# Collaboration between mathematicians and mathematics educators in secondary teacher education

Olov Viirman1, Yukiko Asami-Johansson2, Jonas Bergman Ärlebäck3, Mikael Cronhjort2, Magnus Jacobsson1

1Uppsala University, 2University of Gävle, 3Linköping University

In this symposium, three research and development projects from different Swedish universities will serve as a starting point for a discussion of different ways in which mathematicians and mathematics educators can work together to improve the preparation of prospective secondary mathematics teachers for teaching mathematics.

Comparatively little research has been focused on the education of prospective secondary, and in particular upper secondary, mathematics teachers (USMT’s). There is some consensus among mathematicians and teacher educators (e.g., Dreher et al., 2018; Leikin et al., 2018) that prospective USMTs need solid knowledge of central topics in secondary mathematics, such as calculus and algebra, including some more advanced topics. However, research on USMT education often takes the content and format of mathematics subject matter courses as given, and separates them from the mathematics education courses that are the main focus of the research.

However, many prospective and practicing mathematics teachers fail to see the relevance of their higher-level mathematics studies to their teaching practice (e.g., Wasserman et al, 2018; Zazkis & Leikin, 2010). It has been argued (e.g., Wasserman et al, 2018; Winsløw & Grønbæk, 2014) that one possible explanation is that the higher-level mathematics courses taken by prospective teachers rarely discuss the content from a didactical perspective. In the light of this, research has begun investigating the role that mathematics courses, and the mathematicians who teach them, play in USMT education. For instance, Yan et al. (2020) address the question of how to better connect the mathematics studies of prospective USMTs to their future teaching practice, and Alvarez et al. (2020) present principles for designing tasks that address applications in secondary mathematics education, for use in undergraduate mathematics courses. A recent special issue of ZDM (Buchbinder et al., 2023) is devoted to “exploring and strengthening university mathematics courses for secondary teacher preparation”, where many of the studies touch on the idea of collaboration between mathematicians and didacticians, either in practice or in research. In the Swedish context, however, there is little research on USMT education in general, and even less on what form such collaboration might take. In what follows, we will present three projects from different Swedish universities that address this question in different ways.

## Project 1 – An advanced course in mathematics and didactics for USMTs

One way to understand the demands of USMT education is through the so-called “double discontinuity”, first formulated by the mathematician Felix Klein (e.g., Winsløw & Grønbæk, 2014). The first discontinuity occurs when students enrol at university to become mathematics teachers. The mathematics they then encounter is very different from what they have experienced in school, the difference becoming more pronounced as their studies progress. When they return to schools as teachers, the second discontinuity occurs, as they are expected to teach the “school mathematics” they left behind when entering university. With this as a starting point, and drawing on related initiatives elsewhere (Wasserman et al., 2017; Winsløw & Grønbæk, 2014), at Uppsala University we have developed an interdisciplinary course in advanced mathematics and didactics (Viirman & Jacobsson, 2022), that covers parts of real analysis and abstract algebra relevant to upper secondary mathematics and treats mathematics and didactics in parallel and together. It is taught jointly by a mathematician and a didactician through so-called “team teaching”, with both teachers present in the classroom during all teaching sessions. The course has run twice so far, and in the symposium we will present examples of how the collaboration has created opportunities for learning. For instance, the interdisciplinarity enables us to address not only local relevance, where the same content is taught at university and in school, but also non-local relevance, where specific mathematics content can serve as examples of general didactic relevance, for example concerning the role of examples or classification. Co-teaching allows, among other things, for didactic reflection on the teaching as it happens.

## Project 2 – Mathematical concept formation for USMTs

At the University of Gävle, we have revised the teaching of the course “Mathematical concept formation for subject teachers”. The course has been given by three teachers in cooperation: a didactician, a mathematician, and a theoretical physicist who has focused on didactics during the last decades. Teachers change their institutional positions (regardless of their “actual” positions) in in relation to the emergence of knowledge (Chevallard, 2021). That is, a mathematician may change position to teacher educator, or a didactician to teacher educator, depending on what the didactic stakes of the lesson are. Such an awareness of our own institutional positions enabled fruitful discussions and new perspectives during our revision of the course. When transposing the knowledge for teaching mathematics to USMTs, there are two dimensions, namely, the learning of mathematics as a domain (didactic level), and learning the epistemology of didactics (paradidactic level) (Otaki & Asami-Johansson, 2022). On a didactic level, mathematicians perceive the position of prospective teachers as students and focus on their need to master, for example, mathematical concepts. On a paradidactic level, teacher educators (as an institutional position) perceive the position of prospective teachers as teachers, focusing on their ability to teach future students the mathematical concepts. With these two dimensions in mind, especially the paradidactic level, a guiding principle in our revision was to include mathematical concepts that are known from research and experience to be frequently misunderstood. These include the number system, logical reasoning, proofs, powers, roots, logarithms, and digital tools, including GeoGebra. We also included the concept of distance with some different metrics, as this would allow for examples that we expected would be unfamiliar to the prospective teachers. For example, we used the Manhattan metrics to demonstrate that a circle does not look like the Euclidian circles we are used to. This was based on materials developed at Kleindagarna, which is an arena for meetings between mathematicians and upper secondary mathematics teachers, inspired by Felix Klein (Cronhjort & Hagemann-Jensen, 2021). In the symposium, we will present examples from Kleindagarna of how higher mathematics can enrich conceptual understanding in school mathematics.

## Project 3 – Theoretical foundations of secondary school calculus

At Linköping University, the undergraduate courses in mathematics (large lecture settings) and mathematics education (more intimate class settings) are generally taught separately but in parallel. However, two courses integrate mathematics and didactics: a course on mathematical modelling and "Calculus: Theoretical Foundations for Secondary Mathematics Teaching, 4hp". The main aims of the latter, newly developed course are (i) to provide a theoretical foundation for parts of secondary school mathematics and (ii) to give students the opportunity to relate and practice the content in a way that is relevant to their own future teaching practice. The course actively seeks to engage students in understanding and exploring their dual roles as learners and teachers of mathematics (cf. Bowers & Doerr, 2001). The mathematical content focuses on fundamental theorems about sequences, supremum (infimum), limits, functions, (uniform) continuity, derivatives and integrals. The course examination is closely linked to ways of working as a practising teacher: (i) presenting solutions orally and in writing; (ii) providing comments and feedback on (fellow students') solutions; and (iii) designing a GeoGebra activity intended for the upper secondary level that illustrates a central idea/notion/concept/technique of the course. In the case of this course, the developer and the teacher of the course are one and the same person, and his dual role as a mathematician and a didactician will be discussed and problematised in this symposium.

## Concluding remarks

The projects presented above have obvious similarities but also display clear differences. They all seek to engage prospective teachers in reflection on their dual roles, both as students of mathematics and as future teachers. However, the institutional roles of the academics involved in the projects are different, which has implications for how the collaboration takes shape in the different settings. In the symposium, we aim to use these projects as starting points for a deeper discussion about how to improve USMT education in Sweden, and about the role that mathematicians and didacticians can play in this work. We believe that such a discussion is long overdue within the Swedish mathematics education community.

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