# When implementing the method Thinking classroom, the didactical contract is hard to break

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Efforts have been made to change mathematics teaching from direct instruction towards methods where the students are enticed to think mathematically. This has proven difficult to achieve, partly because of classroom norms and conflicting didactical contracts. Thinking classroom is a teaching method that has been suggested to tackle these challenges. This study investigates the relationship between didactical contract and Thinking classroom. For this purpose, one upper secondary school teacher was invited to implement the method in their teaching. The teacher’s introductions of tasks to the students were analysed in relation to Thinking classroom’s recommendations and the pre-existing didactical contract. Our study shows that although Thinking classroom is designed to bypass norms the didactical contract is hard to break.

## Introduction

For a long time and in many countries, efforts have been made to change mathematics teaching from direct instruction towards, what we call ‘students’ active learning’ (Schoenfeld & Kilpatrick, 2013). These ideas have for instance been labelled as problem-based learning, inquiry-based learning, and reform teaching. Even though these efforts have been underpinned by slightly diverse theoretical ideas (Merritt et al., 2017), there are also many similarities. Problem-solving and mathematical reasoning have been central and one of the purposes has been to foster students to think and reason by themselves actively. However, in changing one’s teaching, teachers may be hindered by different classroom norms (Hofmann & Ruthven, 2018).

In addition to classroom norms, a mathematical classroom also includes a didactical contract (DC) which constitutes what is expected of both the teacher and the students (Brousseau, 1997). In direct instruction, the students expect the teacher to show how the tasks are solved and the teacher expects the students to listen and later use the solving method. A DC is implicit and becomes visible only when the teacher or the students contradict the expected. The DC in direct instruction can also be deliberately broken, of which Thinking classroom is an example (Liljedahl, 2022).

The main reason for using Thinking classroom (TC) is to compel the students to think mathematically by themselves (Liljedahl, 2022). This teaching method consists of several practices with the salient features that the students work in groups that are formed randomly and standing by vertical non-permanent surfaces (for example a whiteboard). The teacher avoids direct instruction by, for example, not answering students’ questions on how to solve the task. Therefore, TC aligns with efforts to change mathematics teaching toward students’ active learning. Thus, we suggest that this prescriptive method, described in Liljedahl (2022), might be considered as a tool for teachers who wish to develop their teaching in this direction.

## Background

First, we describe elements of Thinking classroom relevant to this study as a theoretical background for this paper. Thereafter, we briefly present the theoretical concept of didactical contract.

Thinking classroom (TC) is a method for teaching mathematics that has been studied and developed through a series of smaller studies (Liljedahl, 2018, 2019; Liljedahl & Allan, 2013). In short, TC consists of 14 practices that together can be described as a way to compel students to think mathematically by themselves rather than to mimic the teacher or the teaching material when learning mathematics (Liljedahl & Allan, 2013). The first three practices concern (1) how to choose tasks, (2) use randomised grouping, and (3) that the students are standing and working on erasable surfaces.

The first practice deals with choosing tasks that should compel the students to think mathematically. In the first lessons following TC, the tasks should be non-curriculum[[1]](#footnote-1) (Liljedahl, 2022). This means they are supposed to be a new type of task that is unfamiliar to the students. Furthermore, the tasks should be engaging and easy enough so that every student can begin solving them, while also allowing more complex reasoning. Moreover, to keep the students focused the task should not take more than five minutes for the teacher to introduce. An example of such a non-curricular task is to write expressions using four fours and arithmetical operations that equal for example all numbers 1–20 (Boaler, 2011). This task is easy to explain to the students and for them to begin with but requires creative thinking so that all the expressions can be found. Later, when the teacher and the students have gotten used to TC, the tasks are changed to curricular tasks that need to be covered in the course (Liljedahl, 2022).

The second practice, randomised grouping, implies dividing the students into groups who work together during the lesson (Liljedahl, 2022). The randomisation is to be done openly, that is, the students can see that they really are assigned to a group randomly and not by the teacher. The randomised grouping is done separately for each lesson to avoid students taking specific roles in a fixed group, which might prevent them from thinking freely. The recommended size is three students in each group and if not possible then two. More than three students might lead to smaller groups within the group. The risk of having only two students is that the knowledge variation is not wide enough, or that it is too wide, to engage in fruitful discussions.

The third practice is to have the students work on vertical non-permanent surfaces for example, a whiteboard. The vertical surfaces force the students to stand up, which raises energy levels and supports cooperation since acting passively when standing is more difficult (Liljedahl, 2022). A non-permanent working area has been shown to encourage students to start working on the task faster and more freely since they know that everything easily can be erased. Moreover, to promote cooperation between the students, each group should only have one marker (Liljedahl, 2018).

The first three practices are recommended to be implemented together and the remaining eleven practices[[2]](#footnote-2) can be implemented one by one. The reason for this is that neither of the first three practices is enough to break classroom norms by themselves (Liljedahl, 2022). These three practices function as a signal that the mathematical work is going to be something they are not used to, which implies that the students more freely can step outside their normal roles in a mathematics classroom. We view Liljedahl´s practices, more specifically the first three, as a theoretical background for this paper.

As a theoretical framework, we employ the concept of didactical contract (Brosseau, 1997), which can explain some of the struggles a teacher can face when trying to change the teaching method. A breach of the didactical contract (DC) has been used in other studies, for instance as an explanation for students’ difficulties with mathematical modelling (Jankvist & Niss, 2020). A DC is formed in every mathematical classroom even though it is not explicitly expressed (Brousseau, 1997). Since the DC is implicit, it becomes visible only when one part breaks the contract in some way. Moreover, it is a social agreement that both the teacher and the students are reciprocally responsible for upholding. For example, when the teachers present a task or model on how to solve a specific type of task, in Swedish called ‘genomgång[[3]](#footnote-3)’, it is often made by posing questions to the students (Andrews & Larson, 2017). These questions are often rhetorical and/or closed and seem to be a part of the DC (e.g., Jankvist & Niss, 2020).

Since the DC is mostly implicit the teacher might not be aware of its existence and therefore becomes a hurdle for teachers when changing their teaching methods. If changing one’s teaching involves new demands, such as coaxing students to think rather than mimic, students might react negatively. Not necessarily because they do not want to think by themselves, but because they react to the change in itself. In addition, a teaching method such as TC, that engages students to think actively, is more time-consuming (Serrano Corkin et al., 2019). This might conflict with demands from the curriculum to engage in every mathematical area, thus forming further obstacles to changing one’s teaching.

This paper examines what a teacher can experience when introducing a task while using the first three practices of Thinking classroom. We follow one mathematics teacher when using this teaching method in a Swedish upper secondary classroom. The following questions guide our study: 1) how does the teacher introduce tasks to her students while following the method of Thinking classroom? 2) which part of the pre-existing didactical contract can interfere with the intentions of Thinking classroom during the introduction?

## Method

First, we present the participants, including the researcher’s cooperation with the teacher. This is followed by a description of the pre-existing DC by illustrating how ordinary lessons usually start. Thereafter, data collection and analyses are described and explained. The data presented here comes from a pilot study, conducted by the first author, as a part of a PhD project that implements TC at upper secondary schools.

The researcher presented the project to a group of teachers at one upper secondary school (ages 15–19) and one teacher volunteered to participate. The teacher was experienced with more than ten years working at this school. All 16 students in the group chosen by the teacher gave their consent to participate in the study. The cooperation with the teacher involved two individual meetings where the researcher presented the main idea behind TC and which tasks to use in the six consecutive lessons were discussed. The first three lessons built upon non-curricular tasks chosen from Liljedahl’s book (2022) and the following on growing patterns, both 2D and 3D. Each lesson revolved around one task, thus six tasks in total were chosen.

The pre-existing DC in this classroom during the beginning of ordinary lessons comprises students’ individual work with a task from the textbook, followed by pair work on the same task, which the teacher then solves in front of the students. Thus, the students might or might not work on the task, knowing that they will soon get the solution, which then can be mimicked. The solution in front of the class is often followed by a genomgång where the teacher presents something new, which both can entice students to mimic. The pairs are set by the teacher and only change if the cooperation does not work sufficiently. This implies that the pre-existing DC includes that the teacher can decide who the students are to cooperate with and that mimicking solutions is common. The teacher frequently poses questions to the students during both the presentation and when showing the solution, indicating that questions from the teacher are part of the pre-existing DC. This scenario was described by the teacher and some of the students in interviews and confirmed by fieldnotes.

Since we were interested in visible features such as the teacher’s and students’ gestures and body language, as well as verbal utterances, video recording was conducted, enabling a multimodal analysis (Jewitt, 2012). The six lessons when the teacher used TC, were all filmed as a whole except for lesson five where the recording started after the teacher’s introduction. For this paper, the introduction parts of the video recordings of lessons 1–4 and 6 were analysed.

To understand and explain the findings from the analyses the theoretical construct of DC was used. The reasons for this choice were a) the explicit recommendation of Liljedahl (2022) that it is important to create a breach of classroom norms when implementing TC and b) that the DC is unspoken and reciprocally upheld by both teacher and students thus, difficult to change (Jankvist & Niss, 2020). Both visible breaks of the pre-existing DC, and when it overruled practices of TC became apparent by this theoretical framework. These choices were underpinned by our aim to examine what a teacher can experience when implementing the first three practices of TC.

We analysed the length of the introduction, the students’ positions, the arrangements of the classroom, and the randomisation process since these aspects occur during the introduction. We wanted to investigate what, if any, of TC practices created breaches of the DC visible through these aspects of the introduction, since one rationale behind TC, according to Liljedahl (2022), is to break classroom norms. The data also contained other aspects that were not obtained by this analysis but were salient to us. We found these aspects to be either breaking or confirming the pre-existing DC, hence assisting or challenging the implementation of TC. The following categories, created by inductive content analysis (Krippendorff, 2019) captured these salient aspects: the teacher’s use of a manuscript, questions asked by the teacher, and questions asked by the students. Questions asked by teachers in a mathematics classroom can be categorised in different ways. A common pattern of classroom questions is IRF, initiation, response, and feedback, which might impede mathematical thinking and reasoning (Attard et al., 2018). In this paper though it is more valid to note how often the teacher asks questions than to categorise them since most of them were closed and rhetorical and were part of the pre-existing DC.

## Results

Firstly, an overview of all five videotaped introductions is given. After that, we paint a picture of the teaching situation by providing a thick description of the introduction of lessons 1 and 4 which are chosen as examples of using both non-curricular and curricular tasks. Throughout the results, we demonstrate our conclusions from the analyses by connecting the result with recommendations from TC and/or pre-existing DC.

### Overview of the introduction

An overview of introductions in lesson 1–4 and 6 is shown in Table 1. The time used for the introduction of the tasks ranged from 1:10–6:06 where the two longest were the first two introductions. In TC the recommended timeframe for the introduction is a maximum of five minutes which means that the first two exceeded that timeframe. During the introductions the teacher asked several questions, the greatest number of questions asked by the teacher was 23 in Introduction 1, after that the number of questions decreased. One interpretation of these results is that the prolonged introductions in the first two lessons were due to the use of more questions than in later. In Introduction 3 the teacher asked 20 questions and kept the timeframe which is explained by the fact that almost all questions were rhetorical, which the teacher answered herself. Another interpretation of the prolonged introductions is that the tasks are unfamiliar to the teacher. The few questions the students asked mainly concerned clarifications, for example, “What does factor mean?”. Consequently, the students’ questions did not affect the length of the introductions.

The teacher had the task written on a paper in all lessons but one, hereafter referred to as a manuscript. In the first three introductions, the teacher looked at the manuscript around ten per cent of the time and one third in Lesson 4. Furthermore, she asked the researcher four questions in total during the first two introductions. These actions may be perceived as the teacher’s unfamiliarity with both tasks and teaching method.

Table 1. Overview of the introductions

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Introduction number | 1 | 2 | 3 | 4 | 6 |
| Length of introduction min:sec | 05:32 | 06:06 | 03:18 | 03:01 | 01:10 |
| Number of questions from the teacher | 23 | 15 | 20 | 4 | 3 |
| Number of questions from students | 0 | 4 | 0 | 1 | 0 |
| Looking at manuscript (% of time) | 11,7 % | 8,7 % | 12,1 % | 33,7 % | - |
| Number of times looking at the manuscript + asking questions to the researcher | 14 + 2 | 15 + 2 | 10 + 0 | 9 + 0 | - + 0 |

Another feature concerning the introductions is the randomisation process, not presented in Table 1. A deck of cards was used to randomly divide the students into groups of two or three students each. The researcher had the responsibility to count the number of students present and make sure that the number of cards matched to facilitate for the teacher. During all except one of the lessons, the teacher hands out the cards. No reactions from the students regarding the grouping were visible in the recordings. One possible interpretation is that the pre-existing DC is not breached by the randomisation.

### Lesson 1

The researcher and the teacher prepared the classroom by organising furniture and hanging the vertical non-permanent surfaces (VNPS), brought by the researcher, in strategic places, which took about ten minutes. When opening the door 15 students entered the classroom. They looked at all the desks and chairs that were cramped together to make space around the VNPS (see Figure 1) and left their bags on top of the tables. The teacher stood by one of the VNPS at the back, holding some paper and waiting for the students to gather around her, which they did. Almost all students stood up (see Figure 1) and looked towards the VNPS where the teacher presented the task. The fact that almost all students stood up and how the classroom was organised is a breach of their pre-existing DC. During the next five and a half minutes the teacher explained the task. Throughout that period the teacher asked 23 questions (see Table 1) mostly to engage the students, e.g.:

01:10 Teacher: What number do you want to use?

01:14 Student 1: Three

01:19 Teacher: And what operation do you want to use?

01:23 Student 2: Well… times

01:30 Teacher: What other number do you want to use?

01:32 Student 1: Five

01:36 Teacher: And… what is that?

01:37 Student 1: Fifteen

The use of questions during the introduction can be interpreted as the teacher’s ingrained behaviour and a part of the pre-existing DC. This prolongs the introduction and thereby contradicts the recommendations of TC.

The questions asked by the teacher to the researcher concerned the task itself and when to form the groups. The teacher looked at her manuscript 14 times, equivalent to just over ten per cent of the introduction. As mentioned before this shows that TC is something unfamiliar to the teacher and is illustrating a breach of the pre-existing DC. After the introduction, the researcher showed the students that the workspaces were numbered and handed the cards to the teacher who gave them to the students. The researcher thereby intervenes and thus may override the pre-existing DC. Upon being given a card each student walked to their workspace and started working on the task. There can be several reasons for the students’ lack of reaction, connected to TC, the pre-existing DC, and the researcher’s presence, further discussed later.



Figure 1: A sketch of where the teacher and the students were located in the classroom in Lesson 1, to the left, and Lesson 4, to the right.

### Lesson 4

Once again, the teacher and the researcher arranged the classroom before the students entered. The teacher started the introduction by drawing a 2D pattern on one of the temporary VNPS while she described what she was doing to the students. After one and a half minutes the drawing of the first two figures of the pattern was finished whereas the teacher explained the purpose of the task. The manuscript was looked at nine times for approximately one third of the introduction, most time spent after the drawing was done. Four questions were asked by the teacher, all of them rhetorical. Most of the students sat on benches or a windowsill (see Figure 1) while looking towards the teacher. At the end, one student asked the teacher to repeat the purpose of the task, no other questions were asked by the students. When the introduction was finished approximately three minutes later, the researcher handed out the cards to the students.

There are three differences between this introduction compared to the one in Lesson 1. Firstly, the recommended timeframe is held which might indicate that the teacher has adapted to TC, or the DC has changed. Secondly, the teacher used the manuscript in a larger proportion of the introduction. This is hard to explain through TC or DC, but one assumption is that the teacher wanted to ensure that the purpose of the task was accurate. Thirdly, most students sat down which placed them further away from the teacher. Our interpretation is that the pre-existing DC overruled the recommendations of TC.

## Discussion

Even though many components of Thinking classroom (TC), at a surface level, seem easy to implement as described in Liljedahl (2022), they might involve a more profound change of pre-existing didactical contract (DC). This is suggested to be a cumbersome process (Jankvist & Niss, 2020). A breach of the DC is not only challenging for the students but also for the teacher. As shown in the results some aspects of the pre-existing DC might be ingrained habits that contradict the effort to breach the DC.

For instance, the time spent on the introduction was prolonged by the teacher’s habit of asking questions. According to Andrews and Larson (2017), questions posed by the teacher during a genomgång are very common and applied to this teacher. During the interview, the teacher emphasised that she wants the students to answer her questions to keep the students engaged. However, as shown in the excerpt it can be time-consuming even though the predominant questions were closed and followed the IRF-pattern (Attard et al., 2018), without verbal feedback. We suggest that the pre-existing DC concerning genomgång and questions collide with Liljedahl’s (2022) recommendation regarding the length of the introduction.

Over the sequence of lessons, more and more students chose to sit down during the introduction. During the introduction, Liljedahl (2022) recommends, in practice 6, that the students should stand near the teacher. In Introduction 1 all students but one did just that without any prompts from the teacher or researcher. We presume that the obvious breach of the DC when entering the classroom in Lesson 1, with all the furniture cramped together and the teacher standing at the back of the classroom, caused this. The teacher never encouraged the students to stand up as this recommendation never was presented to her. This demonstrates that it takes more than the first three practices of TC to uphold a new DC. In our opinion, it is important to consider how to present the task including the timeframe and the students’ positions. Therefore, these recommendations ought to be incorporated into practice 1.

We suggest that the use of a manuscript indicates the teacher’s unfamiliarity with the task and the teaching method. It is neither a recommendation in TC nor a part of the pre-existing DC. The amount of time spent looking at the manuscript prolongs the introduction which might influence the introduction to extend the recommended time of five minutes (Liljedahl, 2022). We find it surprising that an experienced teacher uses notes to this extent. However, this can be explained by the extraordinary situation. The tasks were new to her, the method was not only unfamiliar but also introduced briefly. On top of that, a researcher was present in the classroom to videotape the teacher.

One of the practices suggested by Liljedahl (2022) aiming for a breach of DC is randomised grouping. Therefore, one might expect students to react in some way. However, these students did not show any reaction visible in the recordings. We find three possible explanations for this. One is the fact that the researcher was present in the classroom and involved in the randomisation process, thus providing an alteration of the ordinary teaching. However, there are no other situations that can be linked to the presence of the researcher therefore we find this explanation unlikely. Another explanation might be that the first three practices in TC are designed to create a breach of the pre-existing DC. The classroom looks different, the teacher is standing in a different place, there are VNPS on the walls, they must stand, and there is a new kind of task. All this together might create such a breach of the pre-existing DC that the students just accept the randomisation. Although, if this was the case any objections would have been noticed in later lessons when the novelty of TC has worn off, as we can see in the fact that more and more students sat down instead of stood up. However, we connect the lack of reactions to the fact that this teacher uses peer work regularly where the students cannot choose whom to work with, hence a part of their regular DC.

Other challenges when implementing TC of a more practical nature are what kind of VNPS to use, where to hang them, and how to make room around them. In upper secondary school teachers and students normally change classrooms between lessons which entails that the time to rearrange the classroom is limited. We regard these practicalities to be important for schools to consider before implementing TC since it is challenging enough for teachers to change their teaching.

The chosen tasks have neither been presented in detail nor discussed. The reason for this is that the task itself is chosen in beforehand and therefore not a part of the introduction per se. In coming studies, it might be interesting to investigate what impact different tasks have on, for example, student engagement or reasoning possibilities.

We acknowledge that this study has limitations in its design which makes it hard to infer conclusions regarding all aspects of how a pre-existing DC can affect the implementation of TC. To analyse this further, we would need data from a series of whole lessons, including student work on the VNPSs, and more theoretical underpinning. We plan to further explore this phenomenon in coming studies. However, as a first glimpse of what challenges, and possible explanations of the challenges’ nature, we find the results of interest to the research community. Moreover, our findings might support teachers and schools in implementing TC being prepared for the challenges and pursuing the changing of teaching methods.

In upcoming studies, we will provide teachers with a more in-depth introduction to TC in advance and preferably discuss the different practices with other teachers. Other takeaways concerned what kind of VNPS to use, how long it takes to arrange the classroom, and remembering to start the recording on time.

The main conclusion that we draw from this study is that even though the first three practices of TC seem easy to implement they are not; they involve much more than what is seen on the surface. For instance, the presentation of the task demands a more thorough preparation than can be perceived when reading the book by Liljedahl (2022).

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1. In Sweden all tasks can be labelled curricular since the students should learn problem-solving, mathematical reasoning, and communication. Although, we choose to follow the notion presented by Liljedahl (2022). [↑](#footnote-ref-1)
2. These eleven practices include for example how to furnish the classroom, how the teacher answers questions, and how the teacher evaluates. [↑](#footnote-ref-2)
3. No English word fully captures the meaning of the Swedish concept genomgång. [↑](#footnote-ref-3)