# Communication strategies when reading less familiar mathematical expressions aloud

Ulrika Wikström Hultdin

Umeå Mathematics Education Research Centre (UMERC),   
Department of Science and Mathematics Education, Umeå University, Sweden

Spoken mathematical symbols are part of oral communication in mathematics classrooms. The transition from written mathematical symbols into spoken sound is an intricate process that, in some respects, parallels the translation of one language into another. This study focuses on strategy-use in the reading aloud of mathematical symbols and compare them to strategies found in second language communication (SLC). In an analysis of sound recordings of university students who read symbolic expressions aloud, SLC strategies were found to be abundant. This indicates that some obstacles encountered when transforming written symbols into spoken words are similar to problems occurring in SLC. It is suggested that mathematics teaching should include some instructions on how to read symbolic expressions.

## Introduction

Some say that mathematics is a language, and others think that is not the case (Wakefield, 2000). Nevertheless, in mathematics education research, language-rich classroom activities are generally acknowledged as central to learning (Morgan et al., 2014). More specific suggestions on the topic have included teaching mathematics using methods common to second language learning (Wakefield, 2000; Bossé et al, 2018; Ledibane et al, 2018). Because both language use and the use of mathematics are considered social human activities that are learned in communication with others, this is in tune with other current ideas, for example, about the benefits of collaborative learning (Laal & Ghodsi, 2012).

Envisioning the learning of mathematics in a functional collaborative environment typically includes students working together on mathematical problems, discussing and reasoning about concepts and methods, sharing ideas, interpreting different visuals, and so on. It might also include students’ advancements in the use and understanding of mathematical symbols. In such a collaborative environment, the students will probably, now and then, need to transform written mathematical expressions into speech. Intriguingly, little is known about what happens in such situations, and due to the characteristics of the mathematical symbol system, it might involve some challenges.

Mathematical symbols are used to represent values, operations, functions, and patterns. It is a written system, where the signs relate to their content by convention (Pimm, 1987). In contrast to the reading of alphabetical languages, where words can be spelled out syllable by syllable, the reading aloud of mathematical symbols requires the reader to have experience either with the sound, or an alphabetical translation, of the symbols before they can read themselves. An additional peculiarity is that a reader, despite knowing the name of every single symbol included in a mathematical expression, can still be unable to perform the interpretation necessary to express it orally. Differences in the size and spatial location of a symbol can completely change its meaning and how it is read. Consequently, the reading becomes different from the reading of the ordinary Swedish language. For example, the reading order is not necessarily left-to-right and top-to-bottom. Furthermore, there is often more than one option on how to write an expression. Compare, for example, , and (The possibility to read an expression in several ways is also common. Consider, for example, which can be read as “y-double-prime” just as well as “the second derivative of y”. Both are mathematically acceptable. Yet, the latter alternative is a more interpretive form of reading than the former (Pimm, 1987).

At the time I started to develop an interest in reading in mathematics, previous research had shown that silent reading of texts with mathematical symbols interwoven in sentences was more difficult than the reading of texts without any mathematical symbols (Österholm, 2006). Intrigued by those results, I did some work focused on the variation in word use in reading aloud of mathematical symbols to find hints on what could be disturbing the readers (Hultdin, 2013). While listening to some recordings of students that were reading mathematical expressions, I surprisingly found their struggle to be very familiar; were they not behaving in a similar way to persons trying to express something in a second language? Considering the special characteristics of the symbol system, it is not a farfetched assumption to think that students will sometimes experience a lack of vocabulary, or challenges with the interpretation, when transforming a somewhat unfamiliar written mathematical expression into speech. This could cause a stall, which has to be overcome, and to overcome it, some strategy is needed.

## Second language communication strategies

To experience a lack of vocabulary or grammar difficulties that threaten to interrupt or impair communication is common for second language (L2) speakers. In such situations, a plethora of second language communication (SLC) strategies have been found to come into service. Different taxonomies describe different strategy types under different names but, in the end, there seem to be more similarities than differences between these descriptions (Avval, 2012). In 1997, Dörnyei and Scott summarized all the communica-tion strategies proposed to date and divided them into three main groups: direct strategies, interactional strategies, and indirect strategies. Direct strategies concern, for example, change in vocabulary but also self-repair and mumbling. Interactional strategies include asking for help, clarification, and repetition. The indirect strategies are performance-related or related to time pressure and include, for example, marking uncertainty about the message (performance-related), or using filling words to keep the communication channel open (time pressure-related).

Not only in situations of oral communication but also when written language is translated from a first language to L2, SLC strategies come into play (Rabab'ah, 2008). Transforming symbolic expressions into speech bears some resemblance to the situation when one written language is translated to another, at least on the surface. First, the starting point is a written text. Second, the written symbols, and sometimes even the meaning of them, can be more familiar than their “translation”. Third, there is not always a one-to-one transformation; a symbolic expression can often be transformed into several adequate oral expressions.

In the present study, the purpose is to gain knowledge about what happens in a situation where students read out symbolic expressions with which they are not completely familiar. The focus is on the transformation of written symbols into speech, and the question addressed is: What strategies can be identified when students read less familiar mathematical expressions aloud? The aim is to illustrate some of the complexity in the transformation process, not to provide an exhaustive enumeration of all strategies possible to find when students struggle with the reading of symbols. The results could propose ideas for strategies to overcome the struggle.

## Methods

Although reading aloud does not cover the whole spectrum of classroom situations in which students transform mathematical expressions into speech, it allows for the investigation of many diverse expressions in a short time. Since the focus of the study was on the transformation and not on communication in general, the data collection could be limited to sound recordings. All data was originally collected as part of a more extensive project thesis (Hultdin, 2013)1. All participants gave consent for the use of the recorded material in research.

The participants were 18 students (Male = 14, Female = 4, age: 19–25 years), all fluent Swedish speakers, taking a preparatory university course in mathematics. The texts that were read were short Swedish texts, 3–8 lines. Each participant was recorded in a separate session. During the sessions, which were taking place in an office near the students’ ordinary classroom, only the participant and the instructor were present. The data was collected one week before the final course exam. In a break between lessons, potential study participants were informed about the study focus, time and place for participation, confidentiality, and the general procedure of the reading session. The texts read were modifications of ordinary textbook paragraphs or task texts and included mathematics taught in the current course. Of 12 texts read, the six causing the students the most problems (a-c below) were selected for analysis. Three integral expressions, two differential equations, a primitive function, and double angle identities were included in the selection.

At the time, the students were divided into three teaching groups, taught by three course teachers. The analysis included a comparison of participants’ readings to *conventional* reading. For this purpose, a standard was created. By asking the course teachers to write down how they would read a few selected mathematical expressions and specifically asking them to come up with as many alternatives as possible that they would judge to be correct, various “standard readings” were collected.

It was assumed that students would use various communication strategies when struggling with their reading of mathematical symbols. Therefore, strategy use was analyzed when one of the following occurred in the sound recordings: a) longer pauses (compared to reading other text parts), b) use of non-standard vocabulary, or c) a non-standard reading order (compared to the “standard readings”).

### Analysis based on Dörnyei and Scott’s taxonomy

Because of the assumed similarities between the reading situation and L2 communication, the data was classified with the help of Dörnyei and Scott’s taxonomy of SLC strategies (1997). It comprises a summary of relevant SLC strategies, and has a focus on produced verbal interaction, which made it suitable for this first exploration of strategy use for readers of mathematical texts. The choice foregrounds similarities that can be found between the reading situation and a situation of L2 communication. However, also non-SLC strategies were noted during the analysis.

Since the taxonomy concerned oral L2 communication involving a (more active) second person, all the interactional strategies such as “other repair”, “other repetition” and “feigning understanding” had to be excluded. The type of data collected also limited the number of strategies possible to identify in the material. Because all participants were Swedish speaking, strategies used to overcome problems voicing specific L2 sounds were excluded. The strategy where “more words were used to achieve a particular communicative goal than what is considered normal in similar L1 situations” (termed “over explicitness” in Dörnyei and Scott, 1997), turned out to be either literal translations or paraphrasing when symbols were read aloud. Thus, it was not included as a separate category. The final 18 categories are described below.

Direct, resource-deficit-related:

**Message abandonment.** Giving up and skipping the expression/text.

**Message reduction.** Not reading single words or longer text chunks in the middle of a text.

**Message replacement.** When saying anything (instead of nothing), but not at all with the intended meaning. An example would be to say “*and-lah-lah-lah*” instead of the symbols.

Non-standard readings were classified as one of the three above only when no other strategy, for example, “omission”, could be assigned.

**Restructuring.** Abandoning the first communication plan. Continuing with the same message but read in another way. Example from reading *y¨*: *[…] y-prime, y… let’s see, y… the derivative twice.*

**Approximation.** Using a superordinate or relative term as an alternative to a missing one. For example, saying “*area*” instead of “*integral*”.

**Literal translation.** Using a known pronunciation of a symbol(s) in a context where it is normally not used. For example, reading out all parentheses.

**Paraphrasing**. Reading in a more explanatory way after stalling or hesitation. For example, saying “*the primitive function of f-of-x*” when reading *F(x)*. This strategy can include both standard and non-standard vocabulary.

**Word coinage**. Unconventional reading of a mathematical expression based on previous experiences of mathematical syntax and vocabulary. Example: The division sign in in an integral expression is read *“divided by”.*

**Use of similar-sounding words**. Substituting an unknown word (either existing or non-existing) with one that sounds, more or less, like it.

**Use of all-purpose words**. Using all-purpose words, such as “*this thing*” or “*whatever it’s called*” instead of specific ones.

**Mumbling**. Swallowing or muttering a word inaudibly.

**Omission**. Skipping a symbol and continuing as if it had been read.

**Retrieval**. When retrieving a lexical item, saying a series of incomplete or wrong forms before reaching the optimal one.

Direct, performance-problem-related:

**Self-rephrasing**. Repeating a term and, at the same time, changing it (without changing the message). Example: *“the second derivative … the second derivative of f-of-x”*

**Self-repair**. Self-correcting, either directly or after repetition of the incorrect word/phrase.

**Self-repetition**. Repeating a word or phrase immediately after it is said.

Indirect, time-pressure related:

**Use of fillers**. Using all-purpose words, or sounds, to fill pauses.

Indirect, performance-problem-related:

**Verbal strategy marker.** Using verbal marking phrases, such as *“I don’t know what to say.”*, before or after a strategy to signal that the word or phrase might not carry the intended meaning.

For quite a few of the readings, follow-up interviews would have been necessary to report the exact strategy in use. For example, if a participant says the word “function” instead of “integral” it is not possible to know if the person sees the concepts as similar and, thus, uses an approximation, or if they consciously replace the original message just to say something. Yet, many readings were to the point and could easily be classified based on the descriptions. Because of the limitations to the method, only the occurrence—if a strategy was present in data or not—was reported for each strategy, and not its frequency.

## Results

|  |  |  |  |
| --- | --- | --- | --- |
| **Strategy type** | | **Deviation from standard** | **Found**  (Y/N) |
| **Direct strategies** | |  |  |
| *Resource-deficit related* | Message abandonment | message/content | Y |
| Message reduction | message/content | N |
| Message replacement | message/content | Y |
| Restructuring | vocabulary use | Y |
| Approximation | vocabulary use | Y |
| Literal translation | vocabulary use | Y |
| Paraphrasing | vocabulary use | Y |
| Word coinage | vocabulary use | Y |
| Use of similar sounding word | vocabulary use | N |
| Use of all-purpose word | vocabulary use | N |
| Mumbling | vocabulary use | Y |
| Omission | vocabulary use | Y |
| Retrieval | self-correction/  vocabulary search | Y |
| *Performance problem-related* | Self-repair | self-correction | Y |
| Self-rephrasing | self-correction/  vocabulary search | Y |
| **Indirect strategies** | |  |  |
| *Time pressure related* | Self-repetition | vocabulary search | Y |
| Use of fillers | vocabulary search | Y |
| *Performance  problem-related* | Verbal strategy marker | marking uncertainty | Y |

Table 1. SLC strategies in aloud reading of unfamiliar symbolic expressions

When transforming written mathematical symbols into speech, numerous strategies were identified. Strategy use was often associated with the difficulties encountered with specific expressions. When reading integral expressions, about half of the participants exchanged the integration limits. When learning to read in Swedish or English, we are trained to read top-to-bottom. This reading order was also used for symbolic expressions. However, the conventional reading order for integral limits is the opposite, bottom-to-top. The strategy identified was not so much a strategy as a preservation of reading order, and certainly not one of the listed SLC-strategies that were based on oral communication.

Another non-SLC strategy was found for the reading of the symbols α and β, which were symbols unfamiliar to many of the participants. Instead of “alpha”, α was often read like the letter “a”, and in one case, “ex” (x). Beta was read like the letter “b”. The strategy used in these specific cases was to substitute the unknown pronunciation with the known pronunciation of a symbol of a similar shape. For natural languages where the writing is based on alphabetic sounds, the strategy when reading an unknown written word could instead be to spell it out letter by letter. When a symbol is not connected to a specific language sound, the strategy seems to be to focus on the shape of the symbol and approximate it with a similarly shaped one.

A majority of the strategies identified were similar to SLC strategies. Of the 18 potential strategies listed, 15 were found in the recorded material (Table 1). This included both direct strategies that were related to resource deficits and problems with the own performance; as well as indirect strategies related either to one’s own performance or to time pressure.

Strategies transforming the message in such a way that it would be impossible to interpret in the same way as the original print—*message abandonment*, *message reduction*, and *message replacement*—were not very common. *Message abandonment* was almost seen in one case where a participant, who did not find the words, wanted to skip to the following text but did continue after some encouragement from the instructor. The *message reduction* strategy was never found to be the primary strategy. In a few cases, the message was replaced not only by an approximation but by something substantially different. In those cases, it was a matter of saying anything at all, like "blah-bla-bla", just to fulfil the task.

Lack of vocabulary led to the use of several strategies. The resulting message was not unrecognizable but often included some ambiguities. For example, when the word “integral” was *approximated* with “function”. For unconfident readers, the *mumbling* strategy, or the complete *omission* of single symbols, were common. For example, when one of the participants used the term “top value” (Swe: “toppvärde”) when reading the integration limits of an integral expression, that word was read less clearly and in a significantly lower voice compared to other parts of the expression. Examples of the *omission* strategy were found when a participant consistently omitted the integral sign in the reading of integral expressions and when others did not read the “*dx*” at the end of the expressions.

*Word coinage* was a less common, but still present, strategy. One example where reading was based on previous experiences with mathematical syntax and vocabulary was when was read as “the integral *times* f-of-x dee-x from a to b” (Swe: ”integralen *gånger* f-av-x de-x från a till b”). The word choice was assumed to be based on previous experiences with implicit multiplication signs.

The unconventional reading showed a large span in the degree of interpretation of the expressions; both *paraphrasing* and *literal translation* were used. Specific to this study was the participants’ search for words when encountering . When this, after hesitation, was read as “the second derivative” or “the derivative twice”, it was interpreted as *paraphrasing*. Reading as “dee-why divided by dee-tee” (Swe: “de-y genom de-te”) was interpreted as a *literal translation*.

It was common to find simultaneous usage of multiple SLC strategies. For example, the *paraphrasing* strategy was more than once part of a *restructuring* strategy. *Restructuring* was also seen together with *self-repetition* (“from, … from, from”), *self-rephrasing* (“limits, … interval”), and the *use of fillers* (“ehm…ehm …”), strategies used to keep communication going.

In some cases, it was not completely clear which strategy was used. For example, the reading “The equation y-prime-two, no, the derivat… the second derivative of y” (Swe: “Ekvationen y-prim-två, näe, derivat… andraderivatan av y”) can be seen both as a *self-correction* indicated by the “no” followed by a change of the word, and/or as a *retrieval* where the final word is found after saying incomplete and wrong words. Furthermore, *strategy markers*, where participants communicate their uncertainty about message content, often had the additional function of being fillers. Examples were phrases such as “What do you say?” and “What’s the name of this?”, said before reading a difficult passage. An example of a strategy marker that was not used as a filler was the question “If that’ll work?“ (Swe: “Om det går bra?”) said after reading an expression.

The conclusion is that manifold strategies were incited when students read less familiar symbolic expressions aloud, and many of the strategies were similar to strategies used in second language communication.

## Discussion

Seventeen strategies were identified in the recordings of students who were reading symbolic expressions aloud. Two of them were not second language communication (SLC) strategies, and they were both seen in connection with certain symbolic expressions. For integral limits, where the reading order is reversed compared to written language in general, it became obvious that preserving the top-to-bottom reading order was a strategy used when reading symbolic expressions. For singular symbols with unknown names, the strategy was to substitute them with the names of similarly shaped symbols. While top-to-bottom reading might be used without the experience of any communication problems, in specific situations, it is a strategy related to a resource deficit. That is, not knowing the reading order. The symbol shape approximation is more clearly a strategy for overcoming a difficult situation in line with a situation of SLC where vocabulary is lacking. The proper name of a symbol is not known, and that problem needs to be solved. However, the strategy used when solving the problem is not a SLC strategy.

Strategies similar to SLC strategies were abundant in the recorded material. Of the 18 potential strategies listed, 15 were found at least once. The potential SLC strategies that were not found in the data, might be found in situations where a more active communication partner is involved. That may also be the case for many of the strategies that were excluded from the study for methodological reasons.

That the strategic use of similar-sounding words did not occur in this study might be explained by the expressions included in the reading tasks. If the students did not experience any problems finding a word where they already knew a similar sounding one, the strategy could not be used. The limited number of participants is also a factor to consider in relation to the non-occurring SLC strategies. For example, a person who is more prone to using all-purpose words such as “this thing” and “that stuff” in everyday language might also have used them in the reading of mathematical expressions. It is not known whether the 18 participants in this study included such a person, and if so, the reading situation may not have stimulated this type of strategy. That message reduction did not occur as a strategy can be a result of the analysis method, at least partly. In many cases, the resulting message was reduced but it was also possible to find that another, often more specific, strategy was used in the situation.

According to the taxonomy of Dörnyei and Scott (1997), the occurring SLC strategies were related to resource deficits and performance problems, as well as to the experience of time pressure in the communication situation. Although the data was limited to the reading aloud of selected expressions, the results so far imply that the experience of the own performance and the time pressure in the reading situation are similar to what is experienced by L2 speakers in SLC situations. Yet, the resource deficits seem to comprise some additional categories for transformations from symbols to speech. That is not surprising because of the difference between the sign systems. For example, unknown written words can still be read aloud, and for the written Swedish language, the reading order is fixed. Nevertheless, similarities in the use of communication strategies may reflect similarities in the obstacles encountered when reading less familiar symbolic expressions and speaking in L2. Use of similar strategies could also indicate similarities between reading symbols aloud and translating from a first language to an L2, another situation where SLC strategies are activated (Rabab'ah, 2008).

For a teacher to adequately help students learn the language of mathematics—the communication in words about mathematical ideas—it is important to know the many obstacles lined up along the way. As shown, one of the difficulties that causes the need for a communication strategy when transforming written symbols into speech is insufficient vocabulary. Many teachers are already explicitly teaching vocabulary to their math students by explaining and relating different words (Riccomini et al, 2015). If more teachers became aware of the problems associated with the transformation of mathematical expressions into speech, it would not be a large step to include explicit instructions on how to read different symbolic expressions related to the topic currently taught. It is worth noting that some of the unconventional paraphrasing examples identified in the data showed that students can have a sense of the meaning of an expression, or at least its functional use, before knowing a common way to read it. One example would be when the participants are saying “the derivative twice” (see Hultdin, 2013, for more examples).

A final reflection is that human communication is rarely bound to one semiotic system or one type of situation. We communicate by any means available, using the signs we know and the strategies we have acquired, and a common strategy in second language communication could just as well be used when reading a mathematical expression aloud. The aim above all is to deliver the message.

### Note

1. The report is based on part of a project thesis (Hultdin, 2013). At the time, part of the results was reported in Swedish. Examples from data in the present report are the same as in the thesis but translated from Swedish to English. All other text is newly written or further processed (methods, results) for this report and not a mere translation.

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