## Conceptual knowledge of *addition* and *subtraction* when using length models inspired by the El’konin Davydov curriculum

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It can be hard to teach mathematics in a way that facilitates a development of conceptual knowledge.In this presentation we focus on a pilot study which aimedto explore what conceptual knowledge of addition and subtraction, specifically involving the inverse relationship between them, that can be identified when using length models, inspired by the El’konin Davydov curriculum, in teaching students at grade 4 and 5. Three examples of a length model were given to two teachers, who planned a lesson that was observed and video recorded. The analysis used two dimensions: different qualitative levels of how conceptual knowledge of addition and subtraction is communicated, and implicit vs explicit knowledge. Implicit knowledge of all levels could be found in the communication, but explicit knowledge was not as common.

The pilot study, presented here, was planned as an extension of a development project, where we together with teachers had a common problem: that it was not easy to teach students at grade 4 and 5, in a way that facilitated a development of conceptual knowledge. We delimited the content to addition, subtraction and the inverse relationship between them. Moreover, we used a task presented by Skolverket (2023) (Figure 1), which is inspired by Eriksson and Tabachnikova (2022), and the El’konin Davydov (ED) curriculum, aiming at enhancing students’ understanding of mathematical concepts (cf. Davydov, 1982). This curriculum employs learning models, such as for example length models presented in Figure 1, to facilitate reflections on concepts like addition and subtraction.



Figure 1: Three examples of a length model

The aim of the study is to explore what conceptual knowledge of addition and subtraction*,* specifically involving the inverse relationship between them, that can be identified when using length models, inspired by the ED curriculum, in teaching students at grade 4 and 5 mathematics. Among other things, the study seeks to identify whether students can recognize problem situations suitable for addition and subtraction, and explain why these operations are used, and also demonstrate knowledge of the inverse relationship between addition and subtraction (cf. Polotskaia & Savard, 2018).

The task in Figure 1 was given to two teachers at an elementary school. They planned and carried out lessons, which were observed, and video recorded. The teachers first asked the students to compare the models: What is similar and not between the models? After that, the students reflected on which operations could be represented by the models. The data was taken from two of five lessons, and students’ actions when they presented their solutions were used as the unit of analysis. The analyses were conducted in three steps. First the operations used by the students were categorised according to qualitative levels of using addition and subtraction (Polotskaia & Savard, 2018). In a second step, oral and written communication together with gestures, e.g. pointing in the models and written expressions, were added to the analyses to make it clear where and why the students explained their counting. Finally, an interpretation was done regarding if the conceptual knowledge was implicitly or explicitly communicated in the students’ presentations (Crooks & Alibali, 2014). The tentative results reveal a range of conceptual knowledge. While some students used addition in non-suitable ways, other students suitably used both addition and subtraction for the same model, revealing knowledge of the inverse relationship between the operations. Additionally, the results show implicit knowledge of all levels in the communication, but explicit knowledge was not as common.

## References

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