# Cognitive processing of mathematical symbols

Ewa Bergqvist, Bert Jonsson, and Magnus Österholm

Umeå University

This presentation focuses on the use of symbols, which is crucial in the teaching and learning of mathematics, where it is essential to use natural language to describe their meaning. Language is sound based while symbols are primarily not but have also a visual dimension. Cognitive encoding of sound and visual input occurs in different components of working memory. This study shows that mathematical symbols are processed both through sound and visually, but that the sound-based processing plays a more significant role. This is also the case for unknown symbols, which is not in line with some previous research that suggests that new symbols should be processed more visually.

## Introduction

The use of symbols is crucial in mathematics and in the teaching and learning of mathematics. When teaching about symbols, it is essential to use natural language (such as Swedish) to introduce symbols in a meaningful way, e.g., by describing their meaning and use (O’Halloran, 2008). However, symbols are primarily not coding of sounds, like natural languages are. Mathematical symbols, such as “+”, do not have a direct phonological identity. Still, with time and experience, individuals can ascribe phonological traits to such symbols, e.g., “+” might be associated with words like “plus” or “added”.

Therefore, there may be a clearer visual dimension in dealing with symbols. Both phonological and visual aspects of symbols can be actualized through how encoding and storing occurs in long-term memory. This encoding is orchestrated within the working memory, which comprises three components: the central executive, a phonological loop, and a visuo-spatial sketchpad (Baddeley & Hitch, 1974).

From a general developmental angle, processing undergo transformation with age. Younger children lean on visual-spatial encoding but with maturation and increased engagement in specific visuo-spatial content, there is a shift towards phonological encoding (Alloway et al., 2006). However, we do not know much about how symbols are processed cognitively. The purpose of this study is to add empirical evidence about this.

## Method

To examine how mathematical symbols are processed cognitively, we compare the processing of familiar mathematical symbols with unfamiliar (made-up) symbols. To investigate if symbols are processed phonologically or visuo-spatially, we use the dual-task paradigm (Guttentag, 1989), where participants perform two tasks simul­taneously, a primary and a competitive (secondary) task. The goal is to find out what type of secondary task, phonological or visuo-spatial, that disturb the participants’ performance on the primary task the most (i.e., causing more errors). Here, the primary task consists of a visual presentation of a set of 12 symbols, each shown for 4 seconds. In each set, half of the symbols are familiar (mathematical), and half are unfamiliar (made-up). The task is to remember and reproduce as many symbols as possible. The secondary phonological task is to repeat a specific word (“konferensrum”) at constant pace. The secondary visuo-spatial task is to tap a specific sequence, QWOP, on a keyboard at a constant pace. We also use a baseline setting, where the participants first try to remember symbols without any secondary task.

There were 29 participants in the experiment, 21-28 years old. All had completed at least basic upper secondary school mathematics. They were from a range of academic programs. Our analysis used a 3×2 repeated measures ANOVA, with the number of correct responses as the dependent variable. The encoding condition factor was categorized into: Baseline, Phonological, and Visuo-spatial Encoding. The symbol condition factor was categorized into: Math symbols and Fictional symbols.

## Results and conclusions

The ANOVA demonstrated a significant main effect for the encoding condition (F(2,52) = 24.57, p < .001, η2 = 0.16) and the symbol condition (F(1,26) = 21.45, p < .001, η2 = 0.20). No interaction effect was found (F(2,52) = 0.08, p = .92, η2 = 0.0). A subsequent Helmert contrast analysis unveiled significant differences between the baseline and both phonological and visuo-spatial conditions, as well as between the phonological and visuo-spatial conditions, (t(52) = 6.66, p < .001, and t(52) = 2.18, p = .03, respectively).

In relation to previous research (see the introduction) one could have anticipated that the visuo-spatial task would impede the encoding of fictional symbols more than the more familiar mathematical symbols, but there was no such interaction. Instead, both concurrent tasks impede the encoding of symbols in similar ways for both types of symbols. Notably, the phonological loop plays a significant role in the encoding of symbols, with the task impacting the encoding more than the visuo-spatial task.

## References

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