Method use and success on the national test for grade 3 after being taught simplification methods

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This study explores which methods Swedish students use for multi-digit addition and subtraction after being taught the simplification methods regroup and move the difference. The data consist of 30 students' solutions to one subtest of the Grade 3 national mathematics test. Results show that students used a variety of methods, with splitting numbers by place value being the most common, despite having lower success rates. The simplification methods were used less frequently, but more successfully. The findings suggest that conceptually focused teaching of different methods can support students in selecting and using efficient methods, although additional support may be needed to increase actual use. The study also confirms earlier research showing that Stepwise methods outperform Split methods, especially in subtraction.

Methods for solving types of tasks is a key part of all mathematical topics at all levels. One of the first type of multistep methods that students encounter in school are methods for multi-digit addition and subtraction. While there are many methods for calculating sums and differences of multi-digit numbers, teaching in Sweden often focus mostly on the standard written algorithms (in Sweden referred to as *uppställning*). However, research has shown that the introduction of the written algorithm often results in students abandoning their own methods even when they would have been more efficient, and even when students use the written algorithm in erroneous ways (Hickendorff et al., 2019). In this study, we investigate to what extent a group of Grade 3 students, who were taught alternative methods for multi-digit addition and subtraction, used varied methods and to what extent these uses were successful on the Grade 3 national test.

Multi-digit addition and subtraction in Swedish schools

In Swedish Grades 1–3, the only written methods included in the curriculum are methods for multi-digit addition and subtraction. Based on the national tests for Grade 3 (Primgruppen, 2025), this should be interpreted to mean addition and subtraction of numbers with up to three digits. Since 2016, three-digit addition and subtraction have been the focus of one or two of the subtests of the national Grade 3 test, usually including six tasks of which 2–3 are addition tasks and 3–4 are subtraction tasks. For each task, students are given one point for documenting a correct method (*strategy correctness*) and one point for obtaining the right answer (*strategy accuracy*). This subtest has consistently been the subtest with the lowest achievement nationwide, with below 80 %

of students reaching the required level compared to 86–95 % on the other subtests (Skolverket, 2025).

Besides being able to use methods correctly and accurately, researchers and curricula also suggest *strategy flexibility* is an important goal for students (Verschaffel, 2024). Though strategy flexibility is defined in various ways, the core of most definitions is the ability to choose appropriate and efficient methods. This ability implies knowledge of and skill in multiple methods, as well as knowledge of when these methods are efficient. By the end of Grade 3, the Swedish curriculum states that students should be able to choose and use written calculation methods for addition and subtraction with satisfactory confidence (Skolverket, 2024). Though not specified—neither in the curriculum nor the commentary material (Skolverket, 2022; 2024)—being able to choose can be argued to imply that students are supposed to know several methods and have some knowledge of when they are appropriate and efficient, indicating that flexibility is indeed a state mandated goal for Grade 3 students. However, as described above, the scoring of the sub-test of the national Grade 3 test in mathematics that assesses written methods for addition and subtraction does not take flexibility into account. This may encourage both teachers and students to keep to a single method that works the same for every task, such as the standard written algorithm.

Methods for multi-digit addition and subtraction

Previous research has used different terms and classifications of methods for multi-digit addition and subtraction. At the most general level, Hickendorff et al. (2019) and Heinze et al. (2022) differentiate between number-based methods and the digit-based written algorithm, to emphasise that the written algorithm can be used without considering the place value of each digit, treating them as single-digit numbers. A summary of different number-based methods classified by previous research (Fuson et al., 1997; Heinze et al., 2009; 2018; Hickendorff et al., 2019; Kullberg et al., 2024) is given in Table 1. Furthermore, students may use combinations of these methods or record them in different ways than is shown in the examples in Table 1.

The *Split* (also called Decompose) method involves the same steps as the standard algorithm, but records the numbers the different place values represent, rather than just recording digits. This method is rather straightforward for addition bridging over ten but is more complicated for subtraction bridging over ten as it: 1) results in negative results of sub-subtractions, and 2) involves both subtraction (for each place value) and addition (of the results of all sub-subtractions). Several studies have also shown that students have rather low success rate for this method on subtraction tasks; Heinze et al. (2009) report a success rate of 39.0 % in a sample of German third graders and Kullberg et al. (2024) report a success rate of 17.6 % (or 13.3 % if cases where students decomposed into digits are included) in a sample of Swedish second graders. In both studies, students performed substantially better when using the *Stepwise* (also called Jump or Sequential) method on subtraction tasks: 69.6 % and 100 %, respectively.

Method	Description	Addition (45 + 18)	Subtraction (45 – 18)
Split	Decompose both terms in ones, tens etc, do sub-calculations for each place-value and add the results	40 + 10 = 50 5 + 8 = 13 50 + 13 = 63	40 – 10 = 30 5 – 8 = -3 30 + (-3) = 27
Stepwise	Decompose one term (the subtrahend) into numbers that are easily added/subtracted stepwise	45 + 5 = 50 50 + 10 = 60 60 + 3 = 63	45 – 5 = 40 40 – 10 = 30 30 – 3 = 27
Compensation	Add/subtract more and then subtract/add the difference to compensate	45 + 20 = 65 65 – 2 = 63	45 – 20 = 25 25 + 2 = 27
Simplification	Regroup an addition (move part of one term to the other) or move a subtraction (add or subtract equally to both terms) to reach an easier task with the same answer	45 + 18 = 43 + 20 = 63	45 – 18 = 47 – 20 = 27
Indirect addition	Add smaller numbers to the subtrahend until you reach the minuend and then add up all the smaller numbers	–	18 + 2 = 20 20 + 20 = 40 40 + 5 = 45 2 + 20 + 5 = 27
Indirect subtraction	Subtract smaller numbers from the minuend until you reach the subtrahend and then add up all the smaller numbers	–	45 – 5 = 40 40 – 20 = 20 20 – 2 = 18 5 + 20 + 2 = 27

Table 1. Number-based methods for addition and subtraction

Compared to Split and Stepwise, Heinze et al. (2018; 2022) describes *Simplification* (also called Change both numbers) as a more advanced method that students are unlikely to invent by themselves. It is also reported to be used rather seldom by students, especially for subtraction (Heinze et al., 2009). If seen as a method that acts on symbolic expressions—which the name *Simplification* implies—it involves the same steps for both operations: adding a number to one term while subtracting the same number to the other, which will keep the value of the expression the same. This general idea is a property of equalities that is important for students' later development of algebraic thinking (Wilkie & Hopkins, 2024). However, simplifying a symbolic expression is an abstract idea that might be hard to teach directly to Grade 1–3 students. Instead, teaching might be more successful if it is based in the conceptual roots of simplification, which are different for addition and subtraction.

For addition, Simplification can be seen as a regrouping of an additive partitioning by moving one part of one of the terms to the other (Figure 1A). Children are usually able to conceptualise addition as additive partitioning (part-part-whole relationships) even before starting school (Nunes & Bryant, 1996), and the idea of regrouping can be linked to seeing numbers as collections of objects, which is often the first representation of numbers children encounter. This could explain the high success rate (88.1 %) for Simplification for addition reported by Heinze et al. (2009).

For subtraction, on the other hand, Simplification is grounded in the idea of subtraction as difference, which is usually not children's first conceptualisation of subtraction (Nunes & Bryant, 1996). Children usually first encounter subtraction in "take away" situations with collection of objects, for example, having some cookies, eating some of them, and finding out how many that remain. In such situations, it is not obvious that having more cookies but also eating equally many more will result in equally many cookies left. If, however, one thinks about subtraction as the difference between subtrahend and minuend on the number line, Simplification can be conceptualised as moving the subtrahend and the minuend the same distance, which will keep their difference the same (Figure 1B). Considering these links to concrete models of numbers and operations, it is evident that Simplification might better be seen as two different methods: *Regroup* for addition and *Move the difference* for subtraction, where the second has more complex conceptual roots, explaining the comparatively lower success rate (41 %) for Simplification for subtraction reported by Heinze et al. (2009).

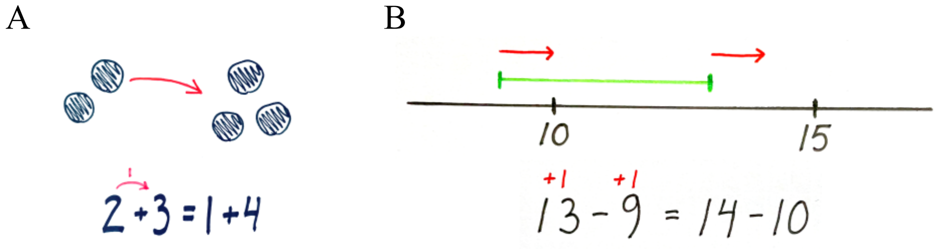

Figure 1. Conceptualisations of simplification for addition and subtraction

Aim and research question

Current teaching can make students overly reliant on digit-based algorithms for multi-digit addition and subtraction, resulting in low achievement. When students are allowed to invent their own methods, it can result in better accuracy, but they may not invent advanced methods that are important for continued learning, such as Simplification. In this study, our aim was to investigate whether teaching students Simplification methods with a focus on their conceptual basis, would result in better achievement for multi-digit addition and subtraction. Our research questions are:

1. What methods did students who were taught the simplification methods Regroup and Move the difference use on the Grade 3 national sub-test?
2. To what extent were students' methods successful, and what errors did students make?

Method

This study is part of a larger design research project that aims to better understand and develop the teaching of whole-number arithmetic in Grades 1–3. In the larger project, teachers and researchers collaborate to design, test and revise a teaching material for Grades 1–3 that aims to promote students' reasoning about number, and to study how teaching affects students' work with and knowledge of number and the four basic operations.

This study focuses on the solutions produced on one of the subtests of the national Grade 3 mathematics test in two classes whose teachers used a first version of the teaching material. The teachers worked at the same school, a school where the socioeconomic background of students is close to an average school in Sweden regarding foreign background and parent's educational level. They started using the material in Spring in Grade 1. Informed written consent for participation was collected from students' guardians, resulting in a total of 30 participating students.

Teaching design

The design of the teaching material emphasizes the development of mathematical understanding, reasoning, and problem-solving abilities. The lesson plans for each week follows a structure developed by Sterner (2015) including the following parts: introduction, pair work, whole class discussion, individual work, and follow-up discussion. In the material, teaching is viewed not merely as the transmission of procedures, but as a process of supporting students in actively constructing knowledge through exploration, discussion, and reflection. The underlying idea of the material is that learning occurs when students engage meaningfully with mathematical ideas, test their hypotheses, and learn from both success and failure. The aim of the material is therefore to support students in not only applying mathematical methods, but also in understanding the underlying principles and structures.

The teaching material used by the teachers in this study was divided into 153 "cycles" each consisting of three lessons, of which five focused on multi-digit addition or subtraction. The aim was not for the students to develop methods themselves. Two methods were introduced to the students: Regroup for addition and Move the difference for subtraction. Regrouping was introduced during Spring in Grade 1 as a way to generate equalities and was later used as a method for two-digit addition in the middle of Grade 2 and for three-digit addition during Spring in Grade 3. Move the difference was introduced for two-digit subtraction early in Grade 2 and extended to three-digit subtraction in the middle of Grade 3. Both methods were introduced conceptually by concrete use of blocks (Figure 2). These representations were then the basis for the documenting the methods with symbols. Below, we describe the teaching of each method in more detail.

Regroup

In Grade 1, the cycle introducing Regroup addressed the method by focusing regrouping of additive partitions of numbers 10–20. Students learnt ways to represent regrouping of terms in a sum and to use regrouping to explain similarities. In Grade 2, the cycle focused on regrouping to simplify additions. The students explored different ways to regroup additions and how regrouping can be used to calculate sums, including using the number bonds to 10 in regrouping. In Grade 3, the cycle focused on regrouping for addition with three-digit numbers. In this cycle students returned to the same method as in Grade 2, but now for additions when one number had three-digits and the other one-, two- or three-digits.

Move the difference

In Grade 2, the cycle introducing Move the difference addressed the method as a way to simplify subtraction. The students explored different ways to reformulate subtractions by adding or removing equal amounts from both the minuend and the subtrahend and got to use ways to document this. Students also discovered the use of number bonds when adding or removing equal numbers. In Grade 3, the cycle focused on Move the difference for subtraction with three-digit numbers. This cycle focused on Move the difference, but students could also use, explain and argue for other methods of their choice.

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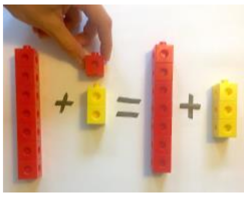

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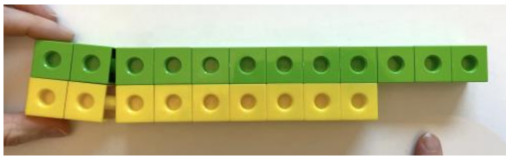

Figure 2. Illustrating Regroup and Move the difference using blocks

Data collection

Data consisted of students' solution to subtest F from the national test in Grade 3. This test focused written calculation methods for addition and subtraction. For the test, the written instruction to students was: *use a written calculation method and write the answer.* The year the data was collected, subtest F consisted of six tasks: two addition tasks and four subtraction tasks. One addition task and one subtraction task were word problems while the other four tasks were stated symbolically. Paper copies of the participating student solutions were collected resulting in 30 tests including solutions to a total of 180 tasks. As the national test is still classified and the particular tasks thus not allowed to be reproduced here, we use examples illustrating students' solutions and errors using other tasks with similar properties as the ones on the test, below.

Coding of students' methods

Each student solution to each task was coded for whether or not a correct answer was reached (simply yes/no), which method the student used, and if any, what type of errors were made. The analysis of methods was carried out using content analysis combining both deductive and inductive elements. We started with predetermined codes based on previous research on methods for addition and subtraction (Table 1), with the exception that we differentiated between the two simplify methods *Regroup* and *Move the difference.* These codes formed the basis for an initial coding of the students' solutions and answers. During the analysis, we identified student responses that could not be fully classified within the existing codes. To preserve the nuances of the data, new codes were created based on the students' actual solutions. Three new codes were introduced: *Take from next place value*, *Counting by ones*, and *No information*. In Take from next place value*,* students divide the minuend into 10 or 100 and the rest, and the subtrahend or part of the subtrahend is then subtracted from the 10 or 100 part. Writing out all numbers between one term and the result or making as many tick marks was coded as Counting by ones and only writing the expression and/or the answer was coded as No information.

The analysis also resulted in dividing the code *Split* into *Split by place value* and *Split other* and the code *Stepwise* into *Stepwise by place value* and *Stepwise other*. In *Split by place value* both terms are decomposed into ones, tens, and hundreds, while in *Split other* both terms are decomposed into parts that *not* exactly made up by place-values. In *Stepwise by place value* one term is decomposed into ones, tens and hundreds, while in *Stepwise other* one term is decomposed into parts that are not exactly by place-values. As we found no examples of the methods *Compensation*, *Indirect addition* and *Indirect subtraction*, these codes were excluded. Once both the deductively and inductively generated codes had been identified, the codes were reformulated, merged, or clarified to better reflect the variation in students' methods. The coding was then revised and difficult cases discussed between the authors to reach a final coding of each method used.

The analysis of errors was based on a first round of analysis of the solutions with a focus on finding and describing all the different error types that occurred (Table 2), conducted by one author. The types were then used to categorize all errors that occurred in each solution to each task, conducted by both authors. In this way, the errors in 57 % of the tasks were analysed independently by both authors. This resulted in no new error types, and thus no revision of the error types. The error types were grouped into four themes *Insufficient number-bond knowledge*, *Insecure method*, *Insufficient systematics* and *Wrong interpretation* (Table 2).

Theme	**Types**
Insufficient number-bond knowledge	The student gets the wrong answer in: • an addition where the sum is ≤18, not 10 • an addition or subtraction with the number bonds to 10 • an addition or subtraction with the number bonds to 100 • a subtraction where the minuend is ≤18, not 10 • a subtraction where the minuend is >18, not a whole ten
Insecure method	The student: • makes a mistake while counting up or down by ones • mixes up units and tens in one or more steps • switched places of the subtrahend and minuend when the difference is less than 0 • adds something that should be subtracted or vice versa. • makes a mistake when in bridging over ten in two sub-calculations • mixes up place-values
Insufficient systematics	The student: • misses some part • forgets that a ten has been bridged and counts it twice
Wrong interpretation	The student draws a wrong conclusion about the operation to use (primarily in a word problem)

Table 2: Error types in each theme

Results

In this section, we present the answer to each of our research questions.

Research question 1: Frequencies of methods used

The students used a total of six different methods on the addition tasks, and eight different methods on the subtraction tasks. The frequencies of solutions using each method is found in Table 3. One can note that the two simplification methods were rather seldomly used, and that some version of Split was still the most common method for both addition and subtraction. Some uses of Split were documented as the standard written algorithm, suggesting that students had somehow been taught this method, by the teacher or outside of school.

	Addition tasks		**Subtraction tasks**	
Regroup	9	15 %	–	–
Move the difference	–	–	6	3 %
Split by place value	18	30 %	41	40 %
Split other	25	42 %	21	17 %
Stepwise by place value	1	2 %	26	18 %
Stepwise other	6	10 %	8	7 %
Take from next place value	0	0 %	6	5 %
Counting by ones	1	2 %	2	2 %
No information	0	0 %	10	6 %

Note. There were twice as many subtraction tasks, and thus subtraction solutions, as addition tasks.

Table 3. Frequencies and percentages of use of different methods for addition and subtraction

Research question 2: success rates and errors

Students' success rates varied over methods, especially for subtractions tasks. The success rates for each of the methods used are found in Table 4. One can note that for addition, several methods were highly successful. The two most successful methods were Regroup and Stepwise other. Regroup was typically used on a task where one addend was close to a whole hundred, in the following way: 203 + 298 = 203(-2) + 298(+2) = 201 + 300 = 501. Stepwise other was typically used to get sub-results of even tens and hundreds, for example: 135 + 68 = (135 + 5) + 60 + 3 = (140 + 60) + 3 = 200 + 3 = 203. Both these methods can thus reduce bridging over ten to number bonds to ten, requiring fewer and easier known number facts. Only seven errors were found in students' solutions to addition tasks, of which four were insufficient number-bond knowledge.

For subtraction, variants of Split were the least successful methods, while Move the difference and Stepwise other were the most successful. Of the 57 errors identified in students' solutions to subtraction tasks, 36 were due to insecure methods, with the most common error type (17 cases) being subtracting the minuend from the subtrahend in a sub-subtraction where the minuend was smaller than the subtrahend when using a variant of Split. An additional nine errors were due to unsecure uses of Split variants. Ten errors were due to insufficient number-bond knowledge, seven due to insufficient systematics and four due to wrong interpretation.

	Addition tasks	**Subtraction tasks**
Regroup	100 %	–
Move the difference	–	100 %
Split by place value	94 %	58 %
Split other	84 %	48 %
Stepwise by place value	0 %	73 %
Stepwise other	100 %	75 %
Take from next place value	–	67 %
Counting by ones	0 %	0 %
No information	–	50 %

Table 4. Success rate of different methods for addition and subtraction

Discussion

In this study, we investigated what methods a group of Swedish Grade 3 students, who had been taught Regroup and Move the difference, used and how successful their methods were. Our results show that students used a variety of methods, and that the most common were variants of Split, while Simplification methods were rather uncommon. These findings are in line previous research in other contexts, such as Swedish Grade 2 (Kullberg et al., 2024) and German Grade 3 (Heinze et al., 2009). However, in our study, students were substantially more successful when using simplification methods than in previous research, especially for subtraction. The high success rate in our study might be a result of students learning these methods separately, based on their differing conceptual bases. Heinze et al. (2009) describe Simplification as one method but does not describe how it was taught to the students in their study.

Simplification of symbolic expressions is an important part of algebraic reasoning (Wilkie & Hopkins, 2024). While some students in our study successfully used of Regroup and Move the difference, there was no wide use of these methods. While teaching these methods conceptually and separately seems like a promising approach, other studies are needed to understand how teaching can support more students in using Simplification methods successfully. Another limitation is that the study design does not allow us to say whether method use affected success rate. One can suspect that use of Simplification methods and success might both be affected by general proficiency, and thus that the higher success rate of Regroup and Move the difference may not be wholly due to the methods in themselves. Further research should investigate the success rate of different methods when used by the same student, and the success rates of different methods when used by different groups of students.

Together with other studies (Heinze et al., 2009; Kullberg et al., 2024), our results support the claim that Split is a comparatively difficult method for subtraction tasks, and not the most successful method for addition. Still, it is the most common method used, and it is the basis for the standard written algorithm, which is commonly the focus of Grade 1–3 teaching. Research, including this study, has consistently shown that Stepwise is at least as successful for addition and substantially more successful for subtraction (Heinze et al., 2009; Kullberg et al., 2024). Based on this, one might suggest Stepwise to be a better choice of main focus in Grade 1–3 teaching than the algorithm.

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