the board, starting with addition, describing the use of Stepwise other, and then the subtraction as  $15 - 9 = 15 - 5 - 4$ , describing Stepwise on tens. However, the teacher linked the steps to the order of the actions on the blocks, not to use of tens as benchmarks. At this point, the students seemed tired.

Next, the students set up and played a few rounds of the game. They were instructed to document how they solved the situations, following the teacher's example on the board. However, the students tended to only record what was thieved or lost, for example, writing  $29 - 3 = 26$ . Three pairs used sentences rather than symbolic notation, which was time-consuming. As on Lesson 1, the sparseness and randomness of tricky situations made it difficult to identify and overview students' methods. One instance of students avoiding a tricky situation by thieving less than the dice showed was observed. This led to problems in the next turn, as their documentation did not match the gameplay. These students got a hint to try to swap some blocks, which was enough for them to see that swapping a 10B for two 5B enabled them to thief 7 from 15 using Stepwise on tens. The observer and the teacher agreed that students needed to encounter more tricky situations and possibly discover methods to provide the teacher with a stronger basis for the subsequent discussion. Therefore, the plan for Lesson 3 was revised.

### ***Lesson 3***

First, students continued playing 2–6 rounds. None of the pairs documented how the thieving was conducted; most only recorded the results after each turn. Again, one pair used sentences rather than mathematical symbols. The observer identified one tricky situation (Figure 1C), where the students collaborated to use Compensate: Student 1 was to thief 12 from 15 (one 10B, one 5B). Student 1 took 10B and then 5B, paused to think, and then stated: "You get two", handing over two 1B to Student 2. Both students then paused to think, then Student 2 took an additional 1B from Student 1, after which both students appeared satisfied.

Since there was no student documentation from the gameplay, the teacher based the whole-class discussion on fictional examples, creating posters illustrating Stepwise on tens for the addition  $23 + 8$  and the subtraction  $15 - 8$ . The teacher asked for alternative methods, and one student demonstrated Compensate, taking one 10B and returning two 1B, but this was not acknowledged by the teacher, who remained focused on constructing the poster of Stepwise on tens. Attempts to connect the symbolic notation to actions on blocks were not entirely systematic, especially concerning swapping. Several ways of swapping were shown after each other, making it difficult to follow which swap was actually used in the method discussed. Subsequently, the teacher showed another example of addition and students suggested several other methods, but the teacher had some difficulty following their reasoning. For instance, the teacher initially interpreted a student's explanation of Stepwise on tens as a "jump of ten". Thus, most of the examples selected by the teacher focused addition rather than subtraction. As a result, the intended focus on methods for subtraction was partly lost.

The last activity was conducted at a later time and not observed, but worksheets were collected and analysed. In total, 62 solutions from 16 students were collected, of which 52 included at least an attempt to document a method, distributed over methods in the following way: One-by-one: 3, Stepwise on tens: 5, Stepwise other: 7, Take from ten: 23, Not determinable: 8. Thirteen students (81 %) documented at least one method, and 7 students documented two different methods (44 %).

### ***Summary Iteration 1***

The game allowed students to quickly get started. However, the sparseness and randomness of tricky situations made it difficult to know to what extent the students were engaged in methods for two-digit subtraction, and, in some cases, students came up with ways to avoid tricky situations. This made it difficult to base the discussions on students' own gameplay. Instead, the teacher had to pick up on students verbally expressed methods during discussions, which was challenging. In the discussions, the teacher started with addition which limited time and focus left for subtraction. Furthermore, the teacher's focus on the order of the steps in Stepwise might not have made the idea of pausing on tens clear for the students. This might explain why more students used Stepwise other than Stepwise on ten in the last activity.

### **Iteration 2**

Based on the outcomes of Iteration 1, two revisions were made. First, it was emphasised that the discussions in Lesson 2 and 3 were to start with subtraction. Second, the gameplay in Lesson 2 was replaced by worksheets illustrating tricky situations with Hare and Tortoise (Figure 2), in order to make sure that all students worked with such situations and to aid the teacher's selection examples for use in the Lesson 3 discussion.

#### ***Lesson 1***

Lesson 1 was conducted in half-classes but otherwise proceeded as in Iteration 1. Two instances of students using Compensate were noted, and two additional cases using swapping followed by Stepwise on tens. In one of these cases, the method closely resembled Compensate, as the swap and the thieving occurred simultaneously: thieving four by swapping one 5B for five 1B, immediately taking four 1B. Another student, having realized that swapping could be helpful, made a swap that did not aid the theft: To thief 9 from 28 (two 10B, eight 1B, Figure 2A), five 1B was swapped with one 5B. The teacher suggested an additional swap, which led them to swap one 10B for one 5B and five 1B, enabling them to proceed. Another student always began by thieving 5B. When set to thief 9 from 27 (two 10B, one 5B, two 1B) they realized they were stuck after taking 5B and came up with swapping one 10B for one 5B and five 1B. One pair asked the observer for support, who merely redirected the question back to the students: "How could you do it? You got to think.". This was enough for them to come up with using Compensate. As in Iteration 1, some students tried to avoid tricky situations which caused problems later. In one such case, the teacher described how a swap could be done, which allowed the students to continue without further issues.

#### ***Lesson 2***

The discussion began asking for situations where it is not easy to thief from a peer. One student explained that he was to thief 8 from his friend having 17 (one 10B, seven 1B). When asked how they handled it, the student explained that they swapped. This was followed by more examples of swaps suggested by students. The teacher then discussed a non-tricky situation, emphasizing the importance of knowing the value of each block, followed by discussing a tricky situation (Figure 1B), focusing on swaps that would enable Tortoise to thief 9 from Hare. Here, the teacher illustrated swaps by drawing arrows. Students explained Compensate and Stepwise on tens twice, and the teacher documented all contributions on the board using symbolic notation.

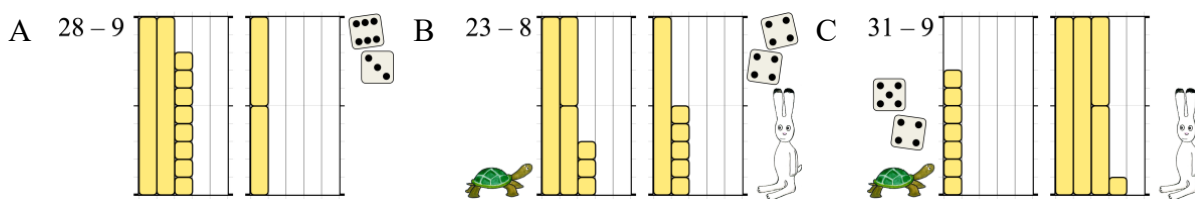


Figure 2. Two tricky situations (A and C) and one non-tricky situation (B)

Next, students worked in pairs with worksheets with tricky situations (such as Figure 2B–C) and the teacher extended the lesson 10 min. The teacher support was mainly in the form of encouragement and prompts to express thinking using “math language”. Two pairs were also guided how to document their solutions when applying the Compensate method. Six of eight pairs attempted to document at least one method, four pairs documenting Compensate on at least one task, and three pairs documenting Stepwise on tens.

### Lesson 3

The teacher demonstrated how to document a swap (one 10B for one 5B and five 1B), in symbolic notation, holding up blocks to illustrate the situation. Next, the teacher presented an example from the worksheet (Figure 2B) and showed how to document Stepwise on tens for subtraction, both on the board and on a poster. The teacher then created three posters, illustrating Compensate for addition and subtraction based on students’ suggestions, starting with taking 9 from 31 (Figure 2C). When discussing Compensate, the teacher showed that nine remains by placing a 1B in front of a 10B.

Next, students worked with subtraction tasks. The teacher gathered six hesitant students at the board and showed Stepwise on tens and Compensate using an example not included in the worksheets. Two of the students were shown an additional example, after which one student was given additional help including number bonds to ten. Most students appeared to write down their answer first, then adding descriptions of their method. The teacher encouraged some students to write down their thinking, stating that it was not enough to write an answer. In total, 80 solutions from 17 students were collected from this activity, of which 56 included at least one attempt to document a method, distributed as: Compensate: 3, Stepwise on tens: 23, Stepwise other: 9, Take from ten: 11, Not determinable: 7. Fifteen students (88 %) documented at least one method, and 7 students documented 2–4 different methods (41 %).

### Summary Iteration 2

As in Iteration 1, students were engaged and came up with methods for handling the tricky situations. The change from continued play to worksheets focusing tricky situations resulted in more focus on methods, and a better overview of what methods the students developed and used for the teacher. There was thus a better basis for the discussion, in which the teacher focused more on subtraction. These revisions may have contributed to students documenting more methods for more tasks, compared to Iteration 1.



## Discussion

In sum, the answer to our research question is that the thieving game can support Grade 1 students' development of the methods Stepwise, Take from ten and Compensation, by means of their own reasoning by engaging students in an activity that gives rise to situations that require methods based on actions on blocks. One may particularly note the method Take from ten, which to our knowledge, have not been reported in previous studies investigating children's subtraction methods (e.g., Heinze et al., 2018).

Previous studies have investigated long-term effects of different teaching approaches such as investigative, problem-solving, and routine approaches (Heinze et al., 2009), explicit and implicit teaching (Heinze et al., 2018) and structural and non-structural approaches (Kullberg et al., 2024b). The studies of Heinze et al. (2009; 2018) have highlighted the advantages of building on students' own ideas but also noted that students' own methods can be limited. Our study complements these studies by investigating particular activities that support students' development of own methods. In both classes, the game was enough for students to come up with ways of handling tricky situations by means of their own reasoning, and after follow-up discussions of how these ways could be used as methods for subtraction, 28 of 33 students (85 %) could document at least one method for two-digit subtraction and 14 (42 %) could document two or more. The developed methods are all based on a "take away" notion of subtraction, which is expected as the game entails thieving blocks. To support students' development of methods based on a "determining the difference" notion of subtraction (Selter et al., 2012), other activities would be needed.

The game was not in itself sufficient for students to extend their actions on the blocks to methods for symbolically represented subtractions of two-digit numbers. As the tricky situations that require development of methods are sparse in the game, letting students think about selected situations was needed for all students to encounter enough tricky situations. When this was included both in the discussions and as a worksheet (Iteration 2, Lesson 2), the teacher got more information about students' methods, and the class had common examples to discuss. It is however likely that students experience of playing the game was important for their understanding of the worksheet. The development from actions on blocks to symbolic methods may also depend on how well students' descriptions of methods are taken up during discussions, and how well they are connected to block actions and symbolic notation. In Iteration 2, where the teacher focused on using tens as benchmarks, more students used Stepwise on tens than Stepwise other, while the distribution was reversed in Iteration 1, where this idea was less clear. This result aligns with the findings of Heinze et al. (2009), that teacher led discussion and comparison of methods gives better opportunities for students to learn to use at least one method accurately, compared to pure problem solving.

While several students came up with their own ideas during playing, some students may have been introduced to methods by the teacher or students during play or discussions. However, our data show that students documented a variety of methods, including methods never described or discussed in class. This indicates that students' methods were not mere imitation, but that the teaching design supported students' own development and selection of methods. Further research is needed to determine if students can retain and extend the methods to three-digit numbers in later grades.

Besides being supporting the development of methods, an advantage of the game is that it entails translating from dice to block representation, which entail practicing number bonds, which is important for developing subtraction methods (Kullberg et al., 2024a; 2024b). A limitation is that it requires specific materials, which are not available for teachers. While it is possible to play the game with regular base-ten blocks, it reduces the frequency of tricky situations. Teachers can create blocks themselves, but that requires time and effort. Further research would be needed to determine if the function of the game extends to other contexts and outweigh its limitations.

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