# Task Frameworks and Teacher Practice:

# A Comparative Analysis

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We compare three different mathematical task frameworks by applying them to a teacher-described task implementation. By means of a case study method, we juxtapose the frameworks against the teacher's own account. We find that certain crucial facets of the teacher's task implementation remain unaddressed by the frameworks. We posit that this discrepancy arises from the inherently specialized nature of researcher-designed frameworks, which stem from the researchers' theoretical orientations, in contrast to teachers' complex experiences of task implementation. Consequently, we advocate for the development of more comprehensive frameworks to facilitate genuine teacher-researcher collaborations in task design.

### Introduction

Tasks play a central role in mathematics instruction. It is often argued that tasks with a certain richness are vital for fulfilling some general goals and needs in mathematics education (e.g. Krainer, 1993). For this reason, and because teachers often orchestrate student learning using tasks, education designers seeking to improve mathematics teaching and learning often adopt a “task-centric” design (Boston & Smith, 2011).

Such task-centric design research efforts require analysis of the practicality and effectiveness of rich tasks in regular classrooms, which in turn entails analysis of the opportunities for cognitive challenge present in the task as it is a) formulated by task designers, b) set-up by the teacher and c) implemented by the teacher and students (Tekkumru-Kisa et al., 2020). Likewise, in a reflection on the articles in an edition of ZDM Mathematics Education focusing on task design, Thanheiser (2017) states that “tasks cannot be considered independently from their enactment”.

The focus on concepts such as intention, alignment, implementation, and enactment, when used in conjunction with the idea of design, draws something of a line between the designer, who is often also a researcher, and the teacher thought of as an implementer of the designer’s ideas. The researcher-teacher divide becomes particularly salient when formulated in terms of fidelity (Century & Cassata, 2016). A fidelity focus may indeed promote something of a defect-oriented perspective on the teacher, as either capable or not capable of carrying out the intentions of the designer. It can be argued that such a divide goes quite deep. The work for the researcher can be summarized as focusing on certain theoretically supported and explicitly stated perspectives and providing specific descriptions of what is focused on. The teacher, on the other hand, must attend to many issues at once in the classroom and “develop implicit theories of action in order to make professional life tolerable [as there are] too many variables to take into account at once” (Eraut, 1994, p. 34).

This brings us to the issue of frameworks developed to analyze tasks. Typically, being developed by researchers, such frameworks should reasonably come with some explicit or implicit foci reflecting the researcher’s interest. It is, therefore, interesting to ask what happens when such frameworks are used to analyze the task choice and implementation of an experienced teacher. In this study we consider the case of a task chosen, modified, and implemented by a teacher, and apply three different researcher developed frameworks to it. Using case studies for “theory testing” (Thomas, 2011, p 516) has been done previously. For example, Phipps ~~a~~nd Merisotis, (2000) used the case study method to investigate the alignment between a framework that describes quality in distance education with the actual practices as perceived by university leaders, teachers, and students. We would like to follow Phipps and Merisotis’ approach, albeit in a much smaller scale, to use a case of a teacher implementing a task in order to test common frameworks of task implementation.

The three frameworks chosen for this investigation were selected based on being a) similar in concerning themselves with cognitively challenging tasks, b) different in how fine-grained analysis they offer, and c) stemming from three different academic contexts. Among the three frameworks, two have a quite analytical focus (COACTIV and Stein’s) and one has an explicit design perspective (CMR). This means that the combined selection of frameworks has the potential to capture different aspects of task characteristics and implementation when applied to a specific case of task implementation.

Could it be that an experienced teacher's use and perspective on rich tasks may contain a selection of issues that established task analysis and task design frameworks will not capture? This is what we will examine in this paper by applying the three task frameworks to a teacher's task formulation and described implementation. The research question we ask is: *What aspects of a teacher’s task implementation can a selection of these three task frameworks capture?*

### Description of the frameworks

Among multiple frameworks available in the research literature, three commonly used frameworks will be considered: Stein’s framework for analysis of mathematical tasks (Stein et al., 1996), Cognitive Activation in the Mathematics Classroom (COACTIV) (Neubrand et al., 2013), and Lithner’s “Creative Mathematically Founded Reasoning framework for task design” (CMR) (Lithner, 2017). In this section, we summarize the main ideas and applications of the frameworks and leave the details of these frameworks for the Results section where they are presented in relation to the teacher’s task formulation and description of implementation.

In Stein’s (1996) framework, tasks can be considered in terms of their inherent features (qualities of the task formulation itself) and cognitive demands (capacities students must call on to successfully solve the task). The task features and cognitive demands are brought to the students through the teacher’s framing of the implementation of the task, which influences but does not determine student enactment of the task. Successful implementation of rich tasks depends here on several teacher-controlled variables of which building on prior learning, scaffolding, appropriate timing, modelling of high-level performance, and sustained pressure for explanations or justifications, are seen as especially important (Henningsen & Stein, 1997). Since its publication in 1996, Stein’s framework has been extensively cited in research on teachers and students as agents in the use of tasks.

Within the COACTIV framework, the task is analyzed according to ten different categories thought to elicit higher levels of cognitive activation in learners that engage with the tasks, with each category evaluated on a numerical scale (Neubrand, 2013). The COACTIV classification framework is focused on the task as intended or formulated (rather than the framing for its implementation) and has primarily been used to analyze written formulations of tasks (most notably in Neubrand, 2013). The classification framework does not explicitly address classroom implementation. Instead, the related COACTIV Quality Instruction model (Kunter & Voss, 2013), developed for and validated by analysis of teaching in lessons, may help with the analysis of implementation of tasks. This model conceptualizes quality of instruction as being based on cognitive activation, classroom management, and individual learning support quite generally, each consisting of multiple smaller variables.

Lithner’s Creative Mathematical Reasoning (CMR) framework (Lithner, 2008)defines CMR as reasoning that is novel, plausible to the reasoner and underpinned by mathematically sound arguments. The CMR framework has been widely applied in analyzing tasks in textbooks and assessments. Lithner’s subsequent work in 2017 further developed the CMR framework for task design, advocating for tasks that challenge students' conceptual, creative, and justificatory skills. Here, effective implementation hinges on teachers resisting the urge to excessively aid students, which would otherwise diminish the challenges inherent in the task.

Collectively, Stein’s framework, COACTIV, and CMR offer a multidimensional perspective on task analysis and implementation.

### Method

Our method can be characterized according to the case study typology provided by Thomas (2011). The *object* of study is the task analysis frameworks’ ability to explicate aspects of task characteristics and task implementation. For this, we chose the three frameworks presented above which we see as useful to study as they are relatively well used by both their developers and other researchers.

The *subject* of study is a teacher’s implementation of a rich task. The teacher was selected as a “local knowledge case” (Thomas, 2011), being a colleague of the first author, who is also a teacher at the same school. The proximity and professional connection enable the first author to ascertain that the teacher has a very well-thought-out view of tasks as well as their implementation in her teaching context. This means that the chosen subject presents a meaningful challenge for the frameworks and could be used to evaluate them in a theory testing approach.

We asked our case teacher to provide both a task and an explanation of why she selected it. The teacher emailed us a description of her task formulation and her framing of the implementation of it with her 6th- and 7th-grade students. We conducted our analysis in two distinct stages. Initially, we applied each of our selected frameworks to the task formulation. Following this, we employed inductive thematic analysis at the semantic level (Braun & Clarke, 2006) to identify recurring themes within the teacher's account of the task's implementation. The first author reviewed the text multiple times, identifying raw data themes such as words and phrases pertaining to implementation. These were then organized into overarching themes. We then shared the analysis with the teacher for a member-check to ensure its credibility. Finally, we compared the themes with the criteria from the three frameworks.

### Results

In this section, we first analyze the task as it was formulated by the teacher. Then we present and analyze the teacher’s description of the implementation of the task.

#### Application of frameworks to the task formulation

The task, presented in Figure 1, is an example of a “puzzle task” (called so by the teacher with her students) used by the teacher.

A white board with writing on it

Description automatically generated

Figure 1: Photo of the problem presented to students in a lesson.

The teacher reported that she found the original task online, and then rephrased and expanded the task significantly. When asked, she described her goals:

It depends on the topic, but since they have to look really hard at something I often try to make it something many kids miss or something usually taught by just telling it to kids, which they don’t digest or retain. [....] Other than that my general goal is helping them learn to think and speak precisely, and to start to use deductive reasoning to show a conclusion, a baby step to proof.

The teacher’s goals for using such tasks thus seem both content-specific and intended to further the gradual development of complex reasoning and communication skills.

Using Stein’s framework, the task features include the requirement to produce multiple solutions and justifications. The use of a calculator could also be interpreted as a form of representation of the fraction in the problem. In terms of cognitive demands, a successful solution requires a choice of procedures which in turn requires conceptual understanding of the grouping of numbers by brackets and divisions.

The scores for COACTIV criteria can be found in Table 2. The scale ranges from 0 to 3 for each criterion. An asterisk (\*) or two (\*\*) indicates that it is difficult or very difficult, respectively, to determine the score for that criterion, according to the authors' opinion. The task formulation is thus likely to receive high scores on some COACTIV categories and low scores on others.

Under the CMR framework, the given task lacks a clear solution process and therefore requires some degree of creativity to be solved. It requires conceptual understanding of the relationship between fraction bars and brackets used on a calculator, and explicitly asks for an explanation based on mathematical reasoning, thus satisfying the CMR criteria.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **COACTIV** | | **Stein** | | **CMR** | |
| *Category* | *Score* | *Task features* | | *Characteristics* | |
| Topic area (1-4) | 1 | Multiple solutions | ✓ | Creativity | ✓ |
| Curricular knowledge level  (1-3) | 3\* | Multiple representations | ✓ | Conceptual challenge | ✓ |
| Type of mathematical activity (1-3) | 2\* | Demand for justification | ✓ | Demand for justification | ✓ |
| Extra-mathematical modelling (0-3) | 0 | *Cognitive demands* | | Reasoning | ✓ |
| Inner-mathematical modelling (0-3) | 3\* | Procedures with conceptual  connections | ✓ |  | |
| Basic concepts  (0-3) | 3 |  | |
| Processing of mathematical texts (0-3) | 0 |  |  |
| Argumentation (0-3) | 2 |  |  |
| Direction of task solution (1-2) | 1\*\* |  |  |
| Number of solution paths required (0-2) | 2 |  |  |

Table 2: Framework elements found in the teacher’s task description.

From the summary in Table 2, it is evident that the task formulation meets the different frameworks’ characterisation of rich tasks that elicit higher-level mathematical thinking.

#### The task implementation by the teacher

The teacher’s description of her implementation of the task is presented below. The text is colour-coded to show the emergent overarching themes:

* Yellow – **timing** considerations
* Blue – **feedback** **or scaffolding**
* Red – **push for justification** or explanation
* Green – **modelling** appropriate engagement with the task
* Brown – **curriculum context, prior learning**
* Purple – attention to **assessment demands**
* Orange – **Sequencing** of multiple tasks over time

Many words and phrases in the teacher’s account encompassed multiple themes present in this analysis. Notably, data coded as Sequencing also connects with Curriculum as well as Scaffolding. The colour code thus reflects only the main focus of each segment.

I tell the students that once per term they should choose two of the B&C puzzles from the curriculum we have completed so far (which they have worked on in class for roughly 30 – 45 minutes per task) and develop them to turn in as part of their summative portfolio.

In the lesson, I put the puzzle on the board and read it. I ask questions to ensure the students understand what is being asked in the puzzle. I may give simplified examples to ensure students understand what is being asked, but I do not tell the students how to start. Then I alternate between a 6 minute timer for working alone, and a 4 minute timer for discussion, and allow the students to attempt to find the pattern/solve the puzzle in roughly 15-20 minutes. I check students’ answers individually, and try to ensure everyone has an answer. If students have the wrong answer, I tell them to keep working on the answer, maybe with a small hint, and if they have the right answer I ask them to write their explanation/justification.

I model an explanation/justification at the start of the year so that students know what this means. Usually, I solve a simple pictorial algebra problem, I solve it, and I write the explanation/justification of my solution, explaining why certain parts (rules I used to solve the problem, why those rules work, relationships between operations or steps, definitions of mathematical terms, etc) are necessary to justify my answer [....] I highlight key words in the explanation/justification which are common in good mathematical justifications, and list good features of them, for example, accounting for all types of numbers (would a rule work with negatives? 0? 1?).

Back to the lesson itself, when students have an answer I ask them to start the explanation. During this process, I may point out good features of explanations when we are working on formative versions of the task. I usually specifically invent an explanation that reads like a story and read it in a silly way to show that a justification is not a story of your journey through a problem. I give regular feedback during the explanation writing process to let students know where they are missing things. For example, you say you did x, but you give no reason for doing x. Why did you have to do that?

In the first few formative versions of the assignment, I write some key words or prompts on the board to help students decide which of their thoughts would be a good thing to include in their explanation. See the example attached. [Figure 1]

[....] I refer to the rubric for more detail, which we have discussed in class. I include the rubric also.

Usually before the due date of the summative we have one or two explanation workshop days where students work on these and I give regular feedback to let them know to what extent they are on track, and if not, why not.

During the member check, the teacher agreed with the results and reported that, in her opinion, modelling and feedback, together with “Socratic questioning” (that could be conceived as a type of feedback, authors’ note), are the most important elements.

#### Application of frameworks to the analysis of task implementation

A summary of the extent to which the frameworks capture the themes in the teacher’s description is presented in Table 3 and explained below.

|  |  |  |  |
| --- | --- | --- | --- |
| **Themes in the teacher’s description** | **Stein** | **COACTIV** | **CMR** |
| Timing | ✓ |  |  |
| Feedback or scaffolding | ✓ | ✓ | ✓ |
| Push for justification | ✓ | ✓ | ✓ |
| Modelling engagement | ✓ |  |  |
| Curriculum context, prior learning | ✓ | ✓ |  |
| Assessment demands |  |  |  |
| Sequencing |  |  |  |

Table 3: Themes from the teacher’s description captured by the frameworks.

Students’ work on solving the task is organised in clearly defined time intervals. Of the three frameworks considered here, only Stein explicitly considers appropriate timing as crucial to successful task implementation.

Scaffolding or supporting student problem-solving is mentioned in all three frameworks, although COACTIV is less explicit than Stein and CMR about what such support looks like. Likewise, all three frameworks agree with the teacher that the demand for justification is an important element of the task.

Modelling features prominently in the teacher’s description of her teaching, as well as in her statement (during member check) that this is among the more important elements in her implementation of the task. Stein’s is the only framework that considers the modelling as an important factor affecting implementation.

Curricular context and building on prior learning are mentioned explicitly in both COACTIV and Stein, although COACTIV does not consider this a factor in implementation but rather a fact of the task formulation itself.

Some important aspects of the implementation of this task were not captured by any of the frameworks: the prominent role of formative and summative assessment and the positioning of the task among other tasks of a similar kind.

### Discussion

We examined the application of three frameworks to a quite advanced and well-thought-out task, chosen, modified, and implemented by an experienced teacher. While the task formulation fit well with the frameworks’ characterization of task design, only Stein’s framework was effective in elucidating critical aspects of task implementation, aligning closely with the teacher’s description. While COACTIV offered a more granular understanding of potential opportunities embedded in the task formulation, some categories proved challenging to assess, and the relevance to the teacher’s specific implementation was limited. The task formulation also aligns with CMR requisites, yet the CMR framework appears comparatively constrained when juxtaposed with the rich, multifaceted perspective provided by the teacher.

The question arises: why does CMR, which is the sole design framework in our selection, capture so few of the crucial facets of implementation identified by our teacher? In light of research indicating the pivotal role of teachers in facilitating students’ enactment of tasks (Henningsen, 1997), it is reasonable to anticipate that research on CMR task design will develop toward formulating more complex design principles to support teachers in CMR task implementation. Indeed, such research efforts appear to be in progress, as evidenced in Sidenvall et al. (2022).

The teacher’s account of her implementation of the task highlighted several considerations that were not explicitly addressed in either framework. These included a focus on assessment requirements, the task's relationship to preceding tasks that differed in content but shared similar cognitive demands, timing that extended over multiple lessons, and a deliberate shift from formative assessment, involving extensive modelling and feedback, to more independently conducted summative assessment at the term's end. Overall, the teacher’s implementation considers more facets of teaching and learning, than did either of the frameworks individually or collectively. This aligns with Eraut's (1994) observation that teachers often navigate numerous complexities in their lessons. Generally, it may be the case that certain behavioural routines that teachers find useful in light of their many priorities may not fit well with the intended task implementation. In the model of implementation of cognitively demanding tasks presented by Tekkumru-Kisa et al. (2020), both the teacher’s set-up of the task as well as the actual classroom work with the task may inflict decline in cognitive demand. Therefore, frameworks that encompass a broader range of factors, particularly those aligned with teachers' priorities such as assessment demands and the relation to long-term skill development, are more likely to support task implementation where cognitive demand is maintained.

This view fits well with Thanheiser’s (2017) emphasis on the importance of evaluating tasks in relation to domain-specific versus domain-transcendent objectives. In this terminology, the task depicted in Figure 1 addresses both domain-specific goals, such as facilitating a more profound grasp of concepts that might otherwise easily be forgotten, and domain-transcendent goals, such as the cultivation of precise communication and reasoning skills. The frameworks discussed in this article appear not to account for these extended objectives, nor do they address how a task can be used to serve both purposes.

Ultimately, the frameworks were applied to the teacher's account of the task's implementation rather than the implementation itself, live in the classroom. It is conceivable that using filmed observations during implementation could have unveiled additional pertinent factors that the frameworks may have addressed to varying degrees. Subsequent research should incorporate direct observations of both the teacher's implementation and the students' execution of the task.

### Conclusion

In this case study, we are grateful to a teacher who generously allowed us insight into her use of a challenging task, the implementation of which turned out to be more complex than three common frameworks could fully capture. The teacher employs the task for multiple purposes: to improve understanding of the curriculum, help students gradually develop complex skills, and to meet assessment requirements. She strategically places the task within a series of similar activities spanning many weeks, choosing the right moment based on student needs and preferences. This study underscores the need for research frameworks to evolve and encompass a broader range of aspects that are pertinent to teachers. Developing such comprehensive frameworks not only benefits our understanding of teaching practices, but also has the potential to enhance our ability to assess the practicality and effectiveness of tasks within design research and implementation studies.

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