Skrifter från SMDF, Nr.8

Editors: Glenda Anthony and Barbro Grevholm

Teachers of Mathematics: Recruitment and Retention Professional Development and Identity

SMDF Svensk Förening för MatematikDidaktisk Forskning
Swedish Society for Research in Mathematics Education

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SMDF Swedish Society for Research in Mathematics Education
For information see web page www.matematikdidaktik.org

Omslag, innbinding og trykk:
Trykkeriet, Universitetet i Agder,
Kristiansand
SMDF:s skriftserie, nr 8
ISBN 978-91-973934-7-8
ISSN 1651-3274

Preface

The idea for this book emerged from discussions that took place after the Thematic Afternoon about mathematics teacher education during The 10th International Congress on Mathematical Education. As participants of the coordinating team we decided to produce a book building on the presentations during the thematic afternoon. Thus all contributors were invited to elaborate their presentation for the session to a full paper and send it so us as editors. The authors were given some time to finish this writing process. A complete peer-review process was then carried out. All contributors reviewed three other papers so each author received comments and suggestions from three colleagues. The chapters were then revised and reviewed by the editors. A technical editing process was carried out by Bente Hellang. We want to express our thanks for the work done by her. After overcoming some challenges concerning the process of finding a publisher and financial support for the printing costs we finally managed to get the book into reality. We express our warm thanks to the board of the Swedish Society for Research in Mathematics Education for allowing us to publish the book in its series. We want to thank the University of Agder for generous contribution to the printing costs for the book. Finally we want to thank the programme committee of The 10th International Congress on Mathematical Education for creating the idea of a thematic afternoon on mathematics teacher education, which gave a basis for the book. We are grateful to all contributing authors for their work and for the careful review process they carried out.

Although much has been published about mathematics teacher education we think this books fills a gap. There is not much literature about recruitment and retention problems and here the chapters by Angier, Grevholm, and Thunberg raise issues around that. Professional development is an important issue for all teachers of mathematics and several authors contribute in unique ways in their chapters to enlighten this area. Examples are chapters by Anthony, Arvidson, Baber, Grevholm, Peterson, and Szajn, White, Hackenberg, and Allexsaht-Snider. The identity of mathematics teachers is an issue in chapters by Anthony, Grevholm, Parker, Proulx, and da Ponte. Finally some chapters deal with the mathematical competency of teachers, for example chapters by Amato, Christiansen, Fraser and Morony, Kaldrimidou, Sakonidis, and Tzekaki, and Oh. Thus we hope that readers will find the contributions in the book worthwhile to study. They represent research, knowledge, reflections and ideas about teachers of mathematics and especially on recruitment, retention, professional development, and identity from many places around the world.

Kristiansand and Palmerston North, 20101124

Barbro Grevholm and Glenda Anthony

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Crucial Issues in Mathematics Teacher Education and Teaching

Barbro Grevholm and Glenda Anthony

This introductory chapter of the book tries to place the book in its context among many other books about mathematics teacher education and professional development of teachers. From 1999 to 2003 an international survey team of mathematics educators and researchers explored research in mathematics teacher education and offered a paper reflecting the current state of the field (Adler, Ball, Krainer, Lin, & Novotna, 2005). Among the conclusions they present is the claim that the field needs to understand better how mathematics and teaching combine in teachers' development and identities (ibid, p. 378). Another suggestion is that we need authentic and interesting stories, both practice-grounded and theory-driven and combinations of reflective papers by teachers with cross-analyses by teacher educators (ibid, p. 378). Some of the chapters in this book meet these wishes presenting unique and interesting stories about mathematics teacher development and identity around the world. In the International Encyclopedia of Education Krainer and Linares (2010) in the section on mathematics teacher education present it as a practical and emerging field of research pointing to the international handbooks. The first international Handbook of Mathematics Education (Bishop et al, 1996) and the first International Handbook of Mathematics Teacher Education (Wood et al, 2008) reflect the increased attention to research on mathematics teacher education. In 1998 the Journal of Mathematics Teacher Education marked the beginning of a new era with the first journal focusing genuinely on research on mathematics teacher education. Krainer and Linares especially point to three dimensions on teacher education. They are the social dimension, teachers' reflections and awareness of the general conditions on which teachers work. Thus the aspects on mathematics teachers' development and identity emphasised in this book complements what is discussed by Krainer and Linares.

Mathematics teachers' knowledge and practices is a theme in the chapter by da Ponte and Chapman in the Handbook of Research on the Psychology of Mathematics Education (2006) and mirrors the interest in PME-conferences over the years for mathematics teachers' work. Other more specific books deal with mathematical tasks used by teachers and in teacher education (Clarke, Grevholm, & Millman) and the professional education and development of teachers of mathematics (Even & Ball, 2009). The richness of publications on teachers of mathematics and teacher education indicates that there are still many unresolved issues worthwhile to deal with and share knowledge and experiences about. In this book a number of new contributions from authors around the world present stories and experiences on mathematics teachers' development and identity and also about the recruitment and retention of new teachers, which is an aspect rarely mentioned in other publications. Issues on recruitment and retention of mathematics teachers are discussed in the chapters by Angier, Grevholm and Thunberg. In the chapters by Anthony, Arvidson, Baber, Grevholm, Peterson, and Szajn, White, Hackenberg, and Allexsaht- Snider focus is on the professional development of mathematics teachers. The mathematical competency of teachers is illuminated in the chapters by Amato, Christiansen, Fraser and Morony, Kaldrimidou, Sakonidis, and Tzekaki, and Oh. The identity of mathematics teachers is an issue in chapters by Anthony, Grevholm, Parker, Proulx, and da Ponte. Below we introduce in short paragraphs the content of each chapter in the book and hope to raise the readers' interest and invite to deeper studies of the chapters.

In the chapter by Solange Amato the research results presented are part of an action research study designed to improve student teachers' understanding of mathematics. The teaching strategies were similar to those suggested for their future use in teaching children and involved the use of multiple modes of representation for each concept and operation in the primary school curriculum. The children's activities included not only using diverse ways of representing mathematical concepts and operations but also using diverse ways of performing operations. Focusing on mathematical relationships and paying attention to the transition from concrete to symbolic were part of the programme activities. The data indicated that most student teachers improved their understanding, but some needed more time to re-learn certain content in the primary school curriculum. The author proposes some practical solutions to improve the situation. Solange Amato has presented more of her work and ideas in line with this chapter in the ICMI Study about mathematics teacher education (see Gellert, 2009) and in a recent book on tasks in primary mathematics teacher education (Amato, 2009).

Corinne Angier in her chapter discusses recruitment problems and the situation of teacher training in England and Wales, where an effort to increase the recruitment of mathematics teachers has seen the introduction of flexible training programmes. By using a narrative methodology, Corinne Angier explores the learning experience of students at Sheffield Hallam University in which the students have opportunities to craft their own learning experiences. In the process of making choices, Angier claims, these students become more aware of their own needs and progress. However, she claims that while it is unlikely that the flexible route will make a significant quantitative contribution to the problem of teacher shortages perhaps these mature and well-prepared teachers will stay longer in the profession. According to the author there are features of the longer and flexible routes into teaching which may promote those skills and dispositions aligned to embracing the complexities and challenges of teaching.

Glenda Anthony in her chapter highlights the ongoing journey to becoming a teacher in the classroom. Her research study set within New Zealand—a country that has a well-established induction programme to support newly qualified teachers—considers what learning is possible in the first year of teaching. More than focus on survival the teachers in her study want to enact those practices promoted in their ITE programmes in a way that builds mathematical relationships with their diverse students. Her findings suggest that newly qualified teachers are offered a variety of support and guidance, but not always in a way that impacts positively on professional learning orientated towards inquiry into practice. For a few newly qualified teachers the induction practices focused on 'fitting in' and achieving initial accreditation requirements mean that learning remained focused on context and management requirements within the existing departmental or school norms. With a move to more flexible initial teacher education programmes, effective induction of newly qualified mathematics teachers continues to be a focus internationally (Wei, Andree, & Darling-Hammond, 2009).

Mark Arvidson's chapter is about algebraic fundamentals and he claims that they are a key issue in the successful preparation of teachers of elementary mathematics. In seeking to illuminate the crucial components needed for success in a course for elementary teachers of mathematics Arvidson claims that the main benefit of learning and doing mathematics is not the specific content; rather it is the fact that it develops the ability to reason precisely and analytically about formally defined abstract structures. Using the Guilford's Structure of the Intellect (SOI) assessment tool and the Learning and Study Strategies Inventory (LASSI) he concludes that pre-service teachers that have a solid foundation in algebraic fundamentals coupled with high motivation and self-discipline experience a higher degree of success in a course for the learning and teaching of elementary mathematics than their counterparts that are not so prepared. While Arvidson did not claim to have established a causal connection between course grade and the ability to teach mathematics, the data suggested that those pre-service teachers who had a solid foundation in algebraic fundamentals experienced a higher degree of success in the learning and teaching of elementary mathematics than their counterparts that are not so prepared. Research on establishing causal links between mathematical knowledge and teaching and learning outcomes continues to be crucial areas of focus for teacher education (see Ball, Thames, & Phelps, 2008).

Sikunder Baber writes about networks of learning. The chapter is a description of a professional education of teachers of mathematics in Pakistan. Baber proposes a variety of approaches for the development of teachers, at pre-service and in-service levels, in recognition of the importance of on-going professional development. Among the approaches, networking among teachers, teacher educators, curriculum developers and policy makers is receiving attention as an innovative and flexible professional development forum. Baber claims that networking can create ownership among stakeholders regarding implementation of change and reforms in the educational landscape. While there is much international interest in networking and community of learning approaches to professional development (see Ponte et al., 2009). Baber presents the unique case of the Mathematics Association of Pakistan (MAP) as a network for learning. The formation and growth of the network can be viewed as offering insights into the improvement of mathematics education in the developing world, especially in Pakistan. The author states that this sharing of the experience may support efforts for creating other networks of learning for implementation of reform in education in different parts of the world. He concludes that the current efforts aim to empower mathematics teachers to become caring and competent professionals, and also to support society to adopt an appropriate learning mode that will improve the quality mathematics education in Pakistan.

The chapter by Iben Maj Christiansen deals with mathematical competencies and awareness in a teacher education practice. Like Ardvison, Christiansen states that the question about how we can best facilitate the learning of mathematics in teacher education programmes so as to affect ambitious mathematics learning in classrooms is far from trivial. The chapter starts from a view on mathematics learning which embraces learning as both acquisition and participation. The chapter describes what happens when learning to 'do mathematics' is an essential motive in a mathematics teacher education programme, and how this learning may be facilitated. Drawing on the perspective of the social theory of learning of Etienne Wenger and others, which stresses learning as social participation in practices, Christiansen describes how student teachers' learning of mathematical competencies can be enhanced through participation in a developing community of practice. The tasks which offered students the opportunity to work with generalizations, conjecturing, proving, symbolizing, representing, and problem handling are further elaborated in another paper (Christiansen, 2006).

Helen Forgasz brings together findings from various studies in her chapter. She notes that while there has been much research on teachers' beliefs about mathematics and about pedagogy, less is known about their views about students and about computer use for mathematics learning, particularly with respect to gender-stereotyping. Since mathematics and computing are generally viewed as male domains, it seems appropriate to explore relevant teachers' and pre-service teachers' beliefs. She presents a summary of recent Australian research findings and observes that some teachers in the studies seemed to be out of touch with students' views with respect to different dimensions in mathematics learning. The teachers' views of boys' and girls' behaviours with computers and their explanations for them reflected traditional gender stereo-typed expectations. Implications for the educational community and for mathematics classroom practices are discussed by the author. Forgasz suggests that inclusive pedagogical practices (as shown in one study on tertiary level) may contribute to reduce gender differences in students' beliefs about technology.

Christopher Fraser and Will Morony contribute a chapter consisting of a description of the teacher developed professional standards for excellence in teaching mathematics (in Australia). The national association of teachers of mathematics in Australia has started work to enhance professional status, and to help re-design teacher education to enable purposeful and owned professional growth throughout teachers' careers. The vehicle for the work is the development of nationally agreed professional standards and assessment of volunteer teachers against these standards. The first phase involved research and development of materials to 'define' quality teaching of mathematics—the AAMT Standards. The second phase of implementation is in two parts. One is a focus on professional development using the Standards. The other is to develop a process for acknowledging outstanding teachers and awarding them the AAMT credential of Highly Accomplished Teacher of Mathematics. AAMT acknowledge that good teaching of mathematics is dynamic, and thus the Standards are viewed as a living document that needs to respond to developing knowledge about the field. In the meantime, Teaching Australia has proposed a national framework for 'National Standards for Advanced Teaching and School Leadership' and the AAMT is contributing to the consultation on that work.

Barbro Grevholm presents research on mathematics teacher education from a Nordic perspective. In many countries mathematical teacher education has been a subject of societal debate and criticism has been expressed. In this chapter the problematique of teacher education in mathematics is presented and issues and concerns related to it are discussed, in a try to understand why there is a problem. A longitudinal study of teacher education in mathematics in Sweden is used as an illustrating example and recent evaluations of teacher education in Denmark, Norway and Sweden serve as sources for finding criticism and problems. A model for teacher education seen as the development of a professional identity is presented and discussed. This leads to the suggestion of solving the problems with teacher education by experiencing it as a lifelong learning and the development of a professional identity.

In the chapter by Maria Kaldimidrou, Haralambos Sakonidis, and Marianna Tzekaki five studies are presented. The studies focus on aspects of teachers' management of the construction of meaning in the mathematics classroom: the ways they handle the epistemological features of mathematics, deal with pupils' work and errors as well as the communicative patterns they adopt. The results show that the teachers tended to treat the epistemological features of mathematics in a unified manner. Their interventions during the pupils' engagement with a mathematical task were very directive, and the communication patterns they followed did not provide space for the mathematical meaning to be negotiated. These findings suggest that the classroom management of the subject matter is likely to distort the mathematical meaning constructed by the pupils, and that it is dialectically related to the communicative practices employed. In conclusion, the authors raise a number of important questions worth studying. How do the mathematical and socio-cultural classroom norms differ? How do they interact with each other? Which is more deterministic in the cycle of their co-existence and how can this cycle be intervened upon? These questions have been taken up by the authors in a range of studies that address the nature of mathematical knowledge in the classroom (e.g., Kaldrimidou, Sakonidis, & Tzekaki, 2008).

Preservice elementary school teachers' knowledge on fractions is the focus of the chapter by Youngyoul Oh. The study analyzed such knowledge in terms of concepts and ability to represent fractions. The sample consisted of 115 preservice elementary school teachers in their third year of a 4-year teacher education programme in South Korea. The investigation used a questionnaire to study the preservice school teachers' knowledge about fractions. The findings suggest that preservice elementary school teachers view fractions primarily as the part-whole relationships and measures. They have a profound understanding of the meaning and representation of fractions as part-whole relationships and measures, but they have significant difficulty with conceptualizing fractions as quotients, operators, and ratios. The result implies that the widely held assumption about elementary mathematics, namely that if one can get the answers correctly then one can teach elementary school mathematics, should be challenged. The author concludes that teachers require deep understanding about what fractions really mean and how fractions are represented with models in order to teach mathematics in a way consistent with reform documents.

The chapter by Diane Parker is concerned with the official pedagogic identities from South African Policy and some implications for teacher education practice. In South Africa the National Curriculum Statements for FET Mathematics (grade 10-12) together with the Norms and Standards for Educators are key policy documents that provide the official basis for mathematics education reform and for the construction of new pedagogic identities. In the chapter Parker uses a framework based on the work of Bernstein to theorise the construction of pedagogic identities. She uses it to build on Graven's description of the new official pedagogic identity of the South African mathematics teacher, and on work by Adler and others to raise questions related to teacher knowledge and the challenges of developing specialist mathematics teacher identities through initial teacher education programmes. Parker argues that teacher educators need to compete for resources in teacher education and they have a responsibility to research and produce criteria for novice teachers to navigate acquisition of the recognition and realisation rules for specialist mathematical pedagogic identities. The work of this chapter is further developed in her doctoral thesis (see Parker, 2009).

Blake Peterson considers mathematics student teaching in Japan in his chapter and asks where the management is. His study followed three Japanese preservice teachers during a 4-week mathematics teaching experience in a Japanese junior high school. Each student teacher taught three lessons during the observation period. Conversations with their cooperating teachers included talking about how to teach mathematics and how students would respond to various tasks. Contrary to their counterparts in the United States they never talked about classroom management issues. Although students at this Japanese junior high school were generally well behaved, management problems did exist but were never discussed. The author asks when do student teachers learn to deal with these classroom management issues. During their first year of teaching, they are closely mentored by other teachers in the school and have opportunities to discuss any problems that arise in their own classrooms. Peterson concludes that in the United States student teachers spend the majority of their time learning about classroom management. In contrast, in Japan student teachers spend much of their time learning how to prepare, teach and reflect upon their lessons. Student teaching has a different focus and purpose in the two countries.

Joao Pedro da Ponte discusses the influence of an in-service distance education course in the construction of mathematics teachers' professional identity, especially regarding their views and practices of reflection and collaboration and their relation with information and communication technology. The course was based on open-learning pedagogy and focused on conducting exploratory and investigative work in the mathematics classroom. Evaluation results show that the perspectives and involvement of the participant teachers depended very much on their previous professional experience and relationship with the Internet. Teachers that used e-mail for collaborative work found this a very stimulating experience whereas others had some difficulty in assuming the roles and values required for this kind of activity. The experience points to the potential for teachers' development of open learning distance teacher education, where collaboration and writing become part of in-service activities and resources. Da Ponte suggests further investigation about tendencies and constraints of teachers changing identities supported by virtual learning communities. Led by further work by Joao Pedro da Ponte and colleagues (2007) in the 15th ICMI study series noted the role of online interactions were a tentative but growing area of inquiry (Ponte et al., 2009).

The chapter by Paola Sztajn, Dorothy White, Amy Hackenberg and Martha Allexsaht-Snider deals with development of trusting relations in the in-service education of elementary mathematics teachers. Research on professional development has highlighted the importance of communities and school-based work for promoting teachers' professional growth. The authors claim that despite discussions on school cultures and learning communities in the literature, not much has been said about how to build trust in a developing community. Trust seems to be taken for granted in professional development projects. The chapter presents issues relating to trust in project SIPS (Support and Ideas for Planning and Sharing in Mathematics Education), which is a school-based professional development initiative aimed at helping teachers improve the quality of their mathematics instruction by building a mathematics education community within their school. The chapter focuses on data from the first year of SIPS and discusses factors that helped build teachers' trust in the mathematics educators (see Sztajn, Hackenberg, White, Allexsaht-Snider, 2007 for later years). The authors conclude that Noddings' care theory could help conceptualize and inform other professional development projects that set out with a goal of creating learning communities in schools. The authors suggest further research to find out how trusting and caring relations in professional development are sustained and how they contribute to changes in teachers' practices and increases in students' learning of mathematics? Sztajn has developed her ideas about caring relations in the education of practising mathematics teachers further in later publications (Sztajn, 2008).

Steve Thornton writes about Standards for Excellence in Teaching Mathematics in Australian Schools developed by The Australian Association of Mathematics Teachers. These Standards outline what teachers believe are the characteristics of highly accomplished teachers of mathematics. The Standards also provide a framework against which teachers can be assessed and which can be used for teachers' on-going professional learning. The assessment should be sustainable in that it equips students with the skills and attitudes that will enable them to meet and monitor their own future learning. The chapter describes how the Standards for Excellence were used to develop an assessment methodology in the context of teacher education that has the potential to develop a powerful and robust sense of teacher identity for prospective teachers. The assessment task was included in a course which also contained extensive instruction, discussion, reading and reflection. Steve Thornton suggests further research on the extent to which the developing sense of identity in these students will grow and develop through their career as teachers.

In the chapter by Hans Thunberg experiences from a combined programme for the education of teachers and engineers is reported. The programme is run by The Royal Institute of Technology and The Stockholm Institute of Education in cooperation. From analysis of the profile and the expectations of the student group it seems that this programme to a very large extent has attracted students that wish to work as teachers in science and mathematics. It is claimed that these prospective teachers would have gone for a traditional engineering education in the absence of this new opportunity. Hans Thunberg emphasises that it is important to communicate the goals of the programme as clearly as possible to the students in order to give appropriate expectations. It is also important to structure the programme in such a way that students' double identity as engineers and teachers is strengthened. The programme was evaluated in 2007 but the report is in Swedish. Some suggestions to improve the integration were indicated and the programme is still operating.

The final chapter of the book contains the report from the thematic afternoon on teachers of mathematics, as it is presented in the ICME10 proceedings (Anthony & Grevholm, 2008). We hope that the book will be of interest to the mathematics education community, to teachers, teacher educators, researchers and also to policy makers working with the professional education of teachers.

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Improving Student Teachers' Mathematical Knowledge

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The research results presented in this paper are a small part of an action research study designed to improve student teachers' (trainees) understanding of mathematics. The teaching strategies enacted in the study were similar to those suggested for trainees' future use when teaching children. These teaching strategies involved the use of multiple modes of representation for each concept and operation in the primary school mathematics curriculum. The data indicated that most trainees improved their understanding, but some needed more time to re-learn certain content in the primary school curriculum. This paper proposes some practical solutions to ameliorate the problem.

Shulman (1986) identified several knowledge components which teachers may use in order to make decisions for the purpose of teaching and to help them promote understanding on the part of their students. One of these components is subject matter knowledge (SMK) which includes both the substantive and syntactic structures of the discipline. The focus of this paper is on teachers' and trainees' acquisition of substantive understanding of the mathematics they will teach. However, pedagogical content knowledge (PCK) which includes "the ways of representing and formulating the subject that make it comprehensible to others and ... an understanding of what makes the learning of specific topics easy or difficult" (p. 9) and general pedagogic knowledge or "knowledge of generic principles of classroom organisation and management" (p. 14) are also mentioned.

For Skemp (1976) relational understanding involves knowing both what to do and why it works, while instrumental understanding involves knowing only what to do, the rule, but not the reason why the rule works. My initial experiences as a novice mathematics school teacher in Brazil, and much later my experiences as a teacher educator, led me to think that both mathematics teacher trainees and primary school teacher trainees do not have an appropriate relational understanding of the mathematics content they are supposed to teach. It did not take long, after I started teaching at schools, to notice that I did not have enough mathematics understanding to teach even the most basic curriculum content to my first class of 11 year olds (5^{th} graders). I could present my students with correct procedures, but could not answer most of their questions concerning the reasons for using certain steps in the procedures (Amato, 2004). These experiences led me to undertake a research project with the main aim of investigating ways of helping primary school trainees to improve their understanding of mathematics in pre-service teacher education. The main research question of the present study became: "In what ways can primary school trainees be helped to improve their relational understanding of the mathematical content they will be expected to teach?"

Theoretical Framework and Related Literature

Teacher trainees' inappropriate understanding of mathematics is not a problem restricted to developing countries. Research on teachers and trainees' understanding of mathematics tends to show that they often do not have sufficient relational understanding of the content they are supposed to teach (e.g., Ball & McDiarmid, 1990; Goulding, Rowland, & Barber, 2002). For Carré and Ernest (1993), SMK shapes the way teachers teach and it is too important "to be acquired incidentally through classroom experiences" (p. 50). Ball and McDiarmid (1990) argue that continued documentation about teachers and trainees' insufficient SMK will not ameliorate the problems encountered in teacher education and teaching; research-based methods of tackling the problem in pre-service education are required.

Fennema and Franke (1992) and Bennett (1993) review early studies which investigate teaching practices as a mediator between teachers' knowledge and student learning. The results suggest that there is a positive relationship between teachers' SMK and their instructional practices. One of the most important factors in such a relationship seems to be teachers' organisation of substantive knowledge. The collective message gathered from those studies is that teachers who possessed more organised and interrelated knowledge tended to be more relational in their teaching, while those with less relational understanding tended to base their teaching mainly on instrumental teaching. Students' learning from school may, therefore, be influenced by their teachers' knowledge of mathematics.

Bennett (1993) recommended that primary school teachers should have the necessary subject knowledge for teaching mathematics to the highest level expected of children at that stage of schooling. Teachers should have relational understanding also at a reflective and formal level (Thompson, 1985). According to Ball (1990), this "includes the ability to talk about and model concepts and procedures" (p. 458). Some teacher educators also suggest that the integration between the teaching of mathematical content and pedagogy is beneficial to teachers' and trainees' acquisition of relational understanding of mathematics (e.g., Ball & Bass, 2000; Stoddart, Connell, Stofflett, & Peck, 1993; Weissglass, 1983). They often associate such integration with re-teaching mathematics to teachers and trainees by using the same methods that can be used to teach mathematics in a relational way to school children. According to Stoddart, Connell, Stofflett, and Peck, teachers tend to teach in the way they were taught. Trainees re-learning of mathematics through children's activities may also be a way of avoiding the future reproduction of traditional ways of teaching based on rote learning. Weissglass (1983) proposes that trainees should first experience in practice the pedagogical theories which will be formalised in the future. Brown (1992) argues along similar lines and sees some similarity between the acquisition of mathematical knowledge and the acquisition of pedagogical knowledge. According to her, theory is constructed by the learners "becoming aware of their own actions" (p. 39). The idea that action precedes theorisation was particularly important for the present study which involved helping trainees to develop simultaneously both types of theory (mathematics content and pedagogy).

In the literature about trainees' SMK, there are some results of teacher educators' efforts to improve trainees' mathematical knowledge. Graeber and Tirosh (1988) mention two small scale, short-term interventions designed to help trainees overcome some of their misconceptions related to division. One intervention involved the use of conflict teaching in individual interviews. The second intervention involved the use of two interactive computer programs to help trainees overcome their difficulties with division word problems with a dividend smaller than the divisor. Tirosh and Graeber (1990) also used conflict teaching as a means of probing trainees' misconception that in a division calculation the quotient must be less than the dividend. Tierney, Boyd, and Davis (1990) attempted to change trainees' misconceptions about area by providing them with similar paper activities to those used for teaching children. These were mainly initial activities for finding areas without formulas. Simon and Blume (1994) used problem solving, group work and whole-class discussions to help trainees improve their knowledge of ratio and of the relationship between area and multiplication. Stoddart, Connell, Stofflett, and Peck (1993) used concrete materials and iconic representations in an attempt to reconstruct trainees' understanding of rational number concepts.

The studies reviewed in this paper were concerned with the re-teaching of particular mathematical contents. More long-term interventions involving the re-teaching of several content areas are needed. Chinn and Ashcroft (1993) write about the dynamic interactions of the parts and the whole in the learning of mathematics: "It is a subject where one learns the parts; the parts build on each other to make a whole; knowing the whole enables one to reflect with more understanding on the parts, which in turn strengthens the whole" (p. 3). I think that a strong relational understanding of mathematics (the whole) for the purpose of teaching involves knowing well its content (the parts) and how the content has been put together (the connections). As the trainees could be teaching any primary school grade in the future, they needed to perceive the curriculum as a more coherent and organic whole. Therefore, I decided to help them develop some relational understanding of a wide range of mathematical content in the primary school curriculum.

Methodology

I decided to carry out an action research project (Amato, 2001) with the aim of improving primary school trainees' understanding of mathematics. Action research is said to be "the research method of preference whenever a social practice is the focus of research activity" (Carr & Kemmis, 1986, p. 165). The main research problem, that is, primary school teachers and trainees' instrumental understanding of mathematics, was identified through my experiences in working with pre-service and in-service courses in teacher education.

The study was performed at University of Brasília, Brazil, through a Mathematics Teaching Course Component (MTCC) in pre-service teacher education. This component consists of a semester (80 hour course) in which both theory related to the teaching of mathematics and strategies for teaching the content in the primary school curriculum are discussed. This is the only compulsory component related to mathematics offered to primary school trainees at University of Brasília. There is an optional course component about mathematics teaching (MTCC2), but it is offered infrequently because of a shortage of mathematics teachers. There were two main action steps and each had the duration of one semester. Thus each action step took place with a different cohort of trainees. As the third and subsequent action steps were less formal in nature and involved less data collection, not many results will be reported from the latter.

In the action steps of the research, the re-teaching of mathematics was integrated with the teaching of pedagogical content knowledge (PCK) by asking the trainees to perform children's activities which had the potential to develop relational understanding of the subject. The children's activities performed by the trainees had four specific aims in mind: (a) promote trainees' familiarity with multiple modes of representation for concepts and operations in the primary school curriculum; (b) expose trainees to several ways of performing operations with concrete materials; (c) help trainees to construct relationships among concepts and operations through the use of versatile representations; and (d) facilitate trainees' transition from concrete to symbolic mathematics. Versatile representations such as straws, part-whole diagrams, and number lines (English & Halford, 1995) were often used in practical and written activities to help trainees relate natural numbers to fractions, decimals and percentages. The idea is to represent together two or more related concepts in order to make their relationships clear (e.g., 35 whole straws and 3 pieces of ¼ to represent the mixed number 35¾). A summary of the activities in the teaching program is presented at the end of this section.

In my previous courses, activities involving translations among and within multiple modes of representation and the use of versatile representations were advocated as a way to help children construct relationships among mathematical concepts and operations, but they were not being used with the teachers and trainees often enough. As trainees needed to improve their relational understanding of the content in the primary school curriculum, they also needed to be treated as learners of mathematics. Therefore, about 90% of the new teaching program became children's activities. Having sufficient teaching time was anticipated to be the greatest problem in this research. An analysis of the MTCC syllabus was performed in order to reduce or exclude certain items and increase the time devoted to the teaching of more complex mathematical content. I decided to: (a) reduce the theoretical content of the syllabus and (b) reduce from the program some mathematical content areas such as the teaching of small numbers (zero to 9) and measurement of capacity, mass and time.

Four data collection instruments were used to monitor the effects of the strategic actions: (a) researcher's daily diary; (b) middle and end of semester interviews; (c) beginning, middle and end of semester questionnaires; and (d) pre- and post-tests. The diary included records of my own thinking and of observations made inside and outside the classroom concerning the research question, the strategic actions and the problems encountered during the action steps of the research. The questions in the questionnaires and interviews focused on trainees' (i) perceptions about their own understanding of mathematics and their attitudes towards mathematics, and (ii) evaluation of the activities in the program. The tests involved open-ended questions related to the context of teaching children. For example: "How would you help your students to understand the reason for the result of ¾ x ½?"

The data analysis was mostly qualitative, but a simple quantitative analysis (frequency and percentages) was also used to describe some of the results.

Summary of the Program Activities

The main children's activities included:

1. Using diverse ways of representing mathematical concepts and operations
   - Activities involving translations among and within multiple modes of representation (contexts, concrete materials, pictures and diagrams, spoken languages and written symbols) for most concepts (in arithmetic, geometry and measurement) and operations in the primary school mathematics curriculum.
2. Using diverse ways of performing operations
   - Practical work and discussion about different algorithms for operations with natural numbers (with hundreds, tens and units) in the concrete mode (no symbols are used).

    - Practical work and discussion using concrete materials and symbols to present and consolidate the traditional algorithms for addition, subtraction, multiplication and division with natural numbers.
3. Focusing on mathematical relationships
    - Activities involving translation between *versatile* representations for each concept with rational numbers. These are representations that can be used for two or more related concepts thus allowing their relationship to become more explicit. Some of the representations that are used for natural numbers (e.g., straws, part-whole diagrams, number lines and symbols) are extended to fractions, mixed numbers and decimals in order to highlight the existing relationships between natural numbers and rational numbers.
    - Practical work and discussion using *versatile* concrete materials and symbols to perform algorithms for addition, subtraction, multiplication and division with rational numbers (fractions, mixed numbers and decimals) that are extensions of the traditional algorithms for natural numbers. For example, in all five types of representation the algorithm for division of two natural numbers (e.g., 24 ÷ 3) is extended to the algorithm for division of a mixed number by a natural number (e.g., 24¾ ÷ 3).
4. Paying attention to the transition from concrete to symbolic
    - Formalisation activities. Through systematic questions asked by me, the trainees are asked to look back at their previous actions with concrete materials and symbols (activities 2c and 3b) and verbalise their past actions (e.g., What did you do next with the tens blocks?). The objective is to construct a symbolic algorithm separated from the concrete materials. Each step in the symbolic algorithm is written by me on the chalkboard after each question is answered.
    - Written exercises involving translations from pictures and diagrams to symbols concerning numbers and operations and the inverse translations (from symbols to pictures and diagrams). The trainees are asked to do the last three items of selected children's exercises.

Other children's activities were performed by trainees, but on a less frequent basis included: (a) reading selected paragraphs extracted from children's books about the history of number systems, fractions, and decimals; (b) playing games in pairs; (c) counting forwards and backwards with mixed numbers and decimals; and (d) performing practical work and discussion using concrete materials and symbols (after children's activity 2a), with the aim of comparing two specific concrete algorithms for addition and division of natural numbers and deciding which was the quickest way of finding the solution and why: (i) starting from the hundreds (left to right), or (ii) starting from the units (right to left).

The main teachers' activities included:

1. Listening to an exposition about some ideas concerning the use of mathematical representations from the theories of Jerome Bruner, Richard Skemp, and Zoltan Dienes. I presented these ideas in the first week of the semester and revised them after some related activities by asking questions such as: "Why did I use bundled straws in previous activities and now I changed to Dienes' base 10 blocks?; What is the theoretical principle behind this change?"
2. Participating in methodological discussions about some of the activities and representations. I started the discussion with questions such as: "How can this

activity be adapted to younger children?", "Which relationships can this activity help children construct?" and "Which is the clearest representation for place value: bundled straws or the abacus? ... Why?"

3. Reading and discussing literature about teaching and learning specific mathematical contents.
4. Recording their actions behind the algorithms performed with concrete materials and symbols (after children's activity 4a).
5. Identifying misconceptions in children's work concerning operations with natural numbers.
6. Completing the tests and questionnaires data collection instruments.

Results and Conclusions

The primary school trainees, mainly females, involved in the research had all passed the entrance examination for the University of Brasília and thus were assumed to have sufficient prior instrumental understanding. Some trainees had previously done a vocational teacher education course at school level and were already qualified as primary school teachers. They were seeking a second qualification at university level. There were also a few teacher trainees from other departments for whom the MTCC was not compulsory.

Experiencing Children's Activities

Only a few trainees demonstrated some relational understanding in the pre-tests; mainly concerning addition and subtraction of natural numbers and a few fraction concepts. The pre-test median mark was 10% and the post-test median mark was 70%. The difference in the two medians indicated a considerable improvement in understanding, as judged by the tests. However, one of my main concerns about the new teaching program was the effect on the trainees of asking them, as adults, to perform many children's activities. The data showed that most of the trainees did not mind experiencing children's activities. They appeared to accept it as a normal strategy in a course component about teaching children. Many trainees mentioned that experiencing children's activities had been a positive aspect of the program in that it had improved their understanding of mathematics. For example, "To experience the activities is very positive, as many times the teacher teaches the content to children without having understood it him/herself" or "The way mathematics was presented, through concrete materials and the relaxed way, led us to conclusions not previously understood" [questionnaire].

In an action research project it is also important to describe the unanticipated problems encountered during the action steps (McNiff, 1992). The focus of the next sections will be on the unanticipated problems which were thought to affect trainees' learning of mathematics. Some of those problems were common to both semesters (i.e., the first and second action steps).

Dealing with a More Practical than Theoretical Course Component

Some trainees commented that they were enjoying the MTCC because they were learning mathematics in a different way from the traditional way they had learned it at school. Having said that it does not mean that there were not problems connected to use of children's activities with trainees. While the practical work with concrete materials was welcomed by most trainees, a few found it difficult to adjust to a more practical rather than theoretical course component. They explained:

> We come from other extremely theoretical course components. We present seminars and there is no difficulty in preparing them. However, when we do a practical course component we find it difficult. [First semester interview]
>
> It is difficult to start reasoning in a different way from what we are used to here at the university. It has always been a very theoretical way of thinking. Your way of working requires a more detailed and sequential reasoning. Sometimes it facilitates because it is less abstract, but it is also very different from what I usually do, such as understanding what is written in a book. ... It is a change of schema. However, if it helps us to understand, I can imagine how much it can help a child. Despite all the resistance, I think it is very good. [Second semester interview]

Another trainee explained that part of the resistance was due to having to abandon their comfortable position as spectators:

> This course component is different from the others because there are many materials. We are not used to practical course components so we show some resistance. [Researcher: Resistance in what sense?] Laziness in manipulating the materials, of putting them on the place value board. We complain when we have to change the materials, when we have to change from the materials to the reports [recording their previous actions with the concrete materials, see teachers' activities (d)]. We are used to sitting down and hearing other people talking. [Second semester interview]

Because the majority of the trainees would soon be teaching and using a more practical way of thinking, their difficulties in dealing with a more practical course component was not thought to be a problem to be solved. In fact, the practical work involved in the MTCC was considered to be one of the main solutions for improving their relational understanding of mathematics. Most of the trainees seemed to have had a very symbolic and instrumental type of teaching at school. Very few of them had worked with less symbolic representations for the mathematical concepts and operations they would have to teach. They needed to learn how to use other forms of representations in teacher education.

Trainees' Difficulties with Certain Mathematical Representations and Content

On the other hand, there were important problems to be solved. A trainee commented about the adult thinking being different from the child's thinking: "I enjoyed the pleasure of working with the concrete materials, keeping in mind their function for a child's learning." However, although we urged trainees to adopt a child's thinking process, an adult's thinking can't be exactly the same as the child's thinking". Indeed, the trainees' thinking, when performing children's activities, was affected by their previous experiences in learning instrumental mathematics at school. The selection of representations to be used by the trainees in the children's activities was based on two pedagogical criteria: (a) clear embodiment of concepts; and (b) versatility. However, some trainees also presented difficulties in learning about multiple representations for certain mathematics content. The most difficult content areas for the trainees were: (a) multiplication by two-digit numbers (e.g., 23 x 48); (b) the distinction between the sharing and the measurement interpretations of division; and (c) operations with fractions and decimals.

Decisions Made During and After the First Semester

The sequence of children's activities in the teaching program was designed so that trainees always experienced practical work with concrete materials or measurement instruments before experiencing activities with iconic representations and activities with only symbols. However, the trainees knew all the symbols in the primary school curriculum and were familiar with most of the symbolic algorithms they were asked to perform with concrete materials. They were, in fact, being asked to translate from symbolic to concrete representations as they already had instrumental understanding of the mathematical content in the program. During a whole class discussion about how to use concrete materials to find a common denominator to add two fractions, Carlos commented: "The child will be learning like that but we are unlearning and re-learning. Everything is too abstract at school." Maria continued by saying: "We are so used to doing things in the abstract that now we are having problems in visualising them in the concrete." In the interview, Daniela mentioned that she was having problems in re-learning mathematics with the use of concrete materials and explained: "I already have everything in my head. Perhaps what I have in my head is easier. I do not have to do much, there is no loss of time."

These trainees' memorised symbolic ways of performing the operations seemed to be interfering with their understanding of more informal representations for these operations. Leandro said that the procedure for dividing two fractions kept coming to the surface whenever he was performing division of fractions with concrete materials. On the other hand, Angela thought she did not have any problems in learning any of the operations with fractions using concrete materials because she had forgotten how to do them with symbols: "I did not remember that I had to multiply by the inverse fraction in division. It was like learning division that day. The things that I had forgotten with symbols I could learn more easily with the concrete materials." At the end of the first semester Angela said that she had not had any problems with most of the representations used in the program. Her only exception had been the understanding of the area representation for multiplication by two-digit numbers (Sugarman & Steward, 1994). She commented that her difficulties were related to the way she had memorised the algorithm by rote, making no relationships with the place value of the digits: "I would verbalise a sum such as 38 x 47 by saying 8 x 7, 8 x 4, jump a place, 3 x 7 and 3 x 4. I did not interpret the three in the tens' place as 30 times and the four as 4 tens or 40."

Both theory and research results tend to support the idea that relational understanding should precede symbolism and automaticity of procedures (e.g., Chinn & Ashcroft, 1993; Hiebert & Carpenter, 1992). The use of concrete materials and iconic representations are more profitable in the earlier stages of concept acquisition, by providing a useful starting-point for the development of relational understanding and abstract thinking. Hiebert and Carpenter (1992) suggest that after achieving automaticity learners become more reluctant to connect their symbolic procedures to other mathematical representations that could provide further links to relational knowledge. They relate this to a phenomenon called *functional fixedness* by Gestalt psychologists where the steps in a procedure may become firmly connected and fixed in the learner's mind, not allowing a more flexible way of thinking about them. Besides, concrete materials and iconic representations carry with them interpretation difficulties inherent in a representational system. They involve visual conventions and, if the learner does not know the conventions, (s)he cannot interpret them (e.g., Hiebert & Carpenter, 1992; Shuard & Rothery, 1984). In order to help in the construction of relationships any type of representation needs to be become familiar to the learner.

Decisions made: Primary school teachers have the social responsibility of helping children learn mathematics. They must develop the ability to work backwards from their symbolic ways of representing mathematics to more informal ways of representation (Ball & Bass, 2000). Representations that proved to be difficult for some trainees, such as the area representation for multiplication by two-digit numbers, are considered to be useful in helping primary school students to understand the multiplication algorithm and in teaching dyslexic students (Chinn & Ashcroft, 1993). Such representations can also help students to make important connections between arithmetic and algebra (Ball & Bass, 2004). It was more appropriate to look for ways of helping trainees learn and be fluent in using these representations than excluding them from the program. The activities for difficult representations and content were started earlier in the second semester. More children's activities were planned for the content that proved to be more difficult for trainees.

Decisions Made During and After the Second Semester

Although the teaching program was modified from the first to the second action step, trainees' difficulties with certain representations and content were still evident in a few cases. Some trainees needed a few activities to understand the content and construct relationships while others needed considerably more activities. Mariana said that the teaching pace during addition and subtraction of fractions was too fast for her. Alice said in the interview that she needed to re-learn fractions at a similar pace provided for children's learning: "I started to understand much more about fractions. However, I have difficulties and I need much more time, to the same extent as a child." Many trainees suggested increasing the teaching time for operations with rational numbers because fractions and decimals were much more difficult for them than place value and operations with natural numbers. Some trainees also suggested increasing the number of activities for geometry and measurement because they were very enjoyable and they had not had enough experiences with these contents as school students.

Decisions made: The trainees who were having more difficulties were advised to seek extra help provided by me and two teaching assistants. The number of children's activities for representations and content that proved to be more difficult for trainees in previous semesters was further increased in the third and subsequent semesters. The activities for rational numbers, geometry and measurement were started at the first week of the semester and they continued throughout each semester. The aim was to provide trainees with multiple opportunities for revising content through activities involving extensions of the content and relationships with other contents.

The number of activities for place value and operations with natural numbers was reduced, but there were still many activities about operations with rational numbers which included a natural number part. Through operations with mixed numbers and decimals (e.g., 35¾+26¼ or 24.75-12.53) trainees experienced further activities related to operations with natural numbers and had the opportunity to make important relationships between operations with natural numbers and operations with fractions and decimals. These changes proved to be effective in helping trainees overcome their difficulties in relearning mathematics relationally within the time available. Taking into consideration the time necessary for a practical approach in teaching large classes, a more appropriate solution would be to offer the MTCC over two semesters with a total of 160 hours.

Discussion

The decision to ask trainees to experience children's activities much more often than had been the case in previous courses was thought to be appropriate. It did not cause any motivation problems for trainees as adult learners. On the contrary, the majority of trainees said that they had enjoyed using children's activities. Although the games and practical activities were time consuming and hard work with large classes they proved to be an appropriate strategy to improve trainees' relational understanding of mathematics as indicated by the post-tests results of improved relational understanding. The strategic actions and teaching activities in the program did not require any changes in nature. Rather, changes involved quantitative and timing adjustments, and were made for the second and subsequent semesters to maximise trainees' learning during a single semester.

Teachers' ability to translate SMK into mathematical representations is considered to be an important part of teachers' PCK (Ball, 1990; Shulman, 1986). It was necessary to help trainees draw clear connections between the symbolic ways of representing mathematics they had in their minds before starting the course and other ways of representing mathematics so that multiple representations could be incorporated in the same schema. Providing trainees with children's activities involving translations among and within multiple modes of representation helped them: (a) improve their own relational understanding of mathematics; and (b) learn an important form of PCK in a tacit way. Acquiring a repertoire of representations and activities that can be transformed by the teacher for classroom use is an adequate and initial form of PCK for a course component about mathematics teaching in pre-service teacher education. Although it is a very basic form of PCK, knowledge of representations was also thought to be the most appropriate knowledge in order to foster trainees' initial feelings of success that would be needed to continue their learning from teaching mathematics. With time and teaching experience trainees would be more able to use such knowledge in combination with more sophisticated teaching strategies.

Some teacher educators believe that working towards developing teachers who are autonomous and seek study groups and other means of learning and growth is incompatible with the idea of learning about SMK and PCK through formal instruction in pre-service teacher education. On the contrary, my own experiences as a novice mathematics teacher, and the ideas about the relationships between knowledge, democracy and autonomy elaborated by social theorists such as Gramsci (1998), led me to think that trainees' acquisition of SMK and PCK in pre-service teacher education is an important precondition for their future autonomy as teachers. My professional autonomy as a novice mathematics teacher was, in many moments, hindered by my instrumental understanding and by my insufficient knowledge of appropriate representations to deal with my students' difficulties.

Novice teachers face many constraints and challenges at the beginning of their careers (e.g., Sullivan, 2004). Classroom settings can be quite stressful for novice teachers whose pedagogical thinking appears to be dominated by concerns of classroom management (e.g., Haggarty, 1995). Learning some initial ideas about PCK was thought to be more easily achieved from trainees' efforts to re-learn mathematical content through strategies which were new to them and from trying to understand how those strategies have helped them to develop relational mathematical knowledge.

Learning some SMK and PCK from my own teaching experiences and from other teachers proved to be a very slow process. It took me a long time and a great effort to acquire some relational understanding and PCK while teaching several large classes simultaneously. Learning mathematics from teaching also seems to be a slow process for primary school teachers, as they have to teach several subjects simultaneously. I think that trainees must acquire in pre-service education SMK and PCK of an adequate level to face the responsibility of providing effective learning experiences to all school students from the beginning of their teaching careers. When teachers find the time to work together in study groups they should be discussing complex problems related to their practice and to their students and not dedicate their precious time trying to acquire SMK and PCK of the mathematics they teach. Such knowledge I consider should be a basic part of their professional knowledge and so the responsibility of pre-service teacher education. An initial knowledge base which I think it is a combination of a strong relational understanding of mathematics (SMK) and knowledge of a repertoire of representations (PCK) must be developed within appropriate programs in pre-service teacher education.

More general theoretical knowledge has been the main focus of teacher education courses at University of Brasília. I believe that the discussion of teaching strategies and materials for teaching particular mathematical content may be interpreted by some academics in a negative way and as providing trainees with recipes or procedures. However, I think that novice teachers' reflections on what happened in the classroom are more important than where teaching strategies and materials they use come from. Slowly they can start combining the ideas gathered from textbooks and from teacher education with their own ideas. Teachers need to be creative, but it is important to know some of the teaching alternatives developed in the past in order to make useful adaptations and informed choices while planning and teaching particular mathematical content. In teacher education it does not seem to be a good idea to tell trainees that their conceptions about teaching acquired during their long experiences in learning mathematics at school are inappropriate. Rather, they must be provided with new experiences to be able to form their own opinions about what may be better.

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From Structures to Stories: Understanding the Experience of a Flexible Teacher Training Route

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Flexible training programmes have been introduced by the Teacher Training Agency in England and Wales to try to increase the recruitment of mathematics teachers. The course at Sheffield Hallam University parallels the full-time one year postgraduate route but the very different mode of delivery has allowed more flexible pedagogies to be developed. The students have opportunities to craft their own learning experiences and in making choices they become aware of their own needs and progress. It seems unlikely that the route will make a significant quantitative contribution to the problem of teacher shortages but perhaps these mature and well prepared teachers will stay longer in the profession?

The shortage of teachers in many developed counties (e.g. England, USA, Australia, and The Netherlands) has led to governments encouraging new routes into the profession. There are now a plethora of such routes in England and Wales which vary; in their cost to the Teacher Training Agency (TTA), their ease of organisation for universities and partnership schools, and more fundamentally in the nature and length of preparation that the beginning teachers experience. I have worked with students preparing to be secondary school mathematics teachers on full-time one and two year post graduate routes, the flexible postgraduate route, two and three year undergraduate routes and a school based training route. In this paper I will draw on the work I undertook for a Masters dissertation (Angier, 2004) in which I used a narrative methodology to explore the learning experience of students on the flexible mathematics route for which I am their academic tutor.

The Flexible PGCE

The flexible Post Graduate Certificate in Education (PGCE) was introduced by the Teacher Training Agency in 1999 as one of a series of measures to increase the supply of teachers. The route is flexible to students' needs and circumstances. It allows exemption from parts of the course where prior academic study and/or relevant teaching experience can be evidenced. The route is able to take account of students' circumstances by allowing them to set their own pace, complete the different parts of the course in any order, and organise their school placements to fit around constraints such as childcare or paid employment.

Students in the mathematics cohort fall roughly into three categories: those who have child care commitments, those who need to maintain some paid employment because they cannot afford to live on the training salary and those who live too far away from the university. In all cases they are unable to attend a full-time course. In total 26 students have enrolled on this route. Seven have completed, four have withdrawn, ten are active and making progress and five are 'dormant'. I took responsibility for the cohort a year after the first five students had enrolled and have therefore worked with all students except three students who withdrew before I met them.

The flexible PGCE at Sheffield Hallam has the same entry requirements and components as the full time PGCE. Students must have a degree in mathematics or a mathematics related subject. They study the same academic units in subject pedagogy and education (now replaced by general professional studies). They undertake the same statutory requirement of 120 days school based practice and must pass the three online tests in numeracy, literacy and ICT set by the TTA.

The flexible PGCE however is delivered completely differently. Whereas the full-time students have a set programme, must attend teaching sessions in the University and submit work for assessment on certain dates the flexible students have none of these constraints. They work through their study units at home and can complete them in any order they choose. They decide when to submit assignments. They can negotiate the timing and to some extent the organisation of their placements. Once a term the mathematics cohort is invited into the University for a Saturday workshop. Apart from these workshops students maintain contact with their tutors, and each other, through an email system and a web based learning environment.

Teacher Education Research

The motivation of teacher educators to research and make sense of the process of becoming a teacher is not necessarily in line with that of government departments whose prime aim is to ensure schools are staffed.

> The pressure on teacher education due to the shortage of teachers leads to a situation of pragmatism where there is no room and time for critical reflection, careful consideration and balanced quality. … There is a strong need for research…The ministry is mainly motivated by the need of short and flexible routes, taking into account prior experiences and schooling, while the institutions are mainly motivated for the development of flexible and individual learning routes by the wish to make students self responsible and capable of managing change. (Snoek & Wielenga, 2001, p. 42-43)

There is some evidence from the United States that the length and nature of pre-service teacher education does affect both the attainment of students in school (Darling-Hammond, 2000) and the beginning teachers' perception of their own preparedness (Darling-Hammond et al, 2002). Whilst there is little comparative research into different routes into teaching there is a vast learning-to-teach literature. One of the most comprehensive reviews of the teacher education literature was undertaken by Kagan (1992) who found a number of common themes one of which was the issue of identity.

> In sum, knowledge of self, classrooms, and pupils does not appear to evolve separately. In this sense, a novice's past and present experiences are ultimately merged, as professional growth encroaches on the novice's most intimate knowledge of self. (Kagan, 1992, p.148)

This theme of identity has continued in the research literature where the frequent use of case studies and personal histories reflects the view that becoming a teacher is an intensely personal journey during which a shift in identity occurs that then has to be reconciled with a person's past and present.

> When teachers talked about their work, they also talked about themselves; the events were filtered through the person of the teacher. (Hasu, 2002, p.11)

Other research has attempted to weave together the students' identity work and the pedagogical stance of the teacher educators. Korthagan (2002) working within the context of 'realistic mathematics education' describes the process of teacher education as highly personalised experience in which the students are guided to reinvent theory for themselves so that no 'gap' is ever allowed to appear between 'theory' and 'practice'. The teacher educators have a clearly defined theoretical framework from which they have crafted a structured course which responds to the diversity of the beginning teachers' life experiences.

Olsen (2003) who worked for two years with four beginning teachers experiencing four different teacher education programmes concluded, ”learning-to-teach is simultaneously an individualized biographical process, a product of sociocultural constructions and constraints, and the result of generalisable epistemological principles” (p. 2). These three strands resonate with the wider teacher education literature and provide a framework within which to undertake and evaluate research. In my work with the flexible cohort I have focused on the first and third elements.

Korthagan and Olsen both suggest a way of understanding teacher education which I recognise from all my teaching experiences; that the teacher, whilst attending to the background and environment in the widest possible sense of her students, has the potential through her decisions and activities to enable transformative learning. I choose to explore a narrative methodology and write stories about each of my students.

Narrative research is a rich but problematic area of the literature. The telling of other people's stories and the description of their developing identities is fraught with methodological and ethical issues. Stories are very powerful, dangerous even, not least because it is very difficult to know on what grounds they can be judged as 'true'.

> At this point it is important to remember that 'truth' means not only 'corresponding to the facts' but also 'trustworthy', and that the term 'fiction' is connected with the idea of being 'shaped' or 'moulded' (from the latin 'fingere'). This enables us to address the complexity of the relationship between truth and fiction by rephrasing our question. Not 'is this narrative 'true'?' but, 'is this narrative shaped and moulded in such a way that we feel it is trustworthy, i.e. does it *persuade* us that we might helpfully rely on the insights it presents about that particular situation to guide our thinking about other situations?' (Winter, 2002, pp. 144-145)

Writing about my students reminded me of composing reports as a school teacher where the process of reflecting and writing about each child gave me insights both into their learning and my teaching. In many instances, now, just as then, I have realised that the story is not quite working out as it should. There are flaws and gaps and I am aware that I have misunderstood and made mistakes.

> It is in the knotty points and moments of disagreement and unpredictability that we gain insights into each other and ourselves and generate the spaces and intersections that are simultaneously uncomfortable and yet satisfying and productive. (Nixon et al., 2003, p. 93)

I think the power of narrative research lies in the potential to attend to the individual and the local, whilst carrying larger themes which we might recognise as theory. The stories are not themselves the knowledge or the theory nor are they the raw data. They are a resource to be drawn on rather like a rich mathematical task which is known by a teacher to engage her pupils and enable them to deepen their understanding and construct new knowledge. In this context I am not able to give the reader that resource but only to report how these stories have enabled me to understand my students' experience better.

> This search for a different kind of knowledge, knowledge which empowers rather than making possible prediction and control, is a significant re-conceptualisation of the purpose of educational research. (Elbaz-Luwish, 1997, p. 78)

My aim in working with their stories was to explore the interaction between their life experiences, the structure of the flexible PGCE, and my decisions as a teacher educator. I wanted to find out how this unusual route determines the space within which students can construct their new identity as teachers. I focus here on what emerged as four key characteristics of this flexible PGCE.

Saturday Workshops

The Saturday workshops provide an unusual learning space for the students because the whole cohort meets together and everyone is at a different stage of the course. I chose to work this way for pragmatic reasons so that I could avoid working only with two or three people at a time and it has turned out to be very beneficial. The days consist of a variety of mathematical and pedagogical activities. Working on mathematical tasks together allows me to model different teaching styles and provides the students with a limited but important opportunity to reconsider their own experience of learning mathematics. I make use of the fact that the students are at different stages of the course. For example, a student may give an account of their placement. Students who have not yet undertaken this part of the course can interrogate the presenter whilst students who have completed a placement can offer their perspectives. Because the experience is relatively recent the post-practice students are able to make sense of where the pre-practice students are 'coming from' and they become more aware of how their own ideas have changed and developed. They are experiencing a learning environment which has some characteristics in common with a mixed attainment classroom. The students come to the pedagogy tasks with different prior experiences which they share to help them understand key issues such as lesson planning and differentiation.

The students make a lot of decisions about their own needs. They spend a lot of time during Saturday workshops comparing and contrasting their progress and questioning each other. There was much talk about Usman's difficult mentor and David applying for a job before he had even begun his first practice. They become aware of each other's priorities and take seriously the task of supporting each other. The cohort has, not surprisingly, developed socially. Lynne grew up and still lives in an all white community and struggled with the parts of the pedagogy units which addressed race and schooling. Becoming part of a very small multi-ethnic cohort has given her a gentle opportunity to get to know and work with students from diverse cultural backgrounds. Sarah is a conscientious and under-confident student. She has chosen the flexible route because she wants to fit her study around the needs of her two small children. Sarah chose to keep a diary throughout her first placement. It was a very detailed and personal account of her responses to the challenges she faced. She was not asked to do this but I think the experience was so overwhelming she needed a mechanism for clearing/storing each day's thoughts. When she fed back her experiences to the cohort during a Saturday workshop she brought her diary and invited the other students to read it.

E-learning Tools

This route would be very difficult to manage without the aid of web based tools. Students are given access to an email system which enables them to contact their tutor and each other. An email conference has been set up for the cohort where public messages and announcements can be posted and at several points in the study units the students are asked to post responses to activities onto this conference. With such a small number of students it is very difficult to stimulate and maintain discussion. As student numbers have risen gradually the conference has started to be used more often. Meeting each other at Saturday workshops may well have helped and there have been some examples of very interesting asynchronous discussion in response to the study units.

The conference site has been used for many different purposes by the students; telling their personal news, giving updates on their progress, describing job applications etc.

Bob decided to post a regular weekly update of his placement on the email conference. By the end he had posted six sides of A4 detailing his problems and successes and how he was making sense of them. No one had asked to him to engage in this public reflection but he clearly felt comfortable to use this virtual space to offer a very honest story of his classroom struggles.

The most drastic decision a student can make about their own needs is that they should leave the course as David did. I asked him to post a brief message on the email conference to let the other students know his decision as I did not feel I should speak "for" him. This prompted three replies the last of which I then responded to. The first two were from Usef and Jane who made it clear in their replies that David's decision had caused them to rethink. I suspect that the same was true for some of the other students who did not post responses.

> Hi everyone
>
> Maybe it wasn't your cup of tea, but I'd like to say this to the rest of the group: just thought I'd write something about my experiences.
>
> I'd start with WOW. I love maths, I love explaining maths, I remember why I liked maths; being able to understand and appreciate concepts … amazing … I want other children to be able to gain this … have fun.
>
> Fun doesn't mean you crack a few jokes, fun means finding learning enjoyable … this is what teachers need to achieve … I think now that the best quality a teacher should have is the love of their subject.
>
> I've enjoyed my lessons at my current placement. Some classes have been challenging … behavioural wise … but that's where my development lies.
>
> Actually today I was talking to some kids from year 7 I haven't even taught and guess what? They said "we've heard you are a very good teacher, they actually gave me the impression that they would be sorry to see me leave.
>
> I've never really thought about this … but I think I like kids…
>
> When I started the placement … truthfully my stomach turned … I was nervous … but now I think I'm gonna miss W High
>
> I hope the rest of you are enjoying it as much as I am…   Usef
>
> Hi Usef,
>
> I'm so glad you submitted this to the conference. I must admit when I read David's very eloquent explanation for leaving the course, it did make me question what sort of person he thought teaching might make him, and would I become that person too. Having read your words however, and related them to my own teaching placement, I can say with confidence that I am doing the right thing and happy to be doing it.
>
> I agree with your opinion that love of your subject is important. On my placement, the biggest surprise came from the amount I enjoyed the maths. I loved rediscovering work I hadn't even

thought about for years, and finding ways of teaching it that I hoped the kids would respond to. I got really excited by a neat way of doing things like converting recurring decimals to a fraction. The best feeling was when some kids said of a subject I had just taught "oh yeah, I've never got that before." Thanks, Jane

I have begun to explore and evaluate the use students make of e-learning tools as part of a project funded by the TTA. One of the conclusions I am quickly drawing is that as the tutor I need to be actively involved and very visible. This may seem overbearing or anti-democratic but it seems to be necessary just as displays on a classroom wall need to be managed and changed regularly. Having such a small number of students, all at different stages I am beginning to realise how important it is that I keep the story going and act as a narrator linking the characters and providing a sense of the journey moving on.

School Placement

Many of the students have negotiated non-assessed school experience where they are essentially volunteers. This time contributes to their statutory 120 days in school but is not part of a formal placement. Working in school without the pressure of being assessed is I think a very valuable experience. It enables different relationships to develop with staff and it provides a safe space for trying different approaches to teaching. Before his first block practice Bob took advantage of his flexi time system at work which, along with the use of some holiday time, enabled him to take one morning a week off. He arranged to visit a school as a voluntary classroom support. This resulted in him working regularly with a small group of disaffected boys who were causing classroom management difficulties. Bob's attitude towards the boys and his understanding of their needs in school has changed significantly over the time that he worked with them. He was influenced by the reading he had been doing for the course and the discussions he has had with fellow students and the staff within the school. Visiting as a non-assessed volunteer one morning a week is very low key compared with the pressures of a full-time PGCE placement but it was a rich learning experience. It gave him some space within which to make sense of young people's attitudes to schooling. Bob grew up in a very poor household and understood that success at school was the route to a secure future. But he did not find school an easy environment and is sympathetic to young people now who don't. He has been challenged and frustrated however by their lack of aspiration or any sense of their own possible futures. Bob is trying to reconcile his intellectual understanding, his own personal history, and his classroom experiences.

Sarah struggled with classroom management on her first placement and when I observed her I found her very distant from the pupils. As she neared the end of the placement she decided that there was one Year 10 class that she had particularly benefited from working with. She had developed a good relationship with the students, and the class teacher was giving her support that she felt helped her make progress. Sarah arranged to continue teaching just this group after her formal placement was over until their next public examination. I was deeply impressed that she had negotiated an arrangement that would allow her to enjoy, and bring to a more natural close, a good relationship with a teaching group. She had a strong understanding of why this was worthwhile. Sarah knew that developing productive relationships with students was a challenge for her and chose to spend more time in the place where she felt she was doing well.

Extended Time Period for Teacher Preparation

Students who opt for the full-time PGCE route complete their course in ten months. This includes 24 weeks in school and four academic assignments. It is almost impossible for these students to engage with cognitive dissonance that arises when classroom experience does not match their expectations. They need to develop a stance, which is often 'borrowed' from the school they are placed in, which will see them through. Students on the flexible route are allowed up to four years and whilst it is too soon to give a meaningful average time to complete it is likely to be at least 18 months. It takes time to engage with all the different discourses through which teaching and learning can be analysed and understood. Students on the flexible route are asked to undertake a considerable amount of reading in place of taught sessions. They build up a rich resource to draw on as they try to make sense of their teaching experiences. It also takes time to understand pupils. Kagan (1992) identifies acquiring knowledge of pupils as a key theme in teacher development.

> Student teachers approach the classroom with a critical lack of knowledge about pupils. To acquire useful knowledge of pupils, direct experience appears to be crucial, particularly extended opportunities to interact with and study pupils in systematic ways. … It is a novice's growing knowledge of pupils that must be used to challenge, mitigate, and reconstruct prior beliefs and images. (Kagan, 1992, p. 152)

Discussion

Having worked with the stories I have become aware that the flexible route is more than just the full-time route delivered by distance learning. It embraces a flexible pedagogy where the students have a great deal of choice and the opportunity to craft their own learning experiences. This spaciousness does not however suit all students and those who have withdrawn or become dormant might have completed the more focused ten month PGCE. The need to provide more structure in terms of tracking and deadlines for some students has been one of the research findings which has influenced the future development of this route.

The flexible route seems to predispose the beginning teachers to offer storied accounts by giving them so much responsibility to assess, and organise to meet, their own learning needs. It begins with discussions before interview about the prospective student's circumstances in which I learn all about their families, their finances, their past experiences and their plans for the future.

> Too often we look at teacher education as separate from the ongoing lives of teachers and student teachers. We pull out the years of teacher education to examine them. In so doing, we separate teacher education experiences from the pasts and futures of our student teachers' lives. We do not create spaces to acknowledge either the ways they have already written their lives prior to teacher education or to the ways they continue to live their stories in the context of teacher education. (Clandinin, 1992, p. 124)

I think the students experience a blurring of the boundary between "this is what I need to do for me' and 'this is what I need to do to complete the course". Having begun to articulate their own relationship with learning, beginning teachers are in a position to consider that their peers may be very different. By establishing a vertical cohort the flexible mathematics PGCE makes these differences more explicit and recasts them as an advantage to the learning community. This is a new model to draw on when they are working in school classrooms. The flexible route allows this meta-cognitive process to be taken one stage further by offering the students some scope to organise their own learning experiences. This is a model not just for their school classrooms, but also for their own ongoing professional development.

It appears at first sight that the highly personalised and individual flexible route could not be as cost effective as the streamlined intensive full-time route. What we cannot take into account because the analysis is yet to be done is the long term contribution these beginning teachers will make to the profession. If those who have had more time and space to prepare for teaching turn out to be those who sustain a long career and become curriculum leaders, whilst those who are rushed into the classroom are found to be most likely to rush back out again, then we would need to rethink our strategies for increasing the supply of mathematics teachers. There are features of the longer and flexible routes into teaching which enable beginning teachers to deconstruct themselves as learners and gradually reconstruct themselves as teachers. In doing so I believe they will have acquired skills and dispositions which will equip them to anticipate and enjoy the complexities and challenges of teaching.

Acknowledgement

I would like to thank Hilary Povey for her help and support in writing this paper and for her supervision of my Masters dissertation which it derives from.

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Professional Learning for Newly Qualified Teachers of Mathematics

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For newly qualified teachers the first few years in the classroom is of vital importance to their professional learning and identity formation. In recognition of the importance to ongoing learning, many education systems provide induction programmes to support the transition phase into the classroom. This report, part of a larger study, reveals the complex learning systems that a group of New Zealand teachers of mathematics needed to negotiate as part of becoming a teacher in the secondary school system. Importantly, we see examples of how the modes of support and guidance variously afforded or constrained newly qualified teachers' learning to enact reform mathematics classroom learning environments.

Introduction

The professional formation of mathematics teachers evolves over a continuum involving initial teacher education (ITE) and the induction period (Feiman-Nemser, 2001; Watzke, 2007). For newly qualified teachers (NQTs) the quality of their professional learning experiences is a crucial influence on the sort of teacher they become (Ingersoll & Kralik, 2004). In addition to providing the NQT with the benefit of expertise—be it a form of intellectual, social, emotional or material resource—the professional community is charged with providing learning spaces and opportunities for the NQT to engage in "serious and sustained professional learning". These opportunities can occur both within a formal induction programme, and within informal arrangements in schools (Little, 2003) or external to a teacher's immediate workplace (Hansen, Haigh, & Ashman, 2003).

Internationally, efforts to support the professional learning and development of newly qualified teachers have seen the growth of induction programmes. Drawing on a recent review of effective induction programmes, Glazerman et al. (2009) claim that comprehensive induction comprises an array of aligned and integrated components which include: carefully selected and trained mentors; a curriculum of intensive and structured support and professional development opportunities; regular meetings with mentors; opportunities to observe experienced teachers; formative assessment tools that permit evaluation of practice; and outreach to wider educational support. We know also, from research by Carver and Feiman-Nemser (2009), that policy documentation that "sets conditions for how induction support is practiced by mentors, and experience by novices" (p. 314) is important and that support should be provided across at least the first two years in the workplace.

However, currently there is considerable variation—both within and between countries—in the professional learning induction programmes and opportunities for beginning teachers. Kardos and Johnson's (2007) recent study of 486 beginning teachers in the U.S. reported that large numbers of teachers work as solo practitioners, and are expected to be prematurely expert and able to work without the support of school-based professional networks. In contrast, newly qualified secondary teachers in New Zealand are supported by a mandated induction programme. A funded 0.2 time allowance in the first year, and 0.1 for the second year, is used to facilitate NQTs' professional learning, provide mentorship, and support their planning and preparation. Suggested elements of the programme (Ministry of Education & New Zealand Council of Teachers, 2006) include: professional discussions, systematic goal setting for teaching and student learning, professional reading time, planning and resource appraisal and development, evaluating student work, professional learning and development activities (e.g., classroom observations of colleagues), and self reflection.

Based on a subset of interviews from a longitudinal national project *Making a difference: The role of initial teacher education and induction on the preparation of New Zealand secondary teachers* (Anthony & Kane, 2008) that explores graduating secondary teachers' experiences of their ITE and their induction, this paper examines the learning experiences and opportunities afforded 15 mathematics teachers[1] within their first year of teaching. Data is drawn from two interviews. The first interview (#1), conducted 6 months after commencing teaching in the classroom, focused on NQTs' views of their ITE preparation, their experiences as a teacher, and their induction experiences. At the end of their first year of teaching, the second interview (#2) focused on their continuing experiences as a teacher, their induction and professional learning experiences, and their career plans.

Conceptual Framework

Findings are framed in relation to the sociocultural literature, with its orientation toward joint enterprise, the centrality of participation and resources, and the notion of trajectories of learning (Wenger, 1998). From a sociocultural perspective, the specific interactions and dynamics of the professional community of the school constitutes an important contributor to a NQT's development (Wilson & Berne, 1999). Kardos and Johnson (2007) note the importance of the professional culture—"the established modes of professional practice among teachers; their norms of behavior and interaction; and the prevailing institutional and individual values that determine what teachers do and how they do it" (p. 2086). From their research studies they found that NQTs are more likely to stay in teaching when they perceive their schools to be places that promote frequent and reciprocal interactions among staff across experience levels, recognise new teachers' needs as beginners, and develop shared responsibility among teachers for the school and its students.

The initial year of teaching is an important phase in any teacher's professional growth. When a NQT enters their own classroom they experience and learn about the complexity of being a teacher—and they find a professional place within the school culture (McCormack, Gore, & Thomas, 2006). In addition to developing a professional identity (Devos, 2010), they need to experiment and construct their own professional practice. Feiman-Nemser (2001) proposed Central Tasks in Learning to Teach (CTLT) for the induction period include: learning the context, designing responsive instructional programmes, creating a classroom learning community, enacting a beginning repertoire, and learning in and from practice. In this paper, we use the CTLT tasks to consider how the specific interactions and dynamics of NQTs' induction experiences provided a resource for learning for the mathematics teachers in their first year of teaching.

[1] Eleven teachers teach only mathematics, and four teach mathematics at the junior level only with responsibility for another subject area.

Teacher Learning

Despite national implementation guidelines (see MoE & NZTC, 2006) and the provision of a NQT time allowance, the teachers in this study reported access to variable induction experiences and support for their professional learning (Anthony et al., 2007).

Learning about the context

In the first 6 months beginning teachers' time allowance was largely used for learning about the context—students, curriculum, and school community. Initially, the extent of this new knowledge seemed, for some, quite overwhelming:

> You don't know the names of any staff, you don't know any of students, you don't know the background of any of your students, you don't know what sort of behaviour to expect from them, you don't know anything about their ability, you don't know anything about their disabilities, that's the big one. (T204#1)

Information about school policies and procedures, reporting to parents, and information related to students was frequently shared in scheduled meetings with other NQTs led by a senior teacher. By the end of the year, the teachers reported a certain familiarity with procedures, and many noted that they had learned to be more patient and tolerant, and sort the 'big stuff' from the 'little stuff':

> I have had to learn about a boundary of reasonableness like I'm a structured person and I like the rules to be consistent and if you are late you are late…I like things to be quite black and white and I have had to learn that I can't be, so I have to work out how late is late enough to give the home detention and how late is late enough to say you are late. So that is something that I have had to learn throughout the year. (T790#2)

Designing a Responsive Instructional Programme

Designing responsive instructional programmes Feiman-Nemser (2001) explains as "the ability to bring together knowledge of content and knowledge of students in making decision about what to teach and how to teach over time and make adjustments in response to what happens" (p. 1028). Addressing diversity and setting realistic expectations based on their developing knowledge of students proved an ongoing challenge for many teachers, with several referring to the need to provide a more structured experience for students.

> …making the work accessible to them. I've had to re-think and probably go back to quite tradition ways of teaching because they find that more accessible, sort of quite processed-based learning. Almost like a formula you can follow to get success seems to be the way they absorb things easier. (T202#1)

However, by the end of their first year most teachers indicated an increased confidence in their ability to recognise and meet the diverse needs of their students in their programmes and through their teaching strategies.

> I feel that I am making a difference with my year 10s. They are a top band group and I really feel like I am extending them…I really like playing with them—'why does this work'—not just this is what we do, but the idea behind it, and their eyes light up and they think through it. (T790#2)

Comments indicated that as the teachers spent time with their students and watched and listened to their students' responses to their teaching, they continued to make adjustments to their programmes and tailor their teaching strategies to maximise learning.

> One of the biggest things I have had trouble with getting to grips with particularly is the way in which they learn mathematics and the level; what students are capable of at different levels. That's something to some extent textbooks and prepared material helps with because you can see the way other authors have organized the work. But until you start teaching the classes and seeing where comprehension is and what proportion of the class is comprehending and what's not, you don't really realize how much of a problem it is. I think that's going to be a big difference [next year] having that experience and knowing how to set work for different levels. (T69#2)

This teacher, with limited ITE mathematics education experiences to draw on, remarked that the classroom observations prompted her to "bring in those things I had applied to science, into the maths teaching".

Day-to-day planning was a challenge for some, especially if preparation also involved learning subject content knowledge; learning that many hoped would pay off in terms of preparation for future years. Several teachers commented that they needed to be realistic about how much time to spend planning:

> So there is a standard that you are expected to produce at Teachers College and then there is what is actually achievable in the real world…what we need more of is what you need to do to get through the week. (T204#2)

Information about resources and mathematics assessment requirements was shared within department meetings and one-on-one meetings with their assigned mathematics mentor. Whilst a few teachers reported that these meetings were based on planned agendas, most reported that mentor meetings were largely responsive to day-to-day needs, for example:

> They've [meetings] helped quite a lot really particularly meeting with my supervisor because that is not particularly structured. If there is nothing else going on we'll go through each of my classes and talk about how they are going. But that is an opportunity to bring up any issues that I have been having during the week or ask questions. Some of them are quite a practical nature, like do you have any good resources for this topic? (T191#1)

However, as the year progressed, several of the teachers expressed an awareness that access to colleagues' time was not to be taken for granted. Those who had regular, as opposed to 'needs-based', meetings appeared more likely to continue productive mentoring arrangements despite the prevailing culture of 'busyness'.

> Mostly the mentoring I have got from people has been really specific because I guess everybody is busy and you need to have an agenda on something specific that you are talking about and achieving through that time. I have really appreciated the fact that my supervisor has time-tabled a regular time to meet and that is our time. (T790#2)

Creating a Classroom Learning Community

Creating a classroom learning community involves teachers maintaining a classroom which is not only productive of students' learning but is also safe and respectful. In accord with the literature (e.g., Johnson, Berg, & Donaldson, 2005) the NQTs collectively reported struggles to motivate students to learn, to implement effective classroom management, and to work in partnership with parents.

> I can't motivate the children and I'm finding that very frustrating. I talk to the other teachers who have got more experience than me.…My HoD doesn't take it personally, he says they have to take responsibility but … I get upset really quickly about students who just don't want to do any work. I just have to accept the fact that kids who are there, 15 of them are not motivated towards maths at least, I just have to accept it. (T417#1)

At 6 months into their teaching, many of the teachers reported grappling with management issues, with the need to establish rules and routines, and manage disruption whilst attempting to undertake quality teaching and learning. The majority of the teachers sought and had been provided with assistance from their more experiences colleagues in tackling issues of classroom behaviour and student motivation. However, one teacher was concerned that seeking advice might be perceived as a weakness by his students:

> On one side if I always ask for help from my colleagues I'm feeling you see there could be a negative effect on what the student may feel about me. So this guy is just unable to control the classroom. Because sometimes I don't know how to handle the situation, I do need help from a colleague so they can show me more examples of how to handle it…. It is a good thing except you see my feeling sometimes if they come too often maybe the students will have the feeling of okay this teacher is not coping. (T551#1)

There was a marked change in focus in the interviews at the end of year, with most teachers reporting that classroom management issues were largely resolved; they finished the year with a sense of order and confidence in the classroom community they had established. Recalling their most enjoyable experience often related to a 'breakthrough' with their 'nemesis' class:

> [At first] I was just really struggling with them and I would go home and I couldn't stop thinking about them and I would be worried about them and thinking how are we going to get through the year … I got through it and the support was really good then and that was a time when [beginning teacher meetings] were really valuable because several times as a group we would just talk about it and people would have different ideas. I had a kid with ADHD in my class and they suggested different strategies which has helped a lot. So I have learnt heaps through that, so although it was a horrible experience I am not sorry I had that class at all. (T790#2)

A few teachers, however, were still focused on creating the positive learning environment, suggesting that work on mathematics teaching per se was on the 'to do' agenda for next year.

> My angry voice I think, I have to get one I think. I find it easier to get to know my students but it's finding that line where I know them as students but they know still know me as 'teacher' and not just friend. So then when it comes to discipline issues and yeah that's sort of the main thing I want to pick up next year. (T6#2)

Enacting a Beginning Repertoire

Enacting a beginning repertoire involves attending "to the purposes not just the management of learning activities and their meaning for students" (Feiman-Nemser, 2001, p. 1029). This requires pedagogical practices which encompass curriculum design, classroom instruction and assessment. A major role of any induction programme is to assist NQTs to enact and broaden their repertoire of teaching skills by developing and extending these skills with an understanding of their new environment and context. However, professional learning in this area appeared to be closely linked with both the NQT's initial capability in terms of confidence and experience of pedagogical practices, and with the school culture.

Some teachers found themselves in schools with a strong 'craft knowledge' culture where more ambitious pedagogies were discouraged in favour of traditional safe approaches to teaching. Several teachers reported explicit awareness of the pull (and sometimes push) to abandon their initially desired practices for safer, less complex activities or actions:

> I find myself actually moving away from what I've been taught. I love the idea of student centred learning, I love the idea of group work. But what tends to be the most effective is actually having a very tight lesson with lots of where students are kept very busy doing work out of a textbook and textbook teaching if you like. So that's not something that I'm entirely comfortable with … (T204#1)

The teacher in this case experienced pressure from both more experienced colleagues and from the community: "That's the advice that I've been given—keep it simple and keep them moving through the work" and "Parents want to see a lot of homework….Parents will be more focused on the amount of homework than the learning that is taking place".

Learning in and from Practice

To develop their practice NQTs must learn to use their practice—be it their own or their colleagues as a site of inquiry. Consistently, those NQTs that had been encouraged and supported to observe experienced teachers reported this as a significant source of professional learning, especially when these experiences confirmed the "privileged teaching repertoire" (Ensor, 2001) promoted within their ITE experience:

> I said [to the mentor] I am having real trouble making this interesting. You know, getting outside the book. She suggested go and watch this other teacher. So I did, and I got some good ideas from it that relates right back to that ITE training, because I found that when I got into the classroom, she was applying some of those outside the square ideas.…Whereas for me, I had reverted back to the way I was taught maths, which was from the book, pen and paper, in the exercise book and had no variety. (T343#1)

The opportunity to watch in other subject areas and interact with teachers outside of their department was also reported as a useful activity by a few teachers. Others reported the process of reflecting back or referring to ITE notes and resources during personal reflection time, to be a valuable source of learning:

> I guess there are big principles that I have really internalised from that [theory], it's not all of the details of who thought of this version of the theory versus this version….Bigger issues around motivation and what makes them succeed and how they learn—it's definitely all going on in the back of my mind. Often having taught something I will sit back and I will think how I might do it better next year for one of those reasons. (T790#2)

Implications and Conclusions

The interviews affirmed Feiman-Nemser's (2003) conclusion that new teachers "long for opportunities to learn from their experienced colleagues and want more than social support and instructions for using the copying machine" (p. 28). Focusing closely on the induction programme embedded in particular school settings revealed complex learning systems that had to be negotiated by NQTs. All teachers sought advice on curriculum implementation, assessment, teaching strategies for specific students' needs, behaviour management and working effectively with parents. They expected and sought to gain insight from colleagues with experience in their subject areas, through regular meetings and classroom observations.

The findings also remind us that a NQT's repertoire of practice is fragile; it needs to be trialled, reflected upon, strengthen and challenged, but challenged in a positive way with guidance within a supportive professional learning community. The push of some colleagues towards structured teacher directed lessons was frequently associated with classroom management issues and 'coping' with low-achieving students. Most of the

NQTs discussed differential practices, expectations, and satisfaction with their teaching of senior students compared with junior students:

> I have some students particularly in Year 9 who their way of trying is just trying to keep their behaviour within acceptable bounds and that's sort of where their priority is rather than their academic work at the moment. (T69#1)

While NQTs reported varied experiences and satisfactions with the formal support programmes, the level of informal support was always highly valued and for the majority of teachers highly accessible. Informal support, in particular, reinforced the role of the 'ethic of care'. The following response indicates a teacher's delight with the school principal's informal observation of her teaching:

> So he comes in and the kids are used to it…He's a Maths person and last time I got him to do an example on the board and the kids just thought it was fantastic, so he has seen me teaching in an informal way which I really appreciate because it makes me feel that he cares about what kind of job I am doing and he doesn't just go on hearsay, he actually takes the time to get out of his office to come and see. (T790#1)

It was clear that despite clear national guidelines for induction not all teachers in this study were necessarily receiving *sufficient* or *appropriate* support and guidance that challenged and furthered their capacity to become more effective in their teaching. Issues of access, focus, and quality, with regard to guidance and support, resulted in differential spaces and opportunities for teacher learning. Moreover, faced with multiple options for support and considerable freedom to plan their non-contact time, the NQTs exercised varied expectations of continued learning, and exhibited varied levels of agency in their participation in the induction programme

NQTs have legitimate learning needs that cannot be properly assessed in advance or outside the contexts of their teaching. Schools need to adapt their advice and guidance programme to suit their situationally relevant context, and to match an individual teacher's levels of experience and preparedness. Equally, NQTs need to be aware of both their non-formal and formal learning needs and be equipped and prepared to take more responsibility for their own professional growth.

For those who rated their induction experiences highly, there was clear evidence that they were involved in relationships with colleagues that both valued them and recognised their special needs as NQTs. There were frequent planned and informal interactions with more experienced teachers involving teaching and learning. The school induction programme was organised and explicit about the available guidance, and NQTs were encouraged to seek help and expected to be learning and improving their teaching practice. Moreover, when their learning needs were not being met to their satisfaction, these NQTs felt comfortable exercising agency in adapting or requesting changes in the nature of support. For these NQTs, the provision of time and sustained learning opportunities enabled them to build upon their initial teacher education experience, to teach in ways that met demanding new standards for student learning and to participate positively in the solution of educational problems. As their mentor teachers remarked, NQTs bring enthusiasm and renewal to our schools—to a point where they may begin to develop a new cultural dynamic.

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Algebraic Fundamentals: A Key to Success in Preparing Teachers of Elementary Mathematics

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Many factors contribute to a general phobia of learning and teaching of mathematics by pre-service teachers. This study sought to elucidate the essential components needed for success in a course for elementary teachers of mathematics. While results indicated that the factors of cognitive ability as determined by Guilford's Structure of the Intellect (SOI) assessment and self-regulation skills as determined by the Learning and Study Strategies Inventory (LASSI) were not statistically robust, significance was determined through correlation analysis and regression. The data seem to suggest, that coupled with high motivation and self-discipline, pre-service teachers that have a solid foundation in algebraic fundamentals experience a higher degree of success in a course for the learning and teaching of elementary mathematics than their counterparts that are not so prepared.

California has been in crisis mode for the last few years, especially in mathematics education. While budget cuts have created additional chaos, mandatory class downsizing has produced an accelerated need for qualified teachers, especially in elementary mathematics. Of particular concern is the number of "provisional" teachers that have been and will be hired. In sum, the issue in question is one of competence in the mathematics discipline. The California content standards for teaching mathematics in the elementary school are explicit in what knowledge base must be mastered. Certainly methods classes are essential, but a proper foundation in the mathematics discipline is indispensable. In the United States, this need becomes even more acute with the renewed emphasis on formally teaching algebra in the 7th and 8th grades (Carré & Ernest, 1993; NCTM, 2000).

Many students in the liberal studies program consider math-related courses as the most dreadful and most feared courses in the curriculum. Bibby (1986) concurs when he cites a typical student's reaction toward mathematics as "it reminded her of a boa constrictor which slobbers its victim before swallowing them" (p. 60). Research indicates that test anxiety, poor study skills, lack of motivation or concentration, and a general phobia toward mathematics are all factors that contribute to the lack of mastery of math content (Arvidson, 1997; Tobias, 1978). However, in spite of the fear and foreboding, the subjects covered in these courses are crucial to academic and professional progress toward a teaching credential.

Palmer (1988) has astutely noted that there exists a symbiotic relationship between the content, the instructor and the learner. While the skirmishes around content standards and under-prepared students are well documented (TIMSS, 1996), the preparation of the pre-service teacher is the critical element for research in this study. According to Meeker (1979), the research clearly indicates that most learning failures occur because the learner is not prepared to learn. The same can be applied to pre-service teachers that are preparing to teach math at the elementary school level. Why do some pre-service teachers succeed in demonstrating competence in K-8 mathematics while others struggle to understand and subsequently fail to competently explain the math concepts they are required to teach?

Fundamentally, the purpose of this study was to investigate the correlations between competence in elementary mathematics and numerical ability, logic and form reasoning, cognition of symbolic systems, evaluation of symbolic systems, convergent production of symbolic systems, information processing, selecting main ideas, test strategies, attitude, motivation, anxiety, concentration, time management, self-testing, study aids, previous success in algebra and attitudes toward mathematics. Considering the myriad of factors that are involved in learning mathematics, a second purpose was to find any identifiable characteristics that contribute to high achievement in the course 'mathematics for elementary teachers'.

Method

Procedure

At the beginning of the semester, students in the Mathematics for Elementary Teachers course were asked to participate in a study investigating cognitive abilities and study skills through a series of assessment instruments. At this point students completed a survey that included information on sex, age, career goal, highest math class completed, attitude towards mathematics and the grade received in their last algebra class. The students were informed regarding the purpose of the study and were asked to identify themselves by their student ID number. It should be noted that research indicates that students with poor cognitive abilities frequently overestimate their performance. This could diminish the reliability of self-reporting regarding their aptitude in mathematics (Kruger & Dunning, 1999).

Using the Structure of the Intellect theory (Guilford, 1967) as the cognitive model, the Structure of the Intellect (SOI) learning abilities test (Meeker, 1975) was used to test each group's specific cognitive abilities. In addition, the Learning and Study Strategies Inventory (LASSI) assessment instrument was also given to assess behaviours, attitudes and beliefs that relate to successful learning (Weinstein, 1987).

Subsequent to both of these assessments, students completed three examinations during the course of the semester to determine their mastery of content required for K-8 teachers of mathematics. In addition, a course grade that also included teaching a math lesson, a written review of the state framework for mathematics, completed homework from a textbook and a monthly mathematics activity packet was used to measure the students performance in the class. The course grade was included in the data set at the end of the semester after the final exam was graded.

Participants

The students participating in this study were undergraduates enrolled in a liberal studies course, Mathematics for Elementary Teachers at Azusa Pacific University in Southern California. These 81 students were enrolled in three sections of an undergraduate course covering elementary school mathematics for pre-service teachers. As previously noted, their performance was measured on three major exams as well as a course grade that included teaching a math lesson, a written review of the state framework for mathematics, completed homework from a textbook and a monthly mathematics activity packet.

Data from students who completed both assessment instruments and all three examinations were retained for analysis. Sixty-eight of the 81 students enrolled met these inclusion criteria. Although students who met the inclusion criteria may have differed from the other students in the class, multivariate analysis of variance revealed no significant differences on examination grades or final grades between students who met and students who failed to meet the inclusion criteria.

Instruments

The Learning and Study Strategies Inventory (LASSI) is a 10-scale, 80 item assessment of students' awareness about and use of learning and study strategies related to skill, will and self-regulation components of strategic learning. Moreover, these skills include behaviors, attitudes, and beliefs that relate to successful learning (Weinstein, 1987).

Specifically, the LASSI scales related to the skill component of strategic learning are: information processing, selecting main ideas, and test strategies. These scales examine learning strategies, skills and thought processes related to identifying, acquiring and constructing meaning for important new information, ideas and procedures, and how they prepare for and demonstrate their new knowledge on tests of other evaluative procedures.

The LASSI scales related to the will component of strategic learning are: attitude, motivation and anxiety. These scales measure students' receptivity to learning new information, their attitudes and interest in college, their diligence, self-discipline, and willingness to exert the effort necessary to successfully complete academic requirements, and the degree to which they worry about their academic performance.

The LASSI scales related to the self-regulation component of strategic learning are: concentration; time management; self-testing and study aids. These scales measure how students manage, or self-regulate and control, the whole learning process through using their time effectively, focusing their attention and maintaining their concentration over time, checking to see if they have met the learning demands for a class, an assignment or a test, and using study supports such as review sessions, tutors or special features of a textbook.

Each scale, with the exception of the Selecting Main Ideas Scale, has 8 items. Selecting Main Ideas has 5 items. Coefficient Alphas for the scales range from a low of 0.68 to high of 0.86 and test-retest correlation coefficients for the scales range from a low of 0.72 to a high of 0.85, demonstrating a high degree of stability for the scale scores.

The second assessment instrument that was used in the study is based on Guilford's (1971) Structure of Intellect theory. Guilford first developed the theory in the United States in the early 1940s. In its first documented application the Structure of Intellect assessment method was used to streamline the selection criteria to better reflect the mental abilities needed to be a pilot. As a result, the failure rate of US Air Corps recruits was reduced from 33% to 3% (Meeker & Meeker, 1999).

Subsequently, Guilford and Hoepfner (1971) developed a wide variety of psychometric tests to measure the specific abilities predicted by the theory. The theory defines intelligence as a juxtaposition of operations, products and contents for processing different kinds of information in various ways. Further, Guilford's structure of intellect model has been applied in programs to enhance thinking skills such as the SOI (structure of intellect), designed by Meeker (1969) and Meeker and Meeker (1999). The SOI is a standardized assessment instrument. The primary purpose of the test is to provide an accurate profile of the examinee's cognitive learning abilities.

Specifically, the SOI encompasses 26 separate tests in 5 different areas. However, this researcher chose to focus on two areas, that of arithmetic and mathematics. In the area of arithmetic, three test were utilized, that of Numeric Facts (CSS), Numeric Judgment (ESS), and Numeric Application (NSS). In the area of mathematics, the Form Reasoning (NSI) test was used.

Specifically, CSS is the cognition of symbolic systems. This is a test of comprehension of numerical progressions. In this subtest, the student must find the rule that is generating a number series. Arithmetic ability is required, but only elementary rote skills. This subset provides information on how well students have mastered rote skills in arithmetic.

ESS is the evaluation of symbolic systems. In this subtest the student is evaluating systems of numbers. The ability to select the correct principle is being tested. Rules are presented and the student examines series of numbers to find the series that has been described by the rule. The task requires math skills and related problem-solving skills.

NSS is the convergent production of symbolic systems. This is a test of the ability to solve complicated arithmetic problems that do not depend on verbal skills. The student is presented with a starting number and a target number to be obtained through a sequence of numerical operations. The task requires skill with signed numbers and selection of correct principles for solutions. Students who have difficulty on this subtest may have problems in seriation or conservation.

NSI is the convergent production of symbolic implications. This is a test of the ability to deduce the solution to a symbolic problem. This subtest involves logic and form reasoning. It requires the student to perform a substitution of a given equivalence or equivalencies to arrive at the correct answer. In the first column, however, substitution is not required; the student simply looks at the top of the column for the correct answer. This subtest predicts the ability to work with commutation and can be used as a screening test for placement into algebra if CSS, ESS and NSS computation skills are good.

An earlier study by Maxwell (1984) confirmed that there were no sex differences in true variance, error/uniqueness variance, or in factor inter-correlations. In sum, this finding suggests that the SOI-LA test is the same for both females and males.

Results

Statistically significant positive correlations between course grade and independent variables were obtained for SOI-ESS ($r = 0.536$, $p < 0.001$), SOI-NSS ($r = 0.353$, $p < 0.01$), SOI-NSI ($r = 0.372$, $p < 0.01$), LASSI-MOT ($r = 0.387$, $p < 0.01$), and last algebra class grade ($r = 0.404$, $p < 0.01$)(see table 1). In addition, paired *t* tests revealed no significant differences between student's final grade in the course and the student's grades on the exams.

Table 1
Correlations between Course Grade and Cognitive Variables

Variable	Course Grade	Algebra Grade
SOI-ESS	0.536***	0.483***
SOI-NSS	0.353 **	0.369**
SOI-NSI	0.372**	0.273*
LASSI-MOT	0.387**	0.289*
Algebra Grade	0.404**	1.000

(*p< 0.05, **p< 0.01, ***p< 0.001)

Overall regression analysis showed that the model with the 5 predictors accounted for 34% of the variance (Rsq = 0.342). Moreover, the F value was significant at 6.977 ($p < 0.001$) with 5 degrees of freedom, validating that the model with the 5 predictors is statistically significant. Regression also reported that SOI-ESS was the strongest predictor ($p < 0.05$) and LASSI-MOT was borderline significant ($p = 0.05$) (see table 2). Interestingly, 'last algebra grade' was not reported to be significant in the regression analysis even though it was the second highest correlated variable to course grade. In sum, when all 5 predictors were controlled for, regression does not show the predictive power of last algebra class grade.

It is hypothesized that since the variables SOI-ESS, SOI-NSS, SOI-NSI, and LASSI-MOT were not controlled for in the correlations, the last algebra class grade contained some of the predictive power of the other variables. Similar results were found in a study of SI components in ninth-grade mathematics achievement (Guilford, Hoepfner, & Petersen, 1965). For example, the abilities of CSS, CSI, NSS, NST, and NSI were found to be relevant to success in Algebra. In addition, according to Guilford (1967, 1982) CSS, ESS, NSS and ESC are used to assess rote arithmetic skills. If all of these and NSI are high, they serve as a good cluster of skills for success in algebra.

Table 2
Regression of Course Grade on the Independent variables

Variable	Beta	T	sig T
Algebra Grade	0.100	0.797	0.428
LASSI-MOT	0.203	1.865	0.067
SOI-ESS	0.337	2.048	0.044
SOI-NSS	0.000	0.002	0.999
SOI-NSI	0.115	0.983	0.329

R = 0.585
Rsq = 0.342
F = 6.977 sig F = < 0.001
N = 67

Discussion

The results of this study suggest that a thorough grounding in algebraic skills and knowledge is the biggest single predictor of success in a mathematics for elementary teachers' course. While SOI-ESS was the strongest predictor for success in the mathematics for elementary teachers' course, it is also an essential component to success in any course requiring algebraic reasoning. ESS, or the judgment of correctness of numerical facts, is the ability to make correct decisions about which of the concepts are needed and the order in which to use them to solve non-verbal math problems. The application of ESS in a course requiring algebra is most easily seen in order of operation problems or in two-step equations. Other requisite skills in algebra are highly diminished in value if the student does not know whether, or when, to add, multiply, divide, etc., in the correct order. In sum, students that are low in ESS have not learned which math rule or principle to use first in solving problems.

A common view of mathematics education is that its main aim is the acquisition of knowledge through the learning of facts. In any given classroom, most measure the effectiveness of mathematics education by testing students' knowledge. But do these tests really measure student's mathematical abilities and understanding? While the learning of facts might be the goal of certain courses, it is not the seminal purpose of mathematics education. Rather the purpose is to improve the mind by acquiring abilities and skills to do things they could not do previously. As Plutarch has astutely noted, "The mind is a fire to be kindled, not a vessel to be filled." Books and files store many more facts than people do. In fact, they are excellent "vessels," but that does not make them smart. Being smart is about doing, not just about knowing (Gardner, 1993). It is not enough to know basic operations in mathematics; the skill is knowing what operation is needed and when to apply it (Arvidson, 1999).

The main benefit of learning and doing mathematics is not the specific content; rather it is the fact that it develops the ability to reason precisely and analytically about formally defined abstract structures (Devlin, 2003; Schoenfeld, 1992). Moreover, many times specific topics in mathematics are not as important as having a high level of mathematical sophistication (NCTM, 1989, 2000). For example, the specificity, rigor and logic found in algebra and geometry provide this sophistication in mathematical understanding and, in turn, form the foundation for subsequent math topics to be fully grasped. As this study seems to indicate, all other measures of mathematical fluency trace their way back to the mastery or failure to master the fundamentals of algebraic reasoning. Authors of the National Academy of Sciences seminal report *Adding it up: Helping children learn mathematics* regard the formal study of algebra as "both the gateway into advanced mathematics and a stumbling block for many students" (Kilpatrick, Swafford, & Findell, 2001, p. 419).

In addition, this study also reported the LASSI-MOT variable as a statistically significant factor in a student's success in the 'Mathematics for elementary teachers' course. The variable represents the Motivation scale that assesses students' diligence, self-discipline, and willingness to exert the effort necessary to successfully complete academic requirements. The maxim, "To learn math is to do math" assumes a strong work ethic of doing and persevering so that learning will happen. For example, a low scoring student may choose when the work is difficult to either give up or study only the easy parts. Typically, students with low scores on the Motivation scale also lack responsibility for their academic outcomes and many do not know how to set and use goals to accomplish specific tasks (Corno, 1992; Weinstein, Schulte, & Palmer, 1996).

While the causal connection between course grade and the ability to teach mathematics cannot be clearly established, the data suggest that, coupled with high motivation and self-discipline, pre-service teachers that have a solid foundation in algebraic fundamentals experience a higher degree of success in the learning and teaching of elementary mathematics than their counterparts that are not so prepared.

> The quality of instruction is a function of teachers' knowledge and use of mathematical content … It depends critically on teachers who understand mathematics, how students learn, and the classroom practices that support that learning … Teachers need to know the mathematics of the curriculum and where the curriculum is headed. They need to understand the connections among mathematical ideas and how they develop. Teachers also need to be able to unpack mathematical content and make visible to students the ideas behind the concepts and procedures. (Kilpatrick, Swafford, & Findell, 2001, p.424-428)

In sum, the implications arising from under-prepared teachers of elementary mathematics cannot be understated. Although children bring important mathematical knowledge with them to class, most of the mathematics they know is learned in school and depends on those who teach it to them. Therefore, improving students' learning in mathematics depends on the capabilities and knowledge of their classroom teachers.

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Networks of Learning: A Professional Association and the Continuing Education of Teachers of Mathematics in Pakistan

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In recognition of the importance of on-going professional development of teachers research proposes a variety of approaches for the development of teachers, both at pre-service and in-service levels. Among them, networking among teachers, teacher educators, curriculum developers and policy makers is recently receiving attention as an innovative and flexible professional development forum. Networking can create ownership among stakeholders regarding implementation of change and reforms in the educational landscape. In this paper, a case of the Mathematics Association of Pakistan (MAP) has been presented as a network for learning. The formation and growth of this network can be viewed as developing insights into the improvement of mathematics education in the developing world, especially in Pakistan. This sharing of the experience may further support efforts for creating other networks of learning for implementation of reform in education in different parts of the world.

What are Networks?

It is difficult to find one suitable definition of a network given the range of purposes for which networks are established. Clarke (1996) in his book *Schools as Learning Communities* provides a useful starting definition:

> Networks constitute the basic social form that permits inter-organizational interactions of exchange, concerted action, and joint production. Networks are unbounded or bounded clusters of organizations that, by definition, are non-hierarchical collectives of legally separate units. Networking is the art of creating and/or maintaining a cluster of organizations for the purpose of exchanging, acting, or producing among the member organizations. (p. 142)

Darling-Hammond and McLaughlin (1995) also stress the importance of networks as powerful tools in teacher learning both at pre-service and in-service levels. The report *Networks @Work* (2002) suggests that:

> Networks provide the 'critical friends' or 'peers' that teachers need to be able to reflect on their own teaching experiences associated with developing new practices in their classrooms. Teacher networking often provides an opportunity for teachers to visit the various schools of participants and to gain 'practical pedagogical clues' from other teachers' classrooms. (Queensland Board of Teacher Registration, 2002)

Also Board of Teacher Registration (1997) writes about the importance of the networks as:

> Professional relationships forged outside the immediate working environment enable teachers to gain valuable insights into new knowledge and practice beyond that gained from interactions with colleagues in their own schools. (pp. 6-7)

Lieberman (1999), while quoting several educational change leaders (e.g. Darling-Hammond & McLaughlin, 1995; McLaughlin & Talbert, 1993; Cochran-Smith & Lytle, 1993), talks about the networks as:

> Networks are becoming popular, in part, because they encourage and seem to support many of the key ideas that reformers say are needed to produce change and improvement in schools, teaching, and learning.

According to Lieberman and Grolnick (1997) networks may provide:

- Opportunities for teachers to both consume and generate knowledge;
- A variety of collaborative structures;
- Flexibility and informality;
- Discussion of problems that often have no agreed-upon solutions;
- Ideas that challenge teachers rather than merely prescribing generic solutions;
- An organizational structure that can be independent of, yet attached to, schools or universities;
- A chance to work across school and district lines;
- A vision of reform that excites and encourages risk taking in a supportive environment; and
- A community that respects teachers' knowledge as well as knowledge from research and reform.

If one can summarize the key literature on networks for learning (e.g., Darling Hammond & McLaughlin, 1995; Lieberman & Wood, 2003; Smith & Wohlstetter, 2001), one can identify two distinctive features of teacher networks that may support teachers' learning on an ongoing basis:

- Personal and social relationships: improved relationships, flexibility, risk-taking, commitment, openness in interacting with each other and clarifying values and expectations.
- Academic and professional aspects: innovation, enriching practice, continual development of teachers focused on professional concerns such as student learning, sharing and getting relevant professional information (dissemination), developing healthy and shared norms, enriching curriculum and influencing policy makers.

Lieberman and Grolnick (1997) also observe several themes regarding the work of networks such as:

> Creating purposes and directions; building collaboration, consensus, and commitment; creating activities and relationships as building blocks; providing leadership through cross-cultural brokering, facilitating, and keeping the values visible; and dealing with the funding problems. (p. 196)

Why are Networks Important in the Context of Pakistan?

Recently Aga Khan University Institute for Educational Development (AKU-IED) based in Pakistan, mandated to improved the quality of education through its innovative programmes and research initiatives, has supported six professional associations; namely, Mathematics Association of Pakistan (MAP), School Head Teachers Association of Development of Education (SHADE), Science Association of Pakistan (SAP), Pakistan Association of Inclusive Education (PAIE), Association of Primary Teachers (APT) and Association of Social Studies Educators and Teachers (ASSET) to form a network called Professional Teachers Associations Network (PTAN). This network has some funding support from the Canadian International Development Agency. The overarching aim of this Network is to promote an enabling environment for the professional growth and development of educators from diverse backgrounds, as a contribution to the improvement of education in Pakistan (PTAN Funding Proposal, unpublished).

In the funding proposal of PTAN an insightful assessment is made about the status of teachers in Pakistan.

> Teaching in the context of Pakistan continues to remain as a neglected profession thus leading to poor status for the teachers within society. This status quo also remains prevalent due to the absence of networking amongst Pakistani teachers and an authentic platform to raise genuine issues to broader audiences as well as to support their own professional development. Pakistani teachers today, find themselves as an ignored identity, in most educational reforms and quality improvement initiatives in the country. This despondency has further perpetuated nonchalance and lack of conviction within their profession leading to the educational system working in a dismal Situation. The main victims, thus being the students, the so-called primary beneficiary of education. (PTAN Proposal, unpublished p. 1)

PTAN, through its constituent members is helping teachers from different sectors (public, private not-for-profit and private for profit) to come together and discuss their professional matters in a more open manner and develop a collaborative strategy to approach their professional matters. The composition of working committees of these professional associations is made up with fair representation of teachers from all the constituencies such as government and private and other nongovernmental organizations that they are serving. This coming together of teachers from different sector schools helps members of these networks to understand their particular issues and develop a holistic approach towards creating greater cooperation to deal with these issues on a more sustained and focused manner.

A case of Mathematics Association of Pakistan (MAP) will be presented to illustrate how this professional association of mathematics teachers has evolved as a network of learning. Within the case a detailed overview of its activities has also been presented.

Case of Mathematics Association of Pakistan (MAP)

Activities and Organization of MAP

MAP was established as a professional association of mathematics teachers to upgrade the quality of mathematics education in Pakistan. Since its inception in July 4, 1997 it has been committed to providing a learning platform for all those related to the field of mathematics education whether directly or indirectly.

MAP has adopted a three-pronged approach to address the continuing professional development of mathematics teachers. First, it has created and structured focused programs for mathematics teachers at both pre-service and in-service levels in order to provide opportunities for them to interact freely with each other on professional matters. For example, MAP organizes a regular workshop every month on various topics such as teaching fractions meaningfully, or geometry—making connections, etc.

Second, for children to develop positive attitude towards mathematics, MAP is very active in organizing separate programs for them. In these programs, the children have opportunities to work in teams to experience mathematics as an interesting and challenging subject. The main focus of these programs has been to help children to see mathematics as a valuable subject to pursue. Moreover, through these programs, MAP is helping mathematics teachers to see how they can teach children according to the new demands of teaching and learning for understanding. For example, MAP has so far organized five

Olympiads for children at different grade levels to work on interesting and challenging mathematics in a collaborative fashion.

Third, in order to create a strong support mechanism for teaching and learning worthwhile mathematics, MAP has been working on various projects where important stakeholders are being encouraged to re-learn mathematics so that they can see the broader role of mathematics in their daily life situations. In this regard, MAP has been actively engaged into the process of rewriting textbooks with the Provincial bodies such as Sindh Text Book Board, a policy level body to design and produce text books for the province of Sindh in Pakistan.

In addition MAP organizes workshops for parents so they can see what it means to learn mathematics and how they would be able to support children's mathematics understanding. This work with the wider society enables MAP to create greater synergy and networking amongst different stakeholders.

If one looks critically at the work of MAP, it is clear that it has created several avenues where mathematics can be conceived of as a human activity and considered as a subject essential in daily life situations and within larger socio-political levels. Within this scenario the learning of mathematics can be seen as an important subject for making informed decisions in today's fast and ever changing world.

Specific Activities of MAP

MAP offers a variety of approaches to upgrading the quality of mathematics education:

_ It assumes the role of champion in furthering the goal of quality of mathematics education in the contexts where it serves.
_ It encourages networking amongst its members and the wider society to deliberate on professional matters and issues in a sustained and effective manner.
_ It is proactive in influencing the policies of government concerning the goals of quality mathematics education within the country and beyond.
_ It has established an Institute of Math Olympiads intended to serve the development of mathematical thinking amongst students at all levels.
_ It develops partnerships for learning with similar professional associations in other countries.

The working committee of MAP is responsible for all its affairs. It comprises members representing various sectors such as private and government schools. Proportionate representation of different sectors enables MAP to cater to the diverse needs in an informed fashion whilst in committee meetings the debates normally canvas professional issues as well policy formulation concerning the activities of MAP. The Chair of MAP is responsible for the overall direction of the association and is accountable to its working committee for all the affairs ranging from policy implementation to setting the strategic directions to achieve the intended goals of MAP.

Veugelers and Zijlistra (2002) have pointed out the importance of the role of the Chair of learning network: "Chairing such group means that all people should get involved, each voice should be heard. The Chair must have the competence to analyze the experiences and ideas and place them in a theory that has clear links to the practice of the schools" (p. 172).

What Qualities does MAP have as a Network?

Leadership opportunities for members of MAP. MAP, as a community of professionals, would not have been playing such a constructive role towards the professional development of teachers without some of the important characteristics of networking. One of those characteristics is improving the quality of relationships among its members and MAP has been pursuing that by developing and working in teams. Teams are necessary for the successful operation of most of its activities. As John West-Burnham (2000) states:

> Effective teams have come to be seen as one of the crucial characteristics of quality organizations and, equally significantly, one of the most powerful catalysts in an organization for implementing change. (p. 15)

Another quality of networking is a culture of sharing. MAP has created a culture where both the active members of MAP and other fellow colleagues share their professional knowledge and concerns in a very open and candid manner. They understand that their views would be well listened to and they would get useful suggestions from their fellow colleagues in a non-threatening environment, which MAP has created so far. This is in sharp contrast with a culture where alternative suggestions are not listened to and valued; this is often observed in the discourse of education in this country. At the moment MAP is operating in Karachi, the largest city of Pakistan having population 10 million people. Being both a private and non-profit network, it is relatively new in the professional development scene of Pakistan. However, with its consistent efforts, MAP has assumed a very influential position in Pakistan. Governments both at provincial and national levels have approached MAP for variety of initiatives. For example, MAP has remained very actively engaged in the processes of the development of textbooks of Sindh province. MAP has also been contributing to the enhancement of the quality of mathematics education in Pakistan in a variety of ways. It has been created as a network to contribute to the development of different areas such as support to its members, involvement in curriculum development initiatives, actively disseminating the research results of various studies being conducted in mathematics education around the world and engaging in dialogue with professional organizations in the world.

MAP´s Contribution to Curriculum Development: Efforts to bring change into mathematics teaching in Pakistan have to begin from the understanding that mathematics teachers are mostly textbook driven. Generally, they teach from the textbook page by page and their focus is on coverage of the syllabus. A learning-for-understanding orientation should be considered important for the development of students to become informed citizens. To achieve that, considerable efforts have been made to devise a progressive curriculum with the involvement of the stakeholders of the school. In Pakistan there is little involvement of teachers in the development of the curriculum. Since they do not have an active involvement in curriculum development, normally mathematics teachers equate curriculum with the textbook and this prevents them from experimenting and implementing new ideas in the classroom. As Barwell (2000) has rightly captured:

> In Pakistan, teachers' practice operates entirely at the implementation level of the curriculum. Teachers have little influence on the intended curriculum in the form of textbooks or government publications and there is no tradition of school-level curriculum planning in the form of schemes of work or similar documents. (p. 37)

In that context MAP has taken up the challenge to change the notion of curriculum as well as the teaching of mathematics. MAP normally plans its workshops in a manner whereby teachers become active learners while working on several diverse mathematical activities designed to enrich meaningful understanding of mathematics. Now the question arises as how teachers can be supported to become more resourceful in implementing these activity-based learning approaches in their respective school contexts. This requires rewriting the curriculum of mathematics for schools. In recognition of this need, MAP, with the support of AKU-IED, played an active role in the review of textbooks of the primary grades of the Sindh Text Book Board (STBB), and Jamshoro, an official body of the province of Sindh in Pakistan established to create, publish and distribute textbooks. After successful review of these textbooks, MAP organized special workshops for mathematics teachers where reviewers shared their experience of reviewing the textbooks. For MAP, it is an exciting challenge to play a proactive role in influencing the design and development of the curriculum of mathematics not only at the school level but also at the national level.

Organizations of Workshops: Another aspect of curriculum development that MAP has been engaged in is the process of introducing Information and Communication Technology (ICT) in the teaching of mathematics. Through various workshops, MAP has encouraged mathematics teachers to learn possible ways to teach mathematics with the software packages such as Cabri Geometre and Excel. The advantage that these packages provided to students is to help them learn different concepts of mathematics in a more meaningful manner. For example, if they wish to explore different properties of angles and sides of a triangle, this can be done with simple dragging of the shape on the screen of the computer. Through dragging the shape they can see what effect it has to stretch the angle measurement of the shape if the vertex of one triangle is fixed etc. In this way students are engaged in the process of developing a conjecturing attitude towards mathematical propositions. This attitude may lead them to prove different mathematical propositions before accepting their truth.

Challenges and Lesson Learned

Since its establishment MAP has been successfully engaged in creating a collaborative culture of doing and investigating mathematics. Its presence is being felt at various levels from schools to governments. It has created several types of professional networking for the development of mathematics teachers in terms of provision of meaningful experiences for children. Despite all its efforts, MAP faces a number of challenges:

- Sustaining a culture of `volunteerism versus commercialism'.
- On-going professional development of MAP leaders and active members.
- How to meet the increasing professional needs of mathematics teachers in Pakistan with implications for resources and outreach.
- Greater networking among sister organizations in the country and in the world.
- Encouraging alternative assessment practices as opposed to heavily emphasized established summative assessment practices in Pakistan.
- Planning and conducting research in mathematics education as all members are volunteers who would take the responsibility for the completion and dissemination of research.

- Establishment of the Math Olympiad Institute devoted towards creating a variety of innovative activities for the children on an on-going basis.
- Having a sound infrastructure (Office space, permanent office secretary).
- Sustained funding until its operations become sustainable through its sources of income.
- Data Base Management System for membership and other relevant categories of the work of MAP.

The leaders of MAP feel that the acceptance of these challenges would not only develop a feeling of accomplishment but also help in creating and sustaining effective networking for mathematics teachers and teacher educators in Pakistan. MAP as a network of professionals is engaged in the process of making contribution towards improving the quality of mathematics education in Pakistan though there remains much to be done. It is essential for a country like Pakistan to encourage networks like MAP to continually grow and sustain their operations. These continual efforts would empower not only mathematics teachers to become caring and competent professionals, but also support society to adopt a learning mode to face the challenges of the Twenty First Century in improving the quality mathematics education in Pakistan.

Acknowledgements

I wish to acknowledge efforts of my colleagues at the MAP without their sincere and untiring efforts MAP would have not achieved what it has achieved so far.

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Mathematical Competencies and Awareness in a Teacher Education Practice

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"How do we facilitate learning of mathematics in our teacher education programmes and relate this to mathematics learning in classrooms?" is far from a trivial question. This paper starts from a view on mathematics learning which embraces learning as both acquisition and participation. It describes how student teachers' learning of mathematical competencies is furthered through participation in a simultaneously developing community of practice. Furthermore, it illustrates the teacher educator's largely implicit shaping of the practice, and thus facilitation of desired competencies and values. The awareness on all components of the practice required from the teacher educator is discussed briefly.

Working in mathematics education implies considering what is meant by mathematics and what we want learners/students to learn. The motives for learning (conceptual understanding, skills, mathematising, applying, etc.) determine the choice of classroom activities which serve to mediate between the learners/students and mathematics. In this paper, I explore what happens when learning to 'do mathematics' is an essential motive in a mathematics teacher education programme, and how this learning may be facilitated.

Theoretical Framework

Hans Freudenthal asserted that mathematics education should first and foremost be about mathematics as a human activity, not as a study of existing mathematical structures. He stressed competencies such as discovering and organising in interplay of content and form, mathematising, abstracting, schematising, formalising, algorithmising, verbalising (Freudenthal, 1991, p. 15 and p. 49).

A recent Danish initiative attempts to describe mathematics education outcomes across schooling levels in terms of *competencies*, including reasoning, representational competencies, symbolic and formalism competencies, modelling, communication competencies and competencies in tool use (Niss, 1999, 2004).

Michael de Villiers (2004) discusses the inter-relations of two specific competencies, namely deductive reasoning and 'quasi-empirical methods'. He claims that the latter are vital in providing students with an understanding of what is involved in doing mathematics, in motivating and in developing mathematical 'intuition'. He focuses on proving, and thus stresses conjecturing, verification, global and heuristic refutation, and the role of proof and proving in developing understanding, though he also mentions the role of experimenting and reflecting. John Mason (2000), amongst others, has considered aspects such as generalising versus specialising, abstracting versus instantiating, etc.

As can be seen from these short descriptions, a focus on 'doing mathematics' implies a stronger emphasis on processes and participation, yet it does not imply shared perceptions of what counts as knowledge, of what matters in learning mathematics, and of how the desired learning takes place. Social learning theories, socialisation theories, or activity theory can offer perspectives on how this learning occurs. Here, I work from the perspective of the social theory of learning of Etienne Wenger and others (Wenger, 1998), which stresses learning as social participation in practices. The participationist view can be seen as not excluding but complementing an acquisition view (Sfard, 2003). In that sense, both are but metaphors for learning, parallel to Activity Theory's claim that learning is both internalisation and externalisation.

The learning theory described by Wenger recognises the historical and social context which structures and gives meaning to our activities/practices—and thus to what is considered competencies. It recognises that participation in practices shapes what we do, who we are, and how we interpret what we do. Therefore, it includes both the explicit and the tacit, such as underlying assumptions, values and shared world views. It stresses that communities and organisations are also learning: through actions and interactions, learning reproduces as well as transforms the social structures in which it takes place.

In order to learn how to generate conjectures, proofs and definitions, to critique conjectures and look for counter-examples, to generalize and symbolize, the learners need to take part in a practice where such activities are prevalent and valued. The generally tacit or implicit components of this practice—its 'common sense'—is worked out through mutual engagement in the practice.

Skott's research on tertiary mathematics education addresses these implicit components. She develops a concept of *potential co-learnings*: "*interactively established possibilities or potentials for learning, which are mainly not communicated explicitly in a given teaching situation*" (2003, p. 13). This proved a useful concept in identifying implicit parts of a mathematics course. She found six categories of potential co-learnings to cover the implicit parts she identified. Her categories are formulated as questions: "*What are valuable mathematical activities? What are mathematical aesthetics? What are interesting mathematical questions? What are mathematical proofs? What are preliminary intuitive mathematical concepts? What are mathematical tools?*" (pp. 14-15).

It is through their engagement of the students in relevant practices that mathematicians (or, as in this study, mathematics teacher educators) can hope to facilitate the increased participation of students in mathematics in a way which implicitly reflects answers to these questions.

Together, the perspectives outlined above provide a framework for considering a teacher education class where the teacher has as an objective to engage the students in mathematical activity. What characterises the practice in which these students participate, and what is the teacher's role in promoting their participation? Which mathematical competencies do students develop and what facilitates this? Are potential co-learnings generated and, if so, what facilitates this?

Method

The data discussed here are part of a larger research and development project, aimed at unveiling relations between experienced mathematics teacher educators' personal theories and what they do (Jørgensen & Geldmann, 1999).

Danish education of teachers for grades 0-9, at the time of the observations, consisted of two general years and two specialisation years (two majors). In this study, a class of third year mathematics student teachers was observed throughout a semester. Their mathematics lessons were recorded and the recordings, as well as various writings by the institution's team of mathematics teacher educators, were discussed in the research team. The teacher educator for this class, Anna Jørgensen, was part of the research team.

Over a period of two to six weeks, lessons are linked together by a theme that unites decontextualised ('pure') mathematics, recontextualised ('applied') mathematics, learning of mathematics, teaching of mathematics, and other components of mathematics teacher education, such as the history of mathematics education, and general pedagogy.

The practice in the college classroom is distinct from the practice of teachers in schools. The research assumes that the teacher educator represents a community of practice of teachers. Through this representation, the teacher brings to the classroom "the concerns, sense of purpose, identification, and emotion of participation" of the practice of mathematics teachers (Wenger, 1998, p. 276). This grants her a strong voice in determining what is valued, what counts as competence, as knowledge, and so forth. The research assumes that many of these aspects remain implicit, but to some extent can be brought to the foreground by questioning/reflecting on practice. For this purpose, selected recordings from the classroom were discussed in detail by the research team. Thus, these discussions constitute both joint analysis and data for the research. Another source of data was interviews with students.

Observations and Analysis

The following observations are from a particular theme titled 'Generalisation' but illustrate general points. Due to space limitations, only the essence of the analysis is presented here. For more details, see (Christiansen, 2006).

Directed by a worksheet called 'Build two towers of equal heights', the students' task was to investigate when it is possible to build two towers of equal heights with a set of rods of lengths 1cm, 2cm, 3cm, ... The first questions addressed instances with the 10 rods which exist in a set, moving onto hypothetical rods with length 11cm, 12cm, 13cm and n cm. The students were asked to find a pattern or rule, and to consider whether their explanation was a proof. They had to determine if it is sufficient that the sum $1+2+3+...+n$ is even. After sharing findings with fellow students, they were to consider the topic in the light of a number of issues related to official and unofficial texts on mathematics education in schools.

The worksheet did not specify *how* to build the towers. The students decided to limit their investigations to towers with the width of one rod (see figure 1).

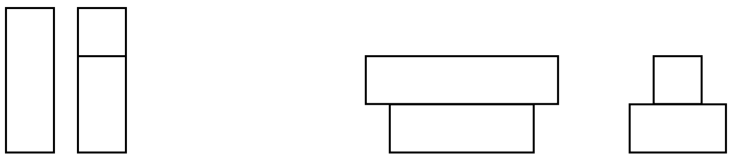

Figure 1. Students only accepted towers 'standing up' as to the left, where the first three rods have been used to form two towers of equal heights. They excluded towers of rods 'lying down', as to the right in the figure.

In that sense, they engaged in delimitation of the problem. This is part of the problem handling competency (Niss, 1999, 2004) and exemplifies what Mason (2000) considers a recurrent theme in mathematics, namely freedom versus delimitation. The openness of the task formulation lead students to engage with these issues (cf. Christiansen, 2006), and thus also contains the potential co-learnings of what constitute valuable mathematical activities and interesting questions.

Proving

Next, some of the students started to 'play' with the rods. Others used a more theoretical approach. All groups concluded that if the sum of the lengths of the rods is an even number, two towers of equal heights can be built.

The students' conclusion rested on an abstraction from dealing with rods to working with their lengths as numbers. In doing so, the students showed that an even sum is a *necessary* condition, but not that it is possible to construct the actual towers without breaking the rods. The teacher was aware, in the situation, that this could be used to touch on the topic of necessary versus sufficient conditions (Jørgensen, 2000). She followed up with the class:

> T: *Well,* until now, we have concentrated on whether... it [the sum] has to be even, because an odd one definitely doesn't work. That is how we, that is how the discussion has been… The topic of my question is on, can one be *certain* that if the sum is even then it'll work,

With her formulation, she embraced the students' conclusion as partially valid, but also treated it as a conjecture. She recognized their 'quasi-empirical' work, yet pushed them in the direction of deductive reasoning/proving ("be *certain*"). She specified her point with examples. A student viewing the problem as related to numbers only was confronted with the physical rods:

> T: But I can divide my number by two, but can I be sure that I can, that I do not have to break the rods? [7 seconds of silence. The teacher looks towards student and smiles]

The teacher represents the community of practice of which the students were working towards becoming members. With this non-trivial question, the teacher opened forms of mutual engagement, which invited the students to participate in the practice she represents. It was also a challenge to the students' 'knowledge', and this invited the students to engage in negotiation of meaning (cf. Wenger, 1998, p. 53). Simultaneously, through this challenge, the teacher provided focus points around which to organize the negotiation of meaning (the teacher herself uses the metaphor *bearings* or *buoys*—with a reference to how buoys are used by sailors to navigate; finding their bearings ensure that they are on a right course without completely determining it). The teacher captures all these aspects of how she promotes participation, when she states that her main task in the beginning of a new year is to create "a space where the students dare to learn and find it worthwhile" (Jørgensen, 2000, p. 1).

The students had to accept the invitation and the challenge, and the teacher kept on asking questions until this happened. The break-through came when a student admitted that he was unsure.

The obvious potential co-learning of what counts as an interesting mathematical question was established through the students' participation in responding to the challenge. The students' discussions turned out also to involve negotiation of perceptions of what mathematics is (Christiansen, 2006).

The students began to develop a proof. In the first stages, this took place in interaction with the teacher. The students offered suggestions to inform the verification of the conjecture, and the teacher challenged these, questioned the certainty, acknowledged what she could accept as given or shown, *but* she did *not* show the students a proof (and there are several), and she did *not* indicate how to construct a proof. Unlike the demonstrations and lectures which prevail in our classrooms, this can again be recognized as an invitation to participate in a practice, as well as the provision of 'bearings' which directed the students to engage in a mathematical practice of verifying, refuting, and clarifying. In that sense, the teacher's responses constituted an important structuring resource around the learning process.

A potential co-learning in this phase was 'what are mathematical proofs'. The students also engaged the interplay between deductive reasoning and 'quasi-empirical methods' (de Villiers, 2004) and the mathematical reasoning competency described by Niss. The latter includes evaluating a chain of arguments, knowing what is special about mathematical proofs, knowing when a chain of arguments is a proof or could become one, being able to think up and construct chains of arguments and develop them into proofs, amongst others.

When the students and the teacher discussed whether something was a proof, or perhaps could be developed into one, they indirectly negotiated the meaning they attach to proofs and proving. The understanding of how to argue convincingly for something was never formulated in words. Yet, through the mutual engagement, it was recognized as a competency which is highly valued in the practice, and which is learned through participation.

The classroom situation illustrates the role of the teacher in establishing a practice which involves these competencies and co-learnings, in particular the importance of mutual engagement and challenging students' 'knowledge'.

Generalising, Symbolising, Visualising

The students found a pattern for when two towers of equal height can be built: '*cannot, cannot, can, can*'. With the first rod alone, you cannot build two towers of equal height; with the first two rods alone, you cannot; with the first three rods, you can; with the first four rods, you can; with the first five rods, you cannot; and so forth.

They queried the connection between this and their conclusion about the necessity of an even sum. This led a group to look for a formula with which to determine the numerical value of $S = \sum_{i=1}^{n} i$ for a given n.

They did so by finding the area of a 'triangle' constructed of the rods aligned in order of magnitude.

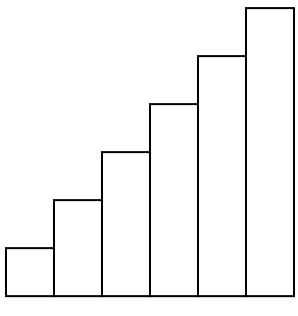

Figure 2: The students lined the rods up in a triangular shape to help them generalise ways of finding the sum of the lengths.

In dialogue between a student at the board, the teacher and the rest of the class, two formulae were developed:

$$S = \frac{n^2}{2} + \frac{n}{2} \qquad \text{and} \qquad S = \frac{n(n+1)}{2}$$

The students convinced themselves that the two expressions are equal.

In our conversations, the teacher mentioned the importance of letting the students learn to use variables through meaningful processes, gradually developing competencies and familiarity. She referred to the above mentioned situation as an example. In that sense, the situation reflects a potential co-learning about what mathematical tools are and how to use them. Symbols make up one such tool. The students were engaged in a practice involving a number of competencies, both concerning representations and translating into symbols, and the teacher ensured the space for this by opening forms of mutual engagement and encouraging negotiation of meaning. From the perspective of social learning theory, we can say that, at all times, the students were involved in learning what it means to communicate mathematically (cf. Lerman, 2005).

In this situation, the teacher gave ample space and time for the students to explore the connection between their two conclusions. She ensured that all the students followed the explanation by the student at the board. She gently guided the process on. She maintained the focus. She asked for links between the visualizations and the symbols. Though the students' focus was on clarifying a particular issue, the teacher's actions both assisted them and provided bearings for what is appropriate in this type of practice, thereby establishing a number of potential co-learnings.

Discussion

It is easy to see the special language, symbols, images, etc. which prevail in this classroom. However, the analysis also indicates the evolving of conventions around what qualifies as mathematical activity. This includes the presence of several of the competencies suggested by Niss (1999, 2004) and of the potential co-learnings discussed by Skott (2003). As the teacher guides the students with her 'bearings', the co-learnings and competencies are both assimilated and negotiated. The students are acquiring identities of participation, which will inform (but not determine) their identity as *mathematics* teachers.

The worksheet introduced a task which offered students the opportunity to work with generalizations, conjecturing, proving, symbolizing, representing, and problem handling. However, the worksheet did not determine the activities. It offered opportunities for engagement, where the students could contribute to the activities and engage with others around the activities in ways which were meaningful to them, and thus the worksheet activities allowed for building of identities within the evolving community.

It was the way in which the teacher interacted with the students around their work, which led to the situation containing potential co-learnings and possibilities for engaging with various competencies. "Teaching must be opportunistic because it cannot control its own effects" (Wenger, 1998, p. 267). The teacher must create possibilities or use existing or evolving possibilities to further the students' mathematical learning; both in the sense of becoming familiar with accepted statements and in the sense of potential co-learnings or familiarity with the mathematical practice and thus increasing ability to communicate mathematically. This requires that the teacher be capable of noting when students' actions and communication contains elements which can inform or be utilized in a mathematical practice. She must try to formulate challenges which contain the invitation to students to engage in mathematical activity. In the planning and particularly in the '*en route* thinking' stages, she must have mathematical awareness which informs her choice of 'bearings' to provide. She must use that mathematical awareness to see when a task or a student's actions contain possibilities to orient the situation in the desired direction—but she must also be prepared to change her ideas of what is the best direction at that time.

On the students' part, this requires taking some level of responsibility for their learning, 'dare to learn' enough to take part in the practice while at the same time acquiring the identity of participation needed in order to learn. Wenger talks about "almost a theorem of love that we can open our practices and communities to others, invite them into our own identities of participation, let them be what they are not, and thus start what cannot be started" (1998, p. 277). This teacher educator is very aware of her role in creating this space for 'daring to learn', for being what they are not. So much so that this is where she puts her attention:

> While I am in the situation, I do not think a lot of theoretical thoughts like "right now, Jeppe is creating his own learning process both of mathematics and mathematics education". Neither do I think "here the dialogue is important, the dialogue which seems to bring out the best in the other [person]." Nor do I simply think that "this thing about moving [the lined up rods] so they fit together can be developed – by Jeppe or someone else – into a proof for the sum of the first n numbers." But I have all of these things in me as a 'readiness'. They take care of themselves; all the while I am thinking "if only I am able to keep the entire group's concentration on Jeppe's narrative". I try to give my energy to his long pauses, his hesitation, his search for words for what he is in the process of realising. What I do and think is directed by an intuitive sense of the most important component of my task: to create the space that allows Jeppe and the others, also the next time around, to dare the vulnerability which comes with the learning process. (Jørgensen, 2000, p. 8)

It was humbling to watch this teacher educator handle the balances of the situation in a way which went far beyond the mathematical aspects of the practice (cf. Christiansen et al., 2003). It illustrates the complexity of practices in which the teacher (educator) invites students to participate through opening forms of mutual engagement, and it illustrates the awareness of mathematical, psychological, sociological, and cultural aspects of the situation it requires of the teacher (educator).

It further illustrated that while we need our teachers equipped with a readiness on all of these aspects, most of all we need them to have an awareness of when something happens which can carry the learning forward for the entire class, and an understanding of what it takes to maintain this 'space'. These 'readinesses', this awareness, and this understanding cannot be taught by explicit means. Our best bet so far is this kind of practice, combined with reflective aspects which can help direct the learners'/students' attention to central features of the practice. Still, while I hope to have illustrated the strengths of this type of practice, we are a long way from knowing and being able to describe what it takes to make such teachers.

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Teachers' and Pre-service Teachers' Gendered Beliefs about Students and Computers

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In this paper findings from various studies are brought together. While there has been much research on teachers' beliefs about mathematics and about pedagogy, less is known about their views about students and about computer use for mathematics learning, particularly with respect to gender-stereotyping. Since mathematics and computing are generally viewed as male domains, it seems appropriate to explore relevant teachers' and pre-service teachers' beliefs. A summary of recent Australian research findings is presented and the implications for the educational community and for mathematics classroom practices are discussed.

Teachers' classroom actions are determined not only by the content they teach, their pedagogical knowledge, or the context in which they work and where learning takes place, but also by their beliefs about mathematics, their students, and pedagogy.

The focus of this paper is on Australian teachers' and pre-service teachers' gendered beliefs about their students and about computer use for mathematics learning. To illustrate what is happening in these areas, I will draw on results from two studies in which I have been engaged in recent years. I will also present the findings from other relevant Australasian research studies. Based on the body of work discussed, implications for classroom practice are put forward.

Previous Research

In the past, mathematics teachers were reported to hold higher expectations for their male than for their female students. Ernest (1975) reported that 41% of the teachers in one study felt that boys were better at mathematics and no teachers said that girls were. When teachers were presented with identical student profiles, some identifying a student as Denis and the others as Denise, and asked for their assessment of what the student would be doing a year later, Denis was considered more likely to be achieving his full potential (Open University, 1986). Parsons, Kaczala and Meece (1982) compared the beliefs and teacher-student interactions of students whose mathematics teachers had high expectancies and low expectancies for their future success. They found that high-expectancy females had the smallest proportion of their interactions praised. McDermott (1983) found that high expectancy students (teachers' expectations for their success in mathematics that year) had more interactions than students of low expectancy, "partly because they initiated more interactions with the teacher" (p.2). Females received more work praise than males, but males had more criticism, more questions that were followed with other questions, and more dyads with brief feedback.

Teachers have also been reported as stereotyping mathematics as a male domain (Leder, 1986), and holding conventional gender-stereotyped attitudes towards the future occupations of their students (Evans, 1982). Teachers of traditionally male-dominated subjects have been shown to have least sympathy towards equal opportunity (Pratt, 1985).

Subtle differences in the way teachers of young children explained the successes and failures of their male and female students were documented by Fennema (1993). Grade 1 teachers also perceived their best male and female students differently with significant differences on several of the 20 traits assessed. Males were considered to display more "competitiveness, logicalness, adventurousness, loudness, volunteering of answers, enjoyment of mathematics, and independence in mathematics" (Fennema, 1993, p.181). Fennema (1993) was cautious about generalising from these and other data about teachers' beliefs. It was suggested, however, that "teachers' beliefs are somewhat negative about females and the learning of mathematics" (p.184).

The findings presented above were all reported some years ago. With research having identified the factors that contributed to the gender differences noted above and many interventions having taken place (see Leder, Forgasz, & Solar, 1996), and with gender equity today expected in all legal, employment, and educational settings, re-examining teachers' gendered beliefs would be expected to yield trivial findings with little evidence of gender-stereotyping. This would be the ideal outcome of such research. In the two studies in which I was involved that are described next, pre-service and mathematics classroom teachers' beliefs about dimensions of mathematics learning were examined for gender-role stereotyping.

Study 1: Pre-service Teachers' Beliefs About Students' Views about the Gender-stereotyping of Mathematics

Recently, the construct *mathematics as a male domain* [the notion that males are more suited to the study mathematics than are females] was re-examined using two new instruments (see Forgasz, Leder, & Kloosterman, 2004; Leder & Forgasz, 2002). A large sample of Australian grade 7-10 students' beliefs were found to be inconsistent with previous research (Forgasz, 2001; Forgasz & Leder, 2000). The vast majority of students was found not to gender stereotype mathematics. However, with respect to certain aspects of mathematics learning, students' beliefs appeared to have changed since earlier times. Girls, for example, were considered more likely than boys to be good at mathematics, to enjoy it, and to find it interesting. Boys were thought more likely than girls to find mathematics difficult and to need additional assistance. Findings such as these appear to challenge notions of mathematics as a *masculine* endeavour. On the other hand, in some respects things had not changed. Boys, for example, continued to be seen as more likely than girls to distract others in class and to tease classmates (male and female) who were good at mathematics.

A slightly modified version of one of the instruments devised by Leder and Forgasz (2002), Who and Mathematics, was administered to a large sample of pre-service teachers in three Australian universities. The pre-service teachers were asked to indicate how they believed students in secondary schools would respond to the items presented. That is, their beliefs about students' gender stereotyping of mathematics were being tapped. The findings suggested that the pre-service teachers believed that students still held the traditional stereotyped views of mathematics (Forgasz, 2000, 2001) including, for example, the complete reverse of the findings mentioned above for students. Similar findings were found with cohorts of US students and pre-service teachers (Forgasz, 2001).

A graph showing the directions of the Australian students' and pre-service teachers' beliefs on each of the 30 items on the Who and Mathematics instrument is shown in Figure 1. Each item on the instrument required the selection one of the following responses:

_ BD: boys definitely more likely than girls
_ BP: boys probably more likely than girls
_ ND: no difference between boys and girls
_ GP: girls probably more likely than boys
_ GD: girls definitely more likely than boys

Scores were assigned to the responses: BD=1, BP=2, ND=3, GP=4, and GD=5, and means calculated. A mean less than 3 indicated that, on average, responses were in the direction "boys more likely than girls", and a score greater than 3 in the direction of "girls more likely than boys". The length of a bar beyond the mid-range value of 3 (the vertical axis of the graph in Figure 1) is a measure of the relative strength of the responses to that item in the "boy" or "girl" direction. For example, it can be seen that Item 16 (distracts others from their mathematics work) had mean scores less than 3 for both students and pre-service teachers, and the long bars in the "boys" direction indicate strong beliefs that "boys are more likely than girls" to "distract others from their mathematics work". A very short bar indicates that, in general, respondents felt that there was no difference between boys and girls (e.g., Item 12, "are encouraged by the mathematics teacher").

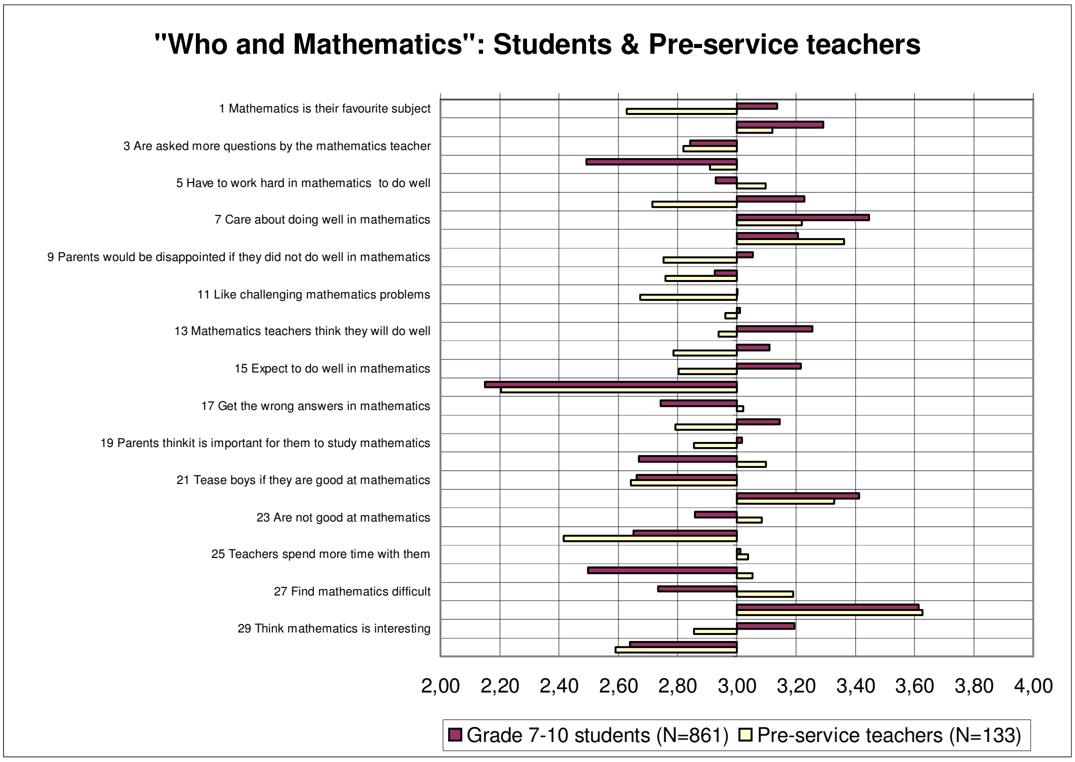

Figure 1. Mean scores on the 30 items of the "Who and Mathematics" instrument: Grade 7-10 students and Pre-service teachers.

Close inspection of the data in Figure 1 reveal that pre-service teachers' responses were frequently in the opposite direction to that of the students. In other words, the pre-service teachers' beliefs about how students would respond to particular items were often completely opposite to how the students actually responded. It was thought that the pre-service teachers' responses actually reflected their own beliefs, thus indicating that they held gender-stereotyped beliefs about mathematics learning that were consistent with findings from earlier research.

Study 2: Teachers' Beliefs about Learning Mathematics with Computers

As part of a larger study on the use of computers for secondary level mathematics learning, grade 7-10 teachers' beliefs about the efficacy of computers for enhancing students' understanding of mathematics were tapped. Teachers were asked if using computers helped students' understanding of mathematics. They responded Yes/No/Unsure to the question. Data were collected from teachers on two occasions: 2001 and 2003. The sample size in 2001 was 96 (F=52, M=44) and 80 teachers (F=40, M=40) responded to the question. In 2003 the sample was 75 (F=41, M=34) and 70 teachers responded (F=40, M=30). The frequencies and percentages of their responses over the two years are shown in Table 1.

The data in Table 1 indicate that about 60% of the teachers in both years agreed that students' mathematical understandings were helped by using computers; about 30% of the teachers was unsure; and less than 10% said "no". In both years there were no statistically significant differences in the response distributions (χ^2 tests were conducted) by teacher gender, although in 2003, a (non-significant) trend for the female teachers to be less positive than the males is apparent – whether this trend continues needs to be carefully monitored in the future.

Table 1
Teachers' responses in 2001 and 2003 about computers helping students' mathematical understandings: Frequencies, percentages, and χ^2 results (by gender)

	2001 (N=96; F=52, M=44)				*2003 (N=75; F=41, M=34)*			
	Yes %	No %	Unsure %	χ^2, p-value	Yes %	No %	Unsure %	χ^2, p-value
All teachers	49 61.3%	7 8.8%	24 30.0%	NA[1]	43 61.4%	4 5.7%	23 32.9%	NA
Females	24 60.0%	4 10.0%	12 30.0%	ns	21 52.5%	4 10.0%	15 37.5%	ns
Males	25 62.6%	3 7.5%	12 30.0%		22 73.3%	0 0%	8 26.7%	

[1] NA – Not Applicable; ns – not statistically significant

The same question was asked of grade 7-10 students in 2001 and 2003, but in terms of their own mathematical understandings. The results are reported in Table 2. The data in Table 2 reveal that in both 2001 and 2003, fewer than 30% of the large samples of students who answered the question agreed that computers had helped their understanding of mathematics; about 40% disagreed and the rest were uncertain. Statistically significant gender differences were found in the response distributions in 2001 and 2003. In each year, a higher proportion of males (2001: 31%; 2003: 33%) than females (2001: 20%; 2003: 25%) believed that computers had helped their understanding.

It was interesting to note that a much smaller proportion of students than of their teachers was positive about the efficacy of computers on their mathematical understanding. In other words, teachers appear to be out of tune with the beliefs of their students. This may be a sign that although the majority of the teachers believe that computers can assist students' mathematical understandings, the computer-based mathematics learning activities they adopt in the classroom do not achieve the expected outcomes.

Table 2

Students' responses in 2001 and 2003 about computers helping their own mathematical understandings: Frequencies, percentages, and χ^2 results (by gender)

2001 (N=2140: F=1015, M=1112, ?=13)									
All students (N=1998)[1]			*Female (N=957)*			*Male (N=1041)*			*χ^2, p-value*
Yes	No	Unsure	Yes	No	Unsure	Yes	No	Unsure	
525	816	668	194	429	334	326	384	331	32.5
(26%)	(41%)	(33%)	(20%)	(45%)	(35%)	(31%)	(37%)	(32%)	<.001
2003 (N=1613: F=810, M=794, ?=9)									
All students (N=1486)			*Female (N=754)*			*Male (N=732)*			*χ^2, p-value*
Yes	No	Unsure	Yes	No	Unsure	Yes	No	Unsure	
427	560	507	185	300	269	239	256	237	12.1
(29%)	(38%)	(34%)	(25%)	(40%)	(36%)	(33%)	(35%)	(32%)	<.01

[1] The number of students answering the question

The teachers were also asked if they believed boys and girls learnt differently when using computers for mathematics. In 2001, 64 (F=33, M=31) teachers responded to the question; in 2003, there were 47 (F=26, M=21) responses. The data are reported in Table 3.

Table 3

Teachers' responses in 2001 and 2003 as to whether boys and girls work differently with computers: Frequencies, percentages, and χ^2 results (by gender)

	2001 (N=96; F=52, M=44)				**2003 (N=75; F=41, M=34)**			
	Yes %	No %	Unsure %	χ^2, p-value	Yes %	No %	Unsure %	χ^2, p-value
All teachers	17	33	14	NA	19	16	12	NA
	27%	52%	22%		40%	34%	26%	
Females	10	20	3	6.5,	9	12	5	ns
	30%	61%	9%	p<.05	35%	46%	19%	
Males	7	13	11		10	4	7	
	23%	42%	35%		48%	19%	33%	

The data in Table 3 indicate that in 2001 27% of the teachers believed there was a difference in the ways boys and girls work with computers; in 2003, 40% of the teachers said there was a difference. This increase in those believing that there was a difference in the ways girls and boys work with computers from 27% in 2001 to 40% in 2003 is a noteworthy finding worthy of future research attention. For the 2001 data, the chi-square test indicated that the male and female teachers' responses were significantly different, with a higher proportion of the female than male teachers saying that there was no difference in the ways boys and girls work with computers, and a higher proportion of male than female teachers being unsure. There was no significant difference in the views of male and female teachers in 2003.

The data gathered from six grade 10 teachers whose mathematics classes were observed in 2002 assist in understanding what the differences in boys' and girls' behaviours with computers are. These teachers completed the same questionnaire as the teachers in 2001 and 2003, as well as being interviewed. They were also asked whether the differences in the ways boys and girls worked with the computers in their mathematics classes affected their classroom teaching approaches. Their responses are summarised in Table 4 (see also, Forgasz, 2003).

Table 4
Teachers' beliefs about their teaching in relation to boys' and girls' computer use in mathematics lessons.

<table>
<tr><td>Teacher: Jack</td><td>Boys/girls different?: Unsure</td><td>Teaching affected?: No</td></tr>
<tr><td colspan="3">Survey: Since I regard computers as a tool, I treat all students the same.

Interview: …in over twenty years of teaching, I think girls tend to be a little bit more careful with the way they're doing things whether… on pen and paper… or doing things on a computer… they will go through the activity in a more methodical manner… whereas I think boys [will] get the job done in a more haphazard fashion… as far as I'm concerned boys tend to think better in 3D, girls find drawing 3D shapes much more difficult than boys do.

With respect to computers specifically: …I've noticed that the girls have got through the program at least as well as the boys have. I would say that a couple of the boys have finished quicker and that may be because…those particular boys…are fairly computer adept, they're only average students… but they picked up the program very quickly and they got finished. The girls… got to the same point a little slower, much more methodically.</td></tr>
<tr><td>Teacher: Kevin</td><td>Boys/girls different?: Yes</td><td>Teaching affected?: No</td></tr>
<tr><td colspan="3">Survey: Boys in this class appear to be the more confident & competent. I'm conscious of it. Let people help each other.

Later, in writing: I feel the girls have less confidence and competence in using computers…I can think of two or three girls… Kate… who struggles in using computers and Lyn who's very quiet… so you don't know whether they're asking or needing help or whether they're getting it from others and… [there are] about two or three girls that seem to seem to hang back, they just idle through things anyway, so you're not too sure whether… that's slowing them down in computers at all… I probably noticed the boys more than the girls… it was just that a few of the boys stood out more as being… able to do things with computers… which didn't make me happy.</td></tr>
<tr><td>Teacher: Fred</td><td>Boys/girls different?: Yes</td><td>Teaching affected?: Yes</td></tr>
<tr><td colspan="3">Survey: Girls need more practice of the concepts learned.

Interview: … my observation that girls naturally are not… as good in mathematics as boys are… [T]hey are better in language skills and they have different strengths than the boys… [It] doesn't apply to everyone, but it's the general trend… [S]o the reason is that because they're…not good in maths as naturally boys are, so I suggest to them to have a bit more practice so the concepts are… more consolidated and they could use it when they need. So I think they need a bit more practice than boys.</td></tr>
<tr><td>Teacher: Irene</td><td>Boys/girls different?: Unsure</td><td>Teaching affected?: No</td></tr>
</table>

<table>
<tr><td colspan="3">Survey: The instructions for the features of the computer software are given to all students. However, it seems that boys tend to NAVIGATE the software more effectively.
Interview: …some of the girls in that class are just sort of helpless types and they don't seem to want to take initiative. Not all of them… some girls who are very independent learners and get on with it. [T]he back row girls, they tend to just put their heads down and do it, and the others… seem to lack the confidence just to do it… I'm just… thinking of the girls that always seem to lag behind and always seem not sure of where to go next.
Probed further about girls' learning of mathematics, Irene added: I think the girls tend to be more reticent… but I'm generalising again, I think the boys tend to be a bit more vocal in sharing some of their ideas…[and] to be a bit physically bigger… there's more of a presence of them… I think girls tend to fade that little bit into the background…. I can think of counter examples where the girls dominate and are vocal and tend to volunteer what they know, maybe more in the junior classes… but I think maybe in the senior classes perhaps the girls tend to be a bit more quiet.
Effect on teaching: Well I have to be aware of drawing the girls in, making sure they're not ignored or feel that I'm only directing my attention to the boys …and not giving the girls a chance…</td></tr>
<tr><td>Teacher: Edna</td><td>Boys/girls different?: No</td><td>Teaching affected?: No</td></tr>
<tr><td colspan="3">Interview: …some of them [students] are very resistant to actually learning mathematics when they're in the computer room. They think it's for playing games. As you noticed I had to tell quite a few of them to get out of the games and get back to the maths… [O]thers will get stuck into what ever it is you give them to do and rise to the challenge, like the girls who were working out how to do the surface area of a cone without being given the slant edge… they were really extending themselves today. But it's difficult because you spend so much time trying to keep the ones who either don't have the skills or aren't interested in doing the work on track but you don't have a lot of time to spend with those kids who are….It would be really great if we could… but it doesn't happen in real life.</td></tr>
<tr><td>Teacher: Kathy</td><td>Boys/girls different?: Yes</td><td>Teaching affected?: Yes</td></tr>
<tr><td colspan="3">Survey: Style of class offered. [Unfortunately, relevant questions were omitted from interview]</td></tr>
</table>

Source: Forgasz (2003, pp. 355-356)

From Table 4, it can be seen that Jack, Kevin, Fred and Irene (pseudonyms) identified differences in the behaviours of boys and girls in their mathematics classrooms and/or in using computers for mathematics learning. Fred's belief that males' had *natural* mathematical superiority over females was the basis of his conclusion that girls needed more practice with computers. Neither Jack nor Kevin had reflected on their observations. They were concerned about the girls' less functional behaviours but did not appear to have strategies to address them. Although not specific to computer use, Irene had made similar observations and was able to describe the actions needed to deal with the situation. Edna's specific observations about the girls getting on with a computer-related mathematics task were consistent with previous research about girls' behaviour in mathematics classrooms. It can be inferred from the teachers' comments that they believe that students who are competent with computers, rather than necessarily mathematically strong, gain most from computer use for mathematics learning. It was boys who were seen as more computer savvy than girls.

In the next section, I summarise findings from a few other Australasian studies that involve gender-stereotyping in mathematics education.

Other Australasian Studies

Lee (2002) observed that early childhood teachers were five times more likely to identify and nominate boys than girls for a mathematics and science enrichment program; these findings echo past research results indicating that boys are more likely than girls to be identified as gifted in fields gender-stereotyped as male (see Gordon & Addison, 1985). Sixteen early childhood teachers who had identified students for the program were interviewed and a model of teachers' conceptions of giftedness among young children was developed. Lee concluded that the teachers' conceptions were overlaid with their beliefs about gender, and that girls were disadvantaged by each of the seven categories of giftedness that guided teachers' behaviours. Lee contended that girls would have significantly fewer opportunities than boys to be identified using the model that emerged.

Wood, Viskic and Petocz (2003) examined gender differences in the use of technology in three tertiary mathematics learning environments. The teachers and students in three different mathematics subjects were involved. The teachers all adopted inclusive pedagogical practices with respect to the learning environment they created, the assessment methods and teaching materials they adopted, and in monitoring their own teaching. Students' attitudes towards the use of computers were gathered. There were no gender differences found in attitudes towards computers or the use of computers. The authors concluded that the use of inclusive practices may be a contributing factor in eliminating gender differences in attitudes to the use of computers, and that group work may account for the positive attitudes among females.

Summary of Findings

There appear to be some common patterns evident in the findings from the studies described above.

- The pre-service teachers in Study 1 and the classroom teachers in Study 2 seemed to be out of touch with students' views with respect to different dimensions associated with mathematics learning. The pre-service teachers believed that students would hold patterns of gender-stereotyped views that are likely to have been prevalent when they were in high school some years earlier. Classroom teachers were more positive than their students about the likely impact of computer use on students' mathematical understandings.
- The classroom teachers in Study 2 whose mathematics lessons had been observed were all very experienced. Their views of boys' and girls' behaviours with computers and their explanations for them reflected fairly traditional gender-stereotyped expectations consistent with beliefs that males are more suited to mathematics (and technology) than are females, that is that mathematics is a *male domain*, as is computing.
- Gender-stereotyped expectations can also be inferred from the identification by early childhood teachers of more males than females for participation in a gifted mathematics and science programme.

- The findings from the study of tertiary mathematics learning settings suggest that pedagogical approaches may contribute to gender differences in students' beliefs about technology.

Conclusions and Implications

For the Educational Community in General

There are some in the wider educational community who no longer believe that gender is an educational issue. Among others there is growing concern about boys' educational disadvantage. Research findings and examination results support contentions that girls excel academically over boys in many subject areas, including mathematics in some contexts. It is also clear, however, that girls are disadvantaged in the hard sciences, mathematics and computing fields, particularly with respect to enrolment numbers; females clearly remain under-represented in many of the relevant academic fields, particularly at the highest levels, and in related careers.

The findings from the studies reported in this paper indicate that it cannot be assumed that mathematics teachers or pre-service teachers hold gender-balanced beliefs about their students or about computer use for mathematics. A second concern relates to teachers and pre-service teachers being out of touch with contemporary students' beliefs which, as has been argued elsewhere (see, for example, Leder & Forgasz, 2002) and is apparent on the graph included in Figure 1, appear to be changing with respect to aspects of the gendering of mathematics.

When dealing with sensitive issues such as gender stereotyping, it is not always easy to identify or address stereotyped beliefs or behaviours. This suggests the continuing need to raise gender-related issues within political circles, and with school administrators, teachers, parents and students. Discussions should be encouraged and appropriate actions taken to eliminate any factors that have been identified as contributing to gender differences at any level of education. In pre-service education programmes, it is important that gender issues are brought to the forefront and that the pre-service teachers are challenged to confront their own belief systems. It is important that they become aware of the potential educational consequences of holding stereotyped beliefs and expectations of students and their capabilities.

For Mathematics Classroom Practice

What teachers believe is a subset of what they know. As can be inferred from the findings in Study 2 described above, and the studies about the selection of gifted young children for science enrichment and of tertiary mathematics teaching with technology, gendered beliefs can lead to actions that favour one group of students over another. In other words, girls are likely to be disadvantaged in the classrooms of mathematics teachers who hold gender-stereotyped beliefs about mathematical aptitudes and/or technological skills in favour of boys. This conclusion is not new. However, considering the extent of knowledge in the field of gender and mathematics, and the many interventions that have been undertaken in many Western societies to try to address girls' identified disadvantages in mathematics learning outcomes (see, for example, Leder, Forgasz & Solar, 1996), gender-stereotyped beliefs and the classroom practices consistent with them are not expected to be widespread in contemporary mathematics classroom settings.

What can be done to address these lingering vestiges of gender-stereotyped beliefs among mathematics teachers? How should efforts to effect further change be focused? Are inclusive classroom practices, as was the case in the tertiary mathematics study described above, a solution? If so, what exactly does this mean for mathematics pedagogy at the various grade levels, and how can inclusive strategies become commonplace in mathematics classrooms? What kind of professional development is needed and what form should it take? Should teachers be asked to monitor their own classroom practices, then confront and challenge what they find?

The main point that has emerged from the studies discussed in this paper is that gender issues in mathematics education have not disappeared. In the continuing pursuit of educational equity generally, and equity in mathematics education in particular, gender needs to remain on the agenda and continued research efforts in the field must be encouraged and supported.

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Teacher-Developed Professional Standards for Excellence in Teaching Mathematics: Progressing Teachers' Professionalism

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The national association of teachers of mathematics in Australia has embarked on a journey to enhance professional status, and to help re-engineer teacher education to enable purposeful and owned professional growth throughout teachers' careers. The vehicle for this journey is the development of nationally agreed professional standards and assessment of volunteer teachers against these standards. The first phase involved research and development of materials to 'define' quality teaching of mathematics—the AAMT *Standards*. The second phase of implementation is in two parts. One is a focus on professional development using the Standards. The other—the focus of this paper—is to develop a process for acknowledging outstanding teachers and awarding them the AAMT credential of Highly Accomplished Teacher of Mathematics.

Introduction

This chapter outlines work being done by the 'profession' of teachers of mathematics (as represented in their national professional association) in response to the fundamental questions of defining and describing high quality teaching of mathematics, and how this can enhance the professional work and lives of teachers of mathematics. It is a report and reflection on work in progress on the implementation of the Australian Association of Mathematics Teachers (AAMT) *Standards for Excellence in Teaching Mathematics in Australian Schools* (AAMT, 2002). The Association has three strands for this implementation:

Recognition—The *AAMT Teaching Standards Assessment Evaluation Project*[2] (TSAEP) was conducted between July 2003 and May 2004. The aim of the project was to pilot the assessment of volunteer teachers against the AAMT *Standards*, and award those who are successful with the AAMT credential of *Highly Accomplished Teacher of Mathematics*. The research and development undertaken by the TSAEP will assist the AAMT to reach its goal of a highly credible and recognised AAMT acknowledgment for those teachers who choose to present themselves for assessment by their peers and who successfully demonstrate that their work is at the standard set by the AAMT *Standards*.

Professional development — The AAMT *Standards* are the profession's consensus view of good teaching of mathematics. Hence the Association promotes the view that the *Standards* are the framework for teachers' career-long professional development. While this strand has not been the focus of a specific project, progress has shed some light on the potential of the *Standards* to be a significant contributor to teachers' professional lives in the future.

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[2] The work of this project was supported by funding from the Australian Government under its Quality Teaching Programme. The views expressed are not necessarily those of the Commonwealth of Australia.

Influence—The AAMT's leading edge work has been informative of wider efforts to enhance teacher professionalism through the development and use of professional teaching standards.

In order to discuss the first two strands more fully it is necessary to outline the background and educational context of the work.

Background and Educational Context

In 1998 the Australian Senate Employment, Education and Training References Committee released *A Class Act;* a report on an inquiry into the status of teaching. In commenting on the issue of standards of professional teaching practice "the Committee insists that establishing … standards of professional teaching practice is possible, unavoidable and absolutely necessary" (p. 16).

Since that time there has been a growing commitment to 'standards' in teaching in Australia. Importantly, there is universal commitment "that [the standards] are determined by the profession itself" (Australian Senate Employment, Education and Training References Committee, 1998, p. 17; see also Commonwealth of Australia, 2003a).

The Australian Association of Mathematics Teachers resolved to take some constructive action in the area, namely to develop a description of good teaching and a means for acknowledging the work of those teachers who achieve this standard.

The Association's resolve was strengthened by two further contextual factors. Firstly, the National Council of Teachers of Mathematics (NCTM) in the USA had published its *Professional Standards for Teaching Mathematics* (NCTM, 1991). This work of the NCTM has been further developed by the National Board of Professional Teaching Standards, the body in the USA that has developed and implemented a system of credentialing high-achieving teachers in a comprehensive set of discipline and age related areas (National Board of Professional Teaching Standards, 1989; 1996).

Also, the Australian Science Teachers Association had been keen to work in this area for several years (Ingvarson, 1998) and this helped persuade the AAMT that work in the area by professional associations was both feasible and essential in the growing climate of interest from employers and others in the development of professional teaching standards.

The AAMT and colleagues from the Education Faculty at Monash University (Melbourne) conducted a research and development project *Excellence in Teaching Mathematics: Professional Standards Project* over the triennium 1999–2001.

The aims of the research project were to:

- determine consensual views on national professional standards for excellence in teaching mathematics in Australian schools (henceforth called the *Standards*), and
- develop an assessment scheme and protocols for certifying this excellence.

The project used a grounded approach (Glaser & Strauss, 1967) that continually sought the views of teachers, synthesised these and reflected the synthesis back to the teachers for confirmation and/or modification. One of the core intentions was to ensure that the *Standards* embrace diversity in teaching mathematics, whether this is in terms of teaching approaches, level of schooling (i.e., age of students), type or location of school. The slogan '*Standards* not standardisation' was adopted within the project as an indication of the commitment to this diversity. The approach in development tried to ensure that voices representing this diversity were represented, although restricted funding limited the direct involvement of teachers from regional and remote locations. The inclusivity of the

Standards is constantly being monitored, and to date no teachers have reported that they do not see themselves reflected in them.

The AAMT's *Standards for Excellence in Teaching Mathematics in Australian Schools* was adopted by the Association's Council in January 2002, subsequently published and widely distributed. The AAMT *Standards* are:

- nationally agreed;
- applicable K–12, and in all teaching contexts in Australia;
- brief (an A3 folded sheet), but there is extensive Web-based supporting material that helps them 'live' in readers' minds.

There are ten professional teaching standards, arranged in three domains (see Table 1).

Table 1
The AAMT Standards *arranged into three domains*

Domain 1 — Professional Knowledge	**Domain 2 — Professional Attributes**	**Domain 3 — Professional Practice**
1.1 Knowledge of students 1.2 Knowledge of mathematics 1.3 Knowledge of students' learning of mathematics	2.1 Personal attributes 2.2 Personal professional development 2.3 Community responsibility	3.1 The learning environment 3.2 Planning for learning 3.3 Teaching in action 3.4 Assessment

The following example is used to illustrate the style of the AAMT *Standards*. It shows that this work—by teachers, for teachers—goes to the very heart of what it means to be a teacher of mathematics.

> **2.1 Personal attributes**
>
> The work of excellent teachers of mathematics reflects a range of personal attributes that assists them to engage students in their learning. Their enthusiasm for mathematics and its learning characterises their work. These teachers have a conviction that all students can learn mathematics. They are committed to maximising students' opportunities to learn mathematics and set high achievable standards for the learning of each student. They aim for students to become autonomous and self directed learners who enjoy mathematics. These teachers exhibit care and respect for their students.

The AAMT *Standards* can be downloaded from www.aamt.edu.au/standards. At this website the *Standards* are supported by a wide range of materials and a full description of the work undertaken by the AAMT on them.

In parallel with the AAMT's work on professional standards there has been a great deal of activity in the area of professional standards in education in Australia generally. Some state governments have created 'institutes of teachers', and a wide range of professional associations and organisations have become interested in professional standards. The issue of codified teaching standards has become extremely political. In this context the AAMT has pursued its goal of creating a comprehensive system that supports teachers' professional growth and provides for recognition in ways that promote their professionalism.

What has distinguished the AAMT work from much of the rest of this activity is the fact that the Association has demonstrably maintained commitment to the principle of 'by the profession, for the profession'. The Association maintains complete ownership of the work, and autonomy of action. Growing as the *Standards* have done from this earlier project conducted using practising classroom teachers of mathematics, the implementation of the *Standards* has been a real developmental effort and there are tangible results. Many others developing similar frameworks appear to have satisfied themselves with merely talking about standards. As a consequence, the AAMT work has high status across the board, and the Association has access to and respect from political and bureaucratic decision-makers on behalf of its members.

The Teaching Standards Assessment Evaluation Project

The Teaching Standards Assessment Evaluation Project (TSAEP) was commenced in July 2003 with funding from the Australian Government. In negotiations with government officials it was apparent that their view is that the AAMT work has the potential to continue to lead the way from the development of teaching standards to their ultimate implementation to enhance the quality and status of teaching. The project was designed to significantly advance knowledge and practical capacity in relation to assessing teachers against professional standards. It has piloted and evaluated the Assessment Model developed during the initial three-year research and development project. The Model has three components that are discussed more fully below. These are a Written Assessment, a Portfolio and an Interview.

Conduct of the Project

In order to be able to assess the candidates at the end of the project using the agreed Assessment Model, a number of tasks needed to be undertaken within the project. Important among these were:

- Developing Guidelines to assist candidates to prepare their Portfolio items. This included commissioning 20 sample items from volunteer teachers to help refine the Guidelines.
- Recruiting the six candidates. They were all from different states; four were female, two male; four primary, two secondary; four teach in government schools, two in non-government schools[3].
- Providing support for the candidates during the process. This included providing general advice, sample Portfolio material and sample Written Assessment items, and enabling candidates to work with a mentor.
- Selecting and training peer Assessors (five teachers from four states).
- Developing the items for the Written Assessment. The work situations of the candidates required only two papers (primary and junior secondary).[4]

The constraints of the project required divergence from the model in several ways. These included only 3–4 months for Portfolio preparation compared with the 6–24 months allowed in the Model; the Written Assessment being held at the end of the process of preparing the Portfolio, rather than the beginning; the Interview being held only hours after

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[3] The sample did not include any teachers from rural or remote locations. Further work is required to determine any impact of a candidate's geographic location on the assessment process.

[4] It is anticipated that separate papers for up to four levels of schooling may be required (early childhood, primary, junior secondary or middle school, senior secondary) in order to target items appropriately.

Candidates received feedback on their Portfolios as opposed to the anticipated 2–3 weeks; the questions to be asked in the Interview being made available in the same time frame, rather than, again, the anticipated 2–3 weeks; and support materials being developed during the process rather than being available from the start.

The Assessment Model

The Assessment Model is based on a commitment from the AAMT to its members and the mathematics teaching profession more broadly that its assessment process will be:

- rigorous and valid;
- adaptable to and applicable in all teaching contexts;
- fair to all candidates no matter what their teaching situation;
- equally accessible to teachers across the country;
- controlled by the candidate insofar as this is possible; and
- oriented towards contributing to professional growth of the candidate—both the process itself and the feedback provided to all candidates.

The Model requires Candidates to:

- respond to unseen questions that simulate teaching decisions[5] through a Written Assessment;
- submit a Portfolio of their work and achievements as a teacher; and
- take part in an Interview.

These are discussed in some detail—with illustrative examples—in the sections that follow a discussion of the overall approach to assessment in TSAEP.

Overall Approach

The AAMT established a committee—nine teachers and three others (a distinguished mathematician, a researcher/teacher educator and its Executive Officer) to manage the project. The committee sought to implement the principles outlined above in the creation of the final assessment process. There were some compromises, of course, and part of the evaluation of the TSAEP identified these and their effects.

However, the two fall-back questions that were used to resolve issues reflect the teacher-oriented spirit of the committee's approach:

- Would this be something I would see as reasonable for myself?
- What would we do if we were trying to assess students in this way?

As a result of protracted discussion and debate, the following approach was decided upon for the TSAEP and a group of assessors trained accordingly.

- Assessment was to be directly against the *Standards*. The evidence presented by the candidate would be accumulated against each standard. There would be at least two assessors for each Candidate, who would reach a consensus decision as to whether

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[5] As an example, from the junior secondary paper:

Having accepted that there are 360° in the interior of a rectangle, Peter is now considering a parallelogram. "I think it will be 360° again, because if you push it a little it will become a rectangle."

The candidates were asked to discuss the background knowledge being demonstrated, the 'appropriateness' of the students' comment, and how the student could be moved on in their knowledge and understanding. See also Fig 1.

the candidate had presented sufficient evidence to convince them that they have reached the standard.

- There were no formal rubrics, checklists, scoring guides or whatever. Assessors were able, however, to use these mechanisms if they assisted their identification and weighing of evidence.
- In the Portfolio the links identified by the candidate between the materials presented and the *Standards* had primacy. Assessors noted instances where what appeared to be positive evidence that could be cited that was not referred to by the Candidate. Assessors made a decision to ignore evidence that they believed was negative, which could be seen as a controversial decision, but not one that had to be implemented in the trial.
- All Candidates received constructive written feedback, prepared and agreed upon by the relevant assessors. This helped candidates—both successful and unsuccessful—to set further learning and development goals.

The Written Assessment

The Written Assessment consists of a series of questions seeking candidates' personal responses. The time allowed for responses is limited—up to three hours was allowed in the trial. The questions simulate teaching decisions and include commenting on student work and responding to hypothetical situations. The context and content of the questions are linked to the candidate's level of schooling.

Generic prompts are used, such as:

- What is mathematically interesting/worrying/unusual in what the student has done?
- Why might s/he have done this? (eg what might be the misconception(s)?)
- How might the teacher respond? What next steps could be useful?

These prompts provide the candidates with appropriate opportunities to frame their responses against the *Standards*, as expected by the process.

Other types of item also expect candidates to draw on their knowledge of mathematics and how students learn mathematics, but in a context that allows them to demonstrate how they would work with others. For example, Figure 1 simulates an interaction with a parent on a topic that remains somewhat controversial in Australia.

At the orientation parent meeting at the beginning of a new school year a parent says that she does not want her child in middle primary to use a calculator until "he knows all his times tables off by heart". How might you respond?

1. You start by outlining some of the ways you might use calculators in your teaching during the year…
2. For one or two of these you go into a bit more detail by identifying why each of the approaches is an advantage for the students' learning of mathematics, how it enhances their learning, etc.
3. The parent has raised the issue of "times tables" and you feel you need to make some comments on the strategies you will be using in this area during the year. (You may or may not talk about strategies that use calculators—it depends on what you actually do.)
4. Are there any other comments you would make, either directly to the parent or as notes to yourself?

Figure 1. An example from the Written Assessment.

In the TSAEP candidates had four questions to consider in a two-hour period. Assessors expected responses that would demonstrate the breadth and depth of candidates' knowledge and experience bases, and their capacity to access these in order to inform their decision-making. That is, it was expected that the evidence drawn from the responses to the Written Assessment items would relate mostly to Domain 1 Professional Knowledge from the AAMT *Standards*[6]. Both of these expectations were met during the trial assessment, with the Assessors confident that these candidates' responses would have resulted in each of them being recommended to proceed with their Portfolios.

The Portfolio

There are five compulsory items that need to be included in the Portfolio. This component of the assessment process is the most substantial and varied. It provides the greatest amount of evidence of the candidate matching the *Standards*.

The Portfolio consists of:

- **Professional Journey**—a brief reflection on the candidate's professional life as a teacher of mathematics;
- **Current Teaching and Learning Practices**—an example of current/recent classroom work;
- **Case Study**—an example of the candidate's efforts over time to address a particular issue(s) with one or a few students;
- **Validation**—some 'objective' material that attests to the candidate as a teacher (video or audio tape of a teaching episode; a report on a structured observation by a peer[7] designed to give a short snapshot of the teacher at work);
- **Documentation**—material that demonstrates the range and quality of the candidate's work and achievements.

This selection has been criticised for not explicitly including an item on the achievement of a whole class over a period of time[8]. Such an entry would provide valuable evidence, but it was decided that it would require too much work in the documentation in order for candidates to do it in line with the comprehensive view of assessment in mathematics in Standard 3.4:

> Excellent teachers of mathematics regularly assess and report student learning outcomes, both cognitive and affective, with respect to skills, content, processes, and attitudes. They use a range of assessment strategies that are fair, inclusive and appropriate to both the students and the learning context. (AAMT, 2002)

Table 2 identifies the *most likely* links between the components of the Portfolio and the ten *Standards*. This table is intended to be used as a guide to assist candidates in the preparation of the items.

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[6] In fact, the evidence is mostly about Standards 1.2 Knowledge of Mathematics and 1.3 Knowledge of Students' Learning of Mathematics.

[7] Difficulties relating to privacy have meant that, in practice, candidates in TSAEP all chose to submit an observation report for this item.

[8] Some evidence of whole class progress is expected in the Current Teaching and Learning Practices entry, however.

Table 2

Likely links between portfolio components and Standards

	1.1	1.2	1.3	2.1	2.2	2.3	3.1	3.2	3.3	3.4
Professional Journey				*	*	*				
Teaching and Learning	*	*	*				*	*	*	*
Case study	*		*	*				*	*	*
Validation			*				*		*	
Documentation				*	*	*				

The instructions to candidates indicate that additional items may be submitted, but these must be accompanied by a rationale that justifies that inclusion. Extra items are likely to be worth the extra time and effort only if they relate to an unusual aspect of their teaching that the candidate feels is important to highlight. It is in this section of the Portfolio that it is possible for candidates to include evidence from parents and students. The HAToM process is open to all teachers across the range of years of schooling, so it was believed that framing and mandating structures for formal student input in particular would be impractical due to the wide range of ages and experiences. In the case of formal parental input there would be other issues, with teachers in schools with a more educated and articulate parent group likely to be at an advantage[9].

The advice to candidates insists that they identify both the actual *Standards* to which they believe the material is relevant and how this evidence demonstrates their achievement of the *Standards*. For example, the following is a small extract from a sample Case Study that describes a teacher who has found that a student has serious deficiencies in relation to graphs and the equation of a straight line (the teacher's linking to the *Standards* in italics):

> It seemed to me that Anthony needed to develop a link between equations, tables of values and their graphs. This type of work would have been encountered in earlier years but for some reason or other this has not been successful for Anthony. I believed that a fresh approach was necessary so I decided to make use of the graphical calculator that Anthony owns to do some work that required him to enter an equation, look at the graph and look at the table of values generated to establish the relationships. This work was done prior to commencing the work on bivariate data as it is necessary background to be able to calculate equations of lines of best fit. This was also good revision for the rest of the class so it was done as a part of the teaching and learning sequence. When ever possible I allowed Anthony to print his graphs from his calculator rather than producing them by hand as I thought that this may appeal to him as he would not have to produce neat work by hand- something that he finds difficult.
>
> *I believe that this demonstrates Standard 1.3 and Standard 3.2 as it establishes an appropriate sequence of learning for this student based on the skills they already have established and the technology available to them. It is important to establish which skills the student already has in order to be able to develop further skills.*

The candidates in the TSAEP understood this expectation to reflect on their work and to discuss it against the AAMT *Standards*, throughout their Portfolio. That they did so and did so successfully in a variety of methods is a strong indicator that the *Standards* provide a

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[9] Nevertheless, it is clear that evidence from students and parents can be very powerful. It is expected that examples of how others have drawn on this type of evidence will assist candidates to do likewise.

framework and a common language for talking and thinking about high quality teaching in the Australian context. This also allowed for a more focussed assessment of the Portfolio, as it identified to the Assessor where the candidate believed the evidence was located.

The Interview

The Interview provides further confirmation of evidence, as well as an opportunity to clarify any matters that were not fully understood from the Portfolio items. Prior to the Interview the Candidate is advised of any particular areas (if any) about which further information and evidence is being sought. The Interview is normally held by teleconference as candidates and assessors are in geographically separate locations.

In the TSAEP Candidates were given only two hours to consider the questions the assessors had developed, which did not prove to be too difficult for them, as they had most of their material with them. The TSAEP assessment occurred over a period of three days and was very intensive in terms of time in a manner that will not be replicated in the future. Rather, assessors will take perhaps a month to assess the Portfolio. Candidates will then be given the Interview questions with two weeks' preparation time.

Results of Assessing the Candidates

Four of the six Candidates in the TSAEP were recommended to receive the Highly Accomplished Teacher of Mathematics (HAToM) award. Two teachers who presenting insufficient evidence in relation to some *Standards* were advised of the areas in which there was insufficient evidence, and a process was established to give them further opportunities to pursue the accreditation.

Findings

An independent external evaluator used a 'participant-observer' methodology to report on the project according to an agreed brief for that work. Observations, document analysis, interviews with participants, participants feedback provided the data for an extensive Evaluation Report. The report contains a wealth of information that can guide the thinking of the AAMT and others in relation to assessing teachers against professional teaching standards. The most significant results of this evaluation follow:

- The TSAEP found that the Assessment Model works—candidates are validly and reliably identified as HAToMs.
- The assessment process is able to discriminate among teachers.
- The Model and the Guidelines are both transparent and flexible in allowing teachers to exercise some control over the form of their submissions.
- The significant time and effort required by teachers to compile a case for accreditation impacts on their personal and professional lives.
- The assessment process is professionally demanding and time-intensive for the Assessors.
- Conducting assessments according to the Model has a significant monetary cost.
- Being assessed using this Model is likely to appeal to a relatively small number of teachers.
- The component of the *Standards* relating to assessment of students' learning (Standard 3.4) was the most difficult to identify. Possibly this was because the

long-term achievement of students is not included in the methods of assessment the candidates undertake.

Is this Approach Sustainable?

The approach can only be sustainable if there is a sufficient pool of candidates for whom the benefits of undertaking the assessment outweigh the costs. The Project provides some insight into likely costs for candidates:

- Time: Likely to be a minimum of 70-80 hours (50-60 hours for the Portfolio; 20 hours to prepare for and take part in the Written Assessment and Interview, and for general preparation and administration).
- Monetary: This has now been set at $AU1550 (allows for payment to Assessors for their work, a contribution to the ongoing training of Assessors, AAMT administration and other personal costs such as stationery, postage etc.).
- The benefits identified by these Candidates are their professional learning, affirmation of their work by their peers and recognition in their school community.
- In the current climate, a monetary cost of over $AU1000 to the individual would outweigh these benefits for the vast majority of teachers. A financial incentive such as promotion, reclassification, a salary increment or similar would make the process more appealing, as would arrangements to subsidise the monetary costs of assessment through scholarship schemes funded by employers, education authorities, unions, institutes of teaching or the private sector.

Professional Development of Teachers of Mathematics

In addition to using its professional teaching *Standards* as the basis for the HAToM recognition system, several more general connections between the *Standards* and teacher professional learning in mathematics are being explored by the AAMT. These are discussed below—the first is the central goal; with the others strategies assisting in reaching that goal.

AAMT Standards *as the Framework for Professional Development*

The *Standards* are the profession's statement of what high quality teaching of mathematics looks like, what highly accomplished teachers know and do, and what they are like. They therefore provide a framework and a common language for talking and thinking about high quality teaching in the Australian context.

An instructive example of the integration of the *Standards* into programs of professional learning is a program entitled *Engaging with Excellence in Mathematics Teaching: Creating Excellence in the Learning Environment*. This pilot teacher professional development program was developed and conducted by the Australian Council for Education Research (ACER) in collaboration with the AAMT in 2004.

The catalyst for the partnership was the release of some classroom videos from the *1999 Third International Mathematics and Science Study* (TIMSS) Video Study (National Center for Education Statistics; 2003). ACER was the Australian partner in this project. The videos are acknowledged as an outstanding resource for teacher development, and it was agreed that these would be used as the framework for the teachers' investigations. Standard 3.1 deals with the learning environment and this is the focus area for this professional development program; the videos capture learning environments (physical, intellectual, emotional) in mathematics classrooms in a variety of countries. In this professional development program, participants used the AAMT *Standards* to self identify their learning needs in mathematics. Another learning activity involved them in analysing, describing and discussing the learning environments represented in selected videos by using the framework and language of the *Standards*. Participants then went on to express their particular learning goals for this program, monitor their progress and note their success.

The pilot program concluded in August 2004. Preliminary analysis of feedback from the 14 or so participants—all of whom had no previous detailed exposure to the Standards[10]—is that they found the document useful to very useful in identifying their professional learning needs. "The Standards self-evaluation form in particular was identified as a most useful instrument" (Peck, Hollingsworth & Morony, 2004, p. 375). Participants reported that the Standards assisted them in working with colleagues who were not in face-to-face sessions and as a means for focussing their learning. "Overwhelmingly, all teachers felt that they had significantly improved their awareness and appreciation of the AAMT Standards and were able to identify ways that their practice (or that of their colleagues) had moved closer to the Standards" (p. 376).

The attempt to focus on a single standard was not successful, however. As soon as real life school initiatives were developed these inevitably reflected a wide range of attributes of good teaching of mathematics beyond the single Standard about the establishment of a positive learning environment. The program is to be revised and piloted again, with a view to being made available across the nation.

Some Other Strategies

The Professional Learning Using the Mathematics Standards (PLUMS) project[11] is trialling the use of the AAMT *Standards* in the context of in-school professional learning programs. Sixteen schools in two clusters (one a capital city, the other in a regional centre) will report in 2006 on the efficacy of the *Standards* for setting targets, guiding actions and monitoring progress of school-based programs for teachers across the spectrum of experience and achievement.

The *Standards* and supporting materials on the website are an immediate source of material for information and inspiration. The self-assessment form of the AAMT *Standards*— is also beginning to be used by individuals and groups of teachers to analyse strengths and weaknesses as a key component of needs analysis.

For those whose role is to provide professional development programs, it is the AAMT's hope that the *Standards* will come to be used to frame and describe their offerings. At a practical level, the Association has begun to link its professional development programs to the areas of the *Standards* that presenters/facilitators expect to address. At the two most recent AAMT biennial conferences[12] each of the parallel sessions (papers, workshops, roundtables) was coded in the program to identify the links to the *Standards*. State and territory associations of mathematics teachers are committed to progressively implementing such a scheme.

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[10] Indeed, some were not even aware of their existence.

[11] The PLUMS project is funded by the National Institute for Quality teaching and School Leadership (NIQTSL), an independent body established by the Australian Government in 2004.

[12] *Mathematics ~ Making Waves* in January 2003 and *Making Mathematics Vital* in January 2005.

This use of the *Standards* as a framework for professional development offerings will, in the AAMT's view, extend beyond the professional development programs of teacher associations to those of education authorities and higher education institutions. While this has yet to occur, there is strong support from the Report of the Review of Teaching and Teacher Education[13] in its Agenda for Action (Commonwealth of Australia, 2003b). This report will be very influential in the next few years in Australian education. Action 39 is the most pertinent:

> The professional learning opportunities provided by employers of teachers, higher education institutions and teacher professional associations be directed to the achievement of the standards to be established for advanced teaching competence… (p. 39)

Materials developed to support the assessment process of the HAToM credential have a natural use in formal and informal professional development of teachers:

- The samples to illustrate the various components of the Portfolio provide stimulus for professional discussion
- Likewise, the sample Written Assessment items are able to be used as stimulus material. The public release sample items are a resource for teacher professional development. They are similar to the Case Methods approach to teacher professional development in mathematics (Barnett et al., 1994)
- The processes for documenting practice in the five Portfolio components could form the basis of a professional learning activity, without any intention of presenting material for assessment.[14]

There is evidence that those who engage with the assessment tasks associated with the HAToM credential find them to be outstanding professional development activities. The teacher who prepared the sample Case Study outlined above commented

> The process (of thinking about the issues, focussing in on a single student, collecting the bits and pieces) is fabulous P.D. and I had no trouble in carrying out that part (other than finding the time!).

One of the teachers, who has received the HAToM credential as a result of the TSAEP, has highlighted how the assessment process supported her professional growth:

> The second and third components[15] were fascinating to undertake. They forced me to become conscious of that which, like most experienced teachers, I do unconsciously and don't usually think about. By raising my level of awareness of what I do both with classes as a whole (as in the unit of work) and with individual students I found myself critically examining my practice… these two sections were some of the most excellent PD I have undertaken. (Reynolds & Morony, 2004)

Issues Relating to the *Standards*

The HAToM process provided the greatest test of the *Standards* since their publication in 2002. This and the other activities underway highlight many issues relating to the *Standards*. The first of these is the need for a process to monitor the *Standards* and to provide a mechanism for ongoing review. The teaching of mathematics is seen as a

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[13] The Review's foci were teachers and teaching of mathematics, science and technology, and creating a 'culture of innovation' in Australian schools.

[14] For example, a faculty could commit to each preparing a case study along the lines in the HAToM process and sharing their work and reflections in an in-school professional learning program that is initiated, supported and managed by the school.

[15] Current Teaching and Learning Sequence and Case Study components of the Portfolio.

dynamic field, and the Standards need to be able to adapt and reflect changes in the field. The input from the HAToM project is particularly relevant here as it was the first opportunity to work through issues of interpretation.

Issues around interpretation have the potential to be very important in Australia where each state jurisdiction is responsible for the education of the children in that state. In addition there are three major schooling systems in each state; the government system, the Catholic system and the private or independent system.

The HAToM process is expensive both in terms of time taken to complete the process for all participants and the subsequent professional costs to the participants. The AAMT is comfortable, on the basis of the evidence from the trial project, that the time expended by candidates will be time well spent, in terms of their personal professional growth. The issue of financial cost is another matter, and in the absence of established rewards in employment (e.g., salary increments, bonuses or promotions) at this time, the cost of assessment would be prohibitive if teachers were expected to bear it themselves. The AAMT is exploring several strategies that will relieve the financial burden on teachers. These include sponsorship by schools, employers, local teacher associations and the private sector. Ultimately, however, the long-term solution will be a system of financial rewards for those holding HAToM credentials.

Another issue for the AAMT, now that there are some HAToMs, is the issue of how they can renew their credential when it expires at the end of 2009. Finally, the Association needs to establish clear processes for those candidates with grievances against the process in future.

The recognition of outstanding teachers through the HAToM process is important to the goals of the AAMT, but the most significant use of the *Standards* is to provide the framework for the professional learning of teachers of mathematics. The Association needs to work towards a time when the professional development provided by employers, universities and others is designed to use the *Standards*. This is a long-term goal, as is the establishment of career structures that provide financial and other recognition of those holding the HAToM credential.

Implications for Australian Teachers of Mathematics

The *Agenda for Action* (Commonwealth of Australia, 2003b) from the Review of Teaching and Teacher Education identifies a number of areas for professional development of teachers of mathematics at both primary and secondary levels. There is an expectation that significant effort will flow—the AAMT's *Standards for Excellence in Teaching Mathematics in Australian Schools* has the potential to drive the policies and programs in the directions identified by the profession.

The successful completion of the TSAEP has seen the AAMT become the first specialist teacher association in Australia to be able to award a standards-based credential for teaching achievement that is directly evidenced in the teachers' work and assessed through a rigorous assessment process. These Highly Accomplished Teachers of Mathematics have been identified as 'champions' in the profession, and ambassadors for the profession. That they are 'people like me' will encourage others to embark on the assessment process. The importance of this development in the Australian context should not be underestimated. The AAMT is a reputable professional organisation that will be in the position of giving peer professional recognition, based on a thorough assessment process. This is analogous to what happens in other established professions.

The implications for the professional development of teachers of mathematics who engage with the assessment process will be profound, and the impact will extend beyond the initial cohort. There will be materials and processes that will easily translate into professional development settings, and this will complement the AAMT's parallel work to promote the uses of the AAMT *Standards* in teacher professional development.

More broadly the continuing achievements of the AAMT in this field will reinforce its influence on the general educational agenda in Australia around professional standards, quality and professionalism.

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Mathematics Teacher Education – a Scandinavian Perspective

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In many countries mathematical teacher education has been a subject of societal debate and criticism. In this chapter I present the problematique of teacher education in mathematics in the Scandinavian countries and discuss issues and concerns related to it, trying to understand why there is a problem. A longitudinal study of teacher education in mathematics in Sweden is used as an illustrating example and recent evaluations of teacher education in Denmark, Norway and Sweden serve as sources for finding criticism and problems. A model for teacher education seen as the development of a professional identity is presented and discussed. This leads to the suggestion of solving the problems with teacher education by experiencing it as a life long learning and the development of a professional identity.

Background and Problematique

Mathematics is a subject that arouses strong feelings in almost everybody. It can create joy to learn or it can create anxiety and unpleasant feelings (Skolverket, 2003). The teacher has a crucial importance for the motivation and progress of pupils' learning. Thus the education of good teachers of mathematics is a matter of concern for both the pupils who succeed and those who reject mathematics. In the efforts of society to educate teachers of mathematics we can discern problems with several aspects. The access to well educated mathematics teachers is rarely enough, the recruitment to the education is problematic and the effect of the education is often questioned. The way society has reacted to this situation has been to implement repeatedly new reforms in teacher education. Why do we have these problems and what are the characteristics of them?

A Longitudinal Study of Swedish Teacher Education in Mathematics

In a longitudinal study of student teachers I followed a group of prospective mathematics teachers for eight years (Grevholm, 2000, 2005). This group began with 48 students in year 1 of their teacher education programme. After 4.5 years when their education was finished 25 of them were eligible to graduate as teachers. Several crucial aspects of the teacher education were identified; namely pre-knowledge of teachers, motivation to study, and engagement in the educational activities.

Importantly, the recruitment of student teachers was difficult as there are often not enough applicants with the pre-requisites that are demanded and needed in order to be successful in the studies. As can be seen from the numbers mentioned above the proportion of students who were able to fulfil the education in the expected time was low (50%). Moreover, when these newly qualified teachers undertook their first jobs in school they were struck by what one could call a reality shock (Grevholm, 2003a). Video-recordings from the classrooms of these teachers and interviews with them gave evidence of several kinds of problems. Mentors that they had been promised as beginning teachers disappeared too early. The lack of experienced colleagues left them with little support in finding relevant teaching material and tasks. The preparations of lessons were time-consuming for newcomers in the teaching profession and they were left with limited opportunities to find time to have a private life. Challenges such as these make many beginning teachers consider leaving the teaching profession early in their career.

The student teachers' conceptual development in mathematics and didactics of mathematics was especially in focus, and it was revealed that initially their concept images of central mathematical concepts (such as numbers, fractions, algebra, equation, function) were vague and meagre and somewhat confused. Repeated observations of the development of mathematical concepts indicated that over a longer period of time, students' conceptions grew more adequate, became richer and included more sub-concepts about teaching and learning, even in periods where the students did not study mathematics (Grevholm, 1998, 2003b, 2005a, 2005b, 2008).

However, many of the student teachers experienced problems with the mathematics courses and failed several times in the examinations. Although they claimed that the study programme was demanding and they worked as hard as they could it was not enough to create success for them. Mathematics and mathematics education takes up only around 20% of the study time in the programme and still it seems as if the learning outcome in mathematics creates significant problems (Grevholm, 2003b). Thus mathematics comes to play a filtering role for students. The student teachers, who felt less successful in mathematics, carried this experience with them out in the classrooms and potentially this might influence their own pupils in less favourable ways (Pehkonen, 2001).

Newly qualified teachers working in classroom situations reported that their professional language was too limited. When they tried to assist pupils during lessons they experienced problems expressing themselves in such ways that pupils understood. The teachers also found it difficult to vary their explanations if the pupil did not follow the first explanation (Grevholm, 2003a). As a result of the longitudinal study, changes took place in the teacher education programme in order to better meet the student teachers' demands to develop a professional language. Natural study groups were introduced where student teachers were given appropriate tasks to discuss in groups without a teacher present. In this way, they could exercise using and developing a professional language (Grevholm, 2004a).

Classroom management is crucial for beginning teachers. Video-recordings from the study provided evidence of different kinds of problems. To be able to help all pupils that need help at the same time is a hard task when you are not skilled in using many different ways of working. To find appropriate tasks on different levels of difficulty adequate for each pupil is demanding and calls for access to and knowledge about different materials and methods. Pupils who get bored because of inadequate tasks can create disciplinary problems and newcomers in the teaching profession have difficulties managing such situations. The competence to judge and diagnose pupils' learning is crucial in order to be able to plan lessons that are appropriate for a specific group of pupils. Beginning teachers naturally are not as skilled in this respect as a more experienced teacher can be. Examples were observed of beginning teachers offering tasks that were too hard for their pupils. The result was confrontation and feelings of failure for both pupil and teacher (Grevholm, 2003a). Other examples showed how chaos can be created in the classroom if work-forms and material were not well chosen for the class.

National Evaluations of Teacher Education

The teacher educations in Sweden, Denmark, and Norway have been evaluated and findings from the reports indicate the same kind of crucial issues (Högskoleverket, 2005, Danmarks evalueringsinstitut, 2003, NOKUT, 2006). The evaluation of the Swedish teacher education, completed in 2004, after a new teacher education was implemented in 2001, affirmed that the reform had contributed to the creation of educations with varied construction and varied content. The evaluators discussed quality and conditions for quality in the education. They noticed that most of the teachers who teach, supervise and examine in teacher education did not have a research education expertise. The teachers who have the most qualified scientific education, for example the professors, are rarely working in teacher education. The new teacher education offered the student teachers greater opportunities to choose and form their own education and take responsibility for it, which placed great demands on them. The evaluators recommended that the plans for the education are made more explicit, that the practicum periods must be guaranteed, that recruiting teacher educators who have a research education background be a priority, that levels of demands in the education be strengthened, and that quality assurance measures be implemented regarding the examination of education theses (Högskoleverket, 2005).

The Danish report (Danmarks evalueringsinstitut, 2003) also evaluated the outcome of new regulations for teacher education, which was implemented by the seminars (where teacher education takes place in Denmark) from 1997. The evaluators concluded that the education worked well in principle and in most areas the content was relevant for the work as teachers in compulsory schools. But in Denmark, as well as in the other countries, the education has double aims—to educate teachers for compulsory school and to prepare the student teachers for further studies on bachelor or higher levels. Here the evaluators claimed that the seminars had not come far enough in giving the students a well defined basis for further studies. The evaluators recommended raising the level of prerequisites to enter the education, and making changes so that student teachers see themselves as students at university level not as pupils in school. The demands for students to participate in the education are not adequate and the evaluation revealed that many students participate rarely. They also recommended strengthening the subject didactics in the studies. From 2001 there are demands to include relevant research in the education.

In Norway the evaluation contained a self evaluation part and external evaluation (NOKUT, 2006). Teacher education was regulated by a ‘Rammeplan’ from 2003 (guiding framework) and thus the evaluation took place in the middle of a process of change. The main impression expressed was that the education had varied quality, but the conditions to offer teacher education were also varied. A number of influencing factors vary between institutions, such as students’ participation in the studies, teacher educators’ engagement in subject didactic teaching, and teacher educators’ participation in the choice of research and development works. It is a great challenge for the teacher education to integrate praxis, subject studies, and subject didactical and pedagogical theories. Theory and praxis seemed to take place in different cycles and there was a lack of connections between different parts of the education. The evaluators suggested improved communication between different actors in the education and discussions that can contribute to common beliefs about what professionalism is in teacher education and in the teacher profession.

Data from mathematics in the teacher education was sometimes taken as example in the evaluations. Thus, it seems, the same kinds of problems sustain and again the suggested solution often seems to be to reform the teacher education. But is it possible to solve problems related to recruitment, pre-knowledge in mathematics, motivation, activity during the education, and success in learning outcomes through reforms of the education? Or are we actually in a situation where the demands on teacher education are too complex and numerous to be handled in a reasonable way during the basic education of teachers?

Some Crucial Aspects in Mathematics Teacher Education

In all three examples above—Denmark, Norway, and Sweden—the plans for teacher education stress the connection of the studies to research and fostering of a competence for change and development. The recommendation is to strengthen the research basis for teacher education. This can be done in many different ways (see Grevholm, 2004b, 2004c). Linking teacher education closer to research is influenced by the historical fact that teacher education has rather recently become part of the academic structure (although not yet in Denmark). But there is a need to make changes more efficient in this area. Relevant research, in for example didactics of mathematics, must be part of the foundation for mathematics teacher education.

Teacher education is a professional education but it is also expected to prepare students for further studies and possibly lead to doctoral studies. These double aims are, as can be seen from above, problematic. What scientific field could be relevant for a teacher, who chooses research studies? I have argued elsewhere for didactics of mathematics as the natural scientific area for a mathematics teacher (Grevholm, 2006). For this to be achievable, the teacher education must lay a foundation in didactics of mathematics solid enough for further research studies. The evaluations referred to above indicated that the subject didactical studies still are weak parts of teacher education.

Another crucial issue common to all three countries was the need for better connection between the different parts of the education. The conception of ‘a didactic divide’ between disciplinary and pedagogical knowledge has been used as an analytic tool to describe the rationale behind designs of reforms in teacher education (Bergsten & Grevholm, 2004). Even if the backgrounds to renewal of teacher education relate strongly to societal changes, including changes in the school system and general views on teaching and learning, it is obvious, that when it comes to the content and organisation of teacher education programmes, the didactic divide between disciplinary and pedagogical knowledge has played a major role. The idea of integration as a solution to problems of connecting different parts of the teacher education is tempting to follow but hard to implement, as can be seen from the evaluations reviewed above. After many attempts in several countries and over a long period of time it might be time to question if this idea can provide a solution to the problems after all. Another solution is to accept that teacher education is so complex and the object for so many different demands that it is not possible in a basic academic education to meet all these demands. A changed perspective is needed. The teacher education must be seen as life long learning and the development over time of a professional identity.

A Model for Teacher Education in Mathematics

Based on the findings of the longitudinal study, a model was created showing how teacher education can be perceived as the development of a professional identity (see Figure 1). The five main elements in the model constitute core parts of the professional identity that is developed in teacher education. They are (i) knowledge in mathematics related to teaching, (ii) competence to judge and diagnose pupils’ mathematical learning, (iii) knowledge about classroom management, methods and materials, (iv) a personal view on and beliefs about knowledge and learning in mathematics, and (v) a professional language for a mathematics teacher. All these five elements are interrelated. The model also indicates the basis for the five main areas and the sources for the knowledge and competencies, and how they are interrelated in a complex system. Student teachers' experiences, earlier knowledge, observations, reflections, practice, research and theoretical studies during the education contribute to the development of the five aspects of the teacher identity.

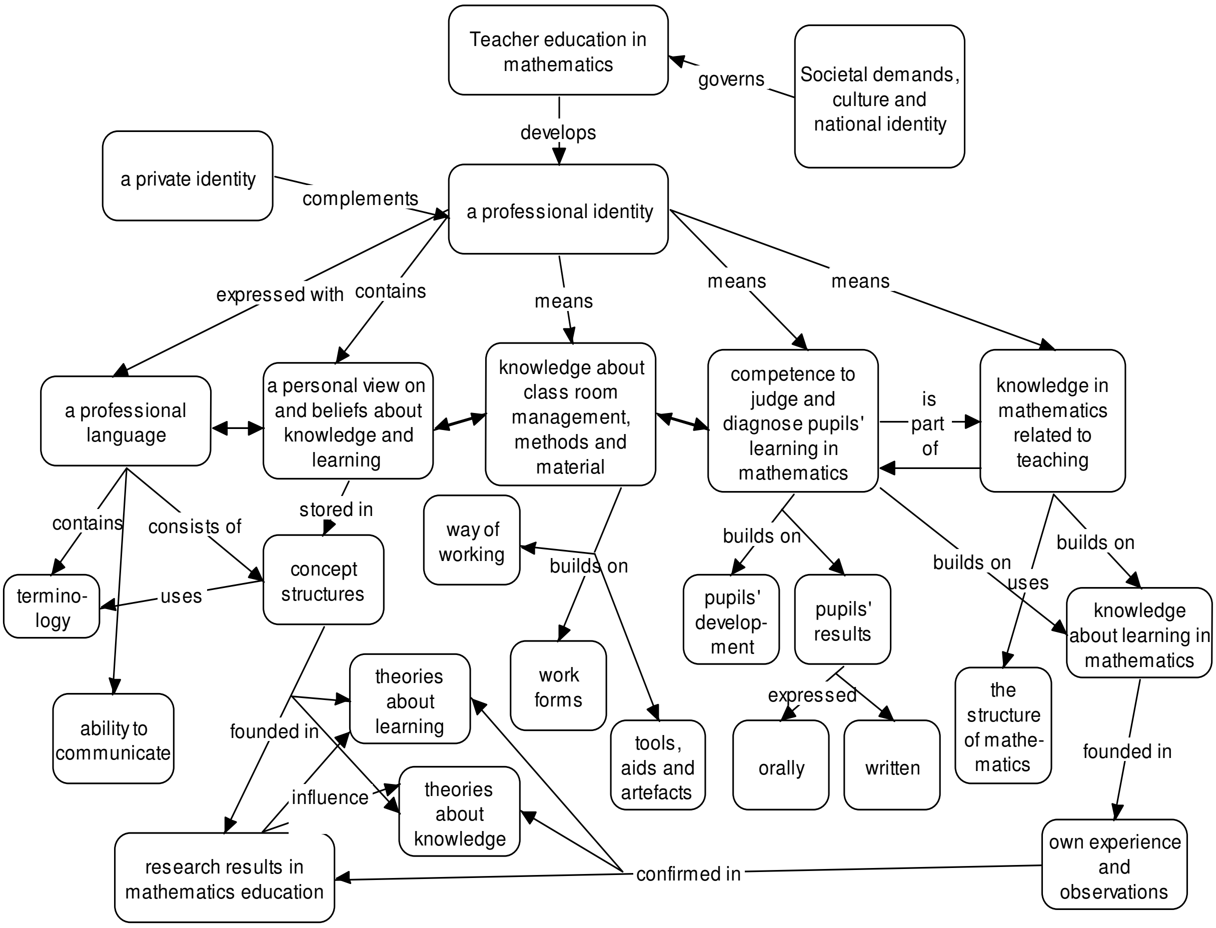

Figure 1 A concept map showing how mathematics teacher education can be seen as the development of a professional identity, with five main elements and their sources (Grevholm, 2006).

The teacher's development of a professional identity is a life-long process and experiences from early childhood and school-years as well as learning and experiences from work in the classroom will add new elements to this professional identity.

One reason as to why mathematics teacher education is problematic is the significant number of competing demands from both outside and inside the education I have argued here and earlier for the view that because of all these incompatible demands, it is only possible to create a basis for the professional work in the pre-service education of teachers (Grevholm, 2006). After the pre-service education, must follow a lifelong learning for the teacher. The development of the professional identity will go on throughout the whole professional life of the teacher. Thus in-service teacher education and different kinds of competence development for teachers are crucial and must be provided generously by society.

What are the Consequences of Problems in Teacher Education?

The Swedish Government published a report in 2004 aiming at changing attitudes to and raising the interest for mathematics and developing mathematics education (SOU 2004:97). With regards to mathematics teacher education, the report recommended that there was a need to improve recruitment to the mathematics teacher education, to develop the basic mathematics teacher education at all levels, to support qualifying competence development and further education, and to increase resources for research on teacher education and competence development.

So far not much has been done in the direction indicated in SOU 2004:97. One outcome is that the government has presented a programme—Lärarlyftet—for competence development aimed at raising the status and competence of teachers (Skolverket, 2007). A substantial amount of money has been invested in this programme until 2010 includiong funding support for unqualified teachers to complete complementing education and for active qualified teachers to complete further education with an 80% salary payment. A number of teachers were able to go into research education under these conditions. It must be expected that many of these teachers will be mathematics teachers. A recent investigation (Statskontoret, 2007) reported that only 35% of the teachers in Swedish upper secondary schools have a teacher education degree for that level and education for the subject (mathematics) they are teaching (ibid, p 33). Thus the need for complementing mathematics teacher education is huge. It remains to be seen if the mathematics teachers have taken this chance for in-service education.

Why the Interest for Teacher Educations?

In Norway preparations are made in order to offer research resources for a comparative study of teacher education in the Nordic countries. In this connection, Peder Haug, professor in pedagogy, presented some important problems in teacher education from a Nordic perspective (2007). He claimed that the interest for teacher education in the Nordic countries is reflected in the intense debate that is going on and that the same questions are central in all places. First, all teacher education programmes in the Nordic countries are under re-organisation to come into alignment with the Bologna-declaration.

The second group of themes, he claimed, came from research and evaluation of the internal activity in teacher education, where a number of questionable areas have been identified, including the relationship between theory and praxis, the lack of professional direction, the weak subject didactical orientation, the tension between subject and pedagogy, the relative absence of links to research, and the low study activity from students.

The third area debated is related to the conditions in the school. This is maybe the most important driving factor for quality in teacher education. International surveys, like TIMSS and PISA, show that the performance of pupils in the Nordic countries is lower than what is expected by society. The political view is that the Nordic countries depend on a high level of knowledge and competence in the global market in order to keep the welfare situation. Human capital is the dominating perspective.

Again, in this description by Haug (2007), we recognise the demands put on teacher education from outside and inside, not only for teacher education to work well but also for teachers to be accountable to outcomes within the school system. Haug expressed that if the wish is to better understand teacher education and professional qualification, it is relevant to carry out comparative studies. The main reason to do a comparison in the Nordic countries is the fact that teacher education programmes in those countries offer manifold and valuable conditions to answer many and wide questions that can be raised about it.

To summarise the situation, it is obvious that teacher education is much debated and many interested parties are putting demands on the education. Repeated changes have taken place in the Scandinavian countries with no firm basis of research to support the direction of changes. Changes have taken place so often that it has not been possible to evaluate one system before the next system has been implemented. Research shows that the development of a professional identity as mathematics teacher is a slow process and it should be allowed to be a life-long process. The pre-service education can only lay a foundation for further in-service education during the whole professional life of the teacher. It could be recommended for the Scandinavian countries to be influence by the conditions in Finland, where all teachers are given a master degree and quick changes of the teacher education system have been avoided.

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Teachers' Management of Meaning Construction in the Mathematics Classroom

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The five studies presented here focus on aspects of teachers' management of the construction of meaning in the mathematics classroom: the ways they handle the epistemological features of mathematics, deal with pupils' work and errors as well as the communicative patterns they adopt. The results show that the teachers tended to treat the epistemological features of mathematics in a unified manner; their interventions during the pupils' engagement with a mathematical task were very directive, and the communication patterns they followed did not provide space for the mathematical meaning to be negotiated. These findings suggest that the classroom management of the subject matter is likely to distort the mathematical meaning constructed by the pupils, and that it is dialectically related to the communicative practices employed.

Theoretical Issues

Research in mathematics education is increasingly focusing on the learning and teaching processes as they are interactively constituted in the classroom. From this perspective, the classroom is usually seen as a social context in which mathematical knowledge is negotiated and constructed while, at the same time, students and teachers are constructed and positioned with respect to that knowledge.

It is widely accepted today that cognitive gains are made in socio-cultural contexts in which teachers enable learning by drawing learners forward in their own ways with appropriate activities. In order to become aware of the dynamics of the mathematics classroom and, in particular, of how students develop their own autonomous mathematical identities, a sociological perspective on mathematical activity is especially valuable. Various researchers have attempted to investigate the activity in the mathematics classroom from such a perspective. For example, Sullivan (1999) focused on classroom tasks arguing that in choosing and using them, teachers should consider the relationship between mathematics content and mathematics context. He suggested, in particular, that in the mathematics classroom, even though the focus may be on particular concepts, the enacting of these concepts requires attention being paid to generalising, problem solving, linking and synthesising as well as to a variety of situations in which the concepts are applicable.

In their attempts to investigate classroom life from a sociological perspective, many researchers have developed theoretical constructs to provide an account of the relationship between pupils' cognitive development and social interactions in the classroom. For example, Voigt (1995), viewing the process of learning mathematics as intrinsically related to the negotiation of meaning in the classroom, focused on the evolution of mathematical themes in the course of this negotiation; he described these as improvisations of *thematic patterns of interaction* to account for students' cognitive development. Sfard (2002), who conceptualised learning mathematics as an initiation to mathematical discourse, advocated that *meta-discursive rules*, that is, rules which regulate the communicative effort, play an important role in students becoming skilful participants in this discourse. Finally, Yackel and Cobb (1996), who conceived of learning as a process directly linked to the social situations in which it occurs, offered two of the most well known theoretical constructs for the analysis of classroom mathematical activity from a sociological point of view. In particular, they differentiated between two aspects of the mathematics classroom microculture, these being *social* and *sociomathematical* norms. The former concerns general social norms, for example, justifying answers, asking for explanations, etc., while the latter are specific to classroom mathematical activity, for example, what constitutes an efficient mathematical solution, what counts as an acceptable mathematical explanation, and so on.

Sullivan and Mousley (2001), adapting the framework suggested by Yackel and Cobb, identified two complementary norms of activity in the mathematics classrooms. The first, named *mathematical norms*, concerns "the principles, generalisations, processes and products which form the basis of the mathematics curriculum" (p. 152). The second, called *socio-cultural norms*, refers to the "usual practices, organisational routines and modes of communication that impact on the approaches to learning teachers choose, the types of responses they value, their views about legitimacy of knowledge produced, the responsibility of individual learners and their acceptance of risk-taking and errors" (p. 152). One aspect of these latter norms is the constraints imposed by students on the mathematics teacher, which shape his/her thinking with respect to planning and teaching. For example, teachers tend to avoid negative student reactions (Shroyer, 1982) and to adjust their teaching to the preferred learning style of the students. In relation to the latter, Doyle (1986) reported that working on a task, pupils try to reduce their risk of failure by seeking to increase the explicitness of the task requirements as well as the level of accountability, thereby narrowing the demands of the task. Reacting to this, teachers tend to select tasks that are familiar and easy to the pupils.

In the studies reported below, an attempt is made to examine aspects of the mathematical, on the one hand, and the socio-cultural classroom norms, on the other. In addition, the way these norms act as dialectic in the construction of meaning within the complex environment of the mathematics classroom is investigated.

The Studies

All five studies presented here utilised the same set of data. The first two (studies 1 & 2) focus on the mathematical norms and, in particular, on the ways teachers tend to handle fundamental epistemological features of the subject matter, both at the primary and the secondary level in two rather different mathematical contexts, those of algebra and geometry. In relation to the socio-cultural norms, the same data were analysed in terms of the ways teachers chose to intervene in pupils' classroom work and their treatment of pupils' errors (studies 3 & 4 respectively). Finally, seeking to understand how the two types of norms come together in the classroom, we looked at the interactions between teachers and pupils, in order to examine how this interaction shapes the epistemological features of mathematics as well as the intervention practices adopted by teachers (study 5).

The data for the studies came from a large project focusing on mathematics teaching in the nine years of the Greek compulsory educational system (6-15 year olds) and an investigation of the possibility of applying alternative, pupil-centred mathematics teaching approaches in the Greek school. The data collected consisted of 48 mathematics lessons (28 primary and 20 secondary) given by 23 teachers (11 primary and 12 secondary), observed in various classes of the last two primary school grades and all three of the high school grades, for a period of over a month in the northern part of Greece. For each teacher, at least two 45 minutes sessions on different topics were observed; these were then videotaped and transcribed.

In the following sections, for each of the five studies, some theoretical issues, the research question(s), the analysis of the data and some basic conclusions are briefly presented. Finally, an attempt to draw some general conclusions with respect to the mathematical and the socio-cultural norms of the mathematics classroom is made.

It should be noted that, in all five studies, in order to analyse the transcripts, an interpretive approach was adopted: identifying and coding classroom episodes related to features of interest, observing similarities and differences between them, and clustering and re-clustering them into sensible groups. Furthermore, it is worth pointing out that the illustrative classroom episodes provided for each of the studies below have been selected for their potential to clarify and explain, and are not meant to be exemplary or reflect ideal classroom practice.

Studies 1 and 2: Management of the Epistemological Features of Mathematics in two Contexts

Pupils learn what is important in mathematics via interpreting the classroom events, attributing to each of them a value proportional to its usefulness to the mathematics lesson (Sierpinska & Lerman, 1996). These interpretations concern the meaning of concepts and processes as well as their nature and value. In this respect, the study of the nature and the organisation of the mathematical content assume particular importance. Such a study requires an analysis of the ways in which the epistemological elements of mathematics, that is, the nature, meaning and definitions of the mathematical concepts, the theorems and the solving, proving and validation procedures emerge in the mathematics classroom.

In study 1, the analysis carried out focused on two dimensions (Ikonomou, Kaldrimidou, Sakonidis, & Tzekaki, 1999): (a) the organisation and interrelationships of the various elements of the mathematical content (concepts, definitions of concepts and theorems) and (b) the organisation and selection of the elements of the mathematical activities (solving, proving and validation processes). The results showed that both in primary and secondary classes, the epistemological features of mathematics are presented and dealt with in a homogeneous manner, which does not allow students to distinguish them from one another with respect to their meaning and function in mathematics. Furthermore, the activity developed in the mathematics classroom is often, almost entirely, deprived of the characteristics of mathematical processes, which have to do with the pursuit of solving and proving processes, as well as checking and confirming.

The following episodes substantiate the preceding points. In episode 1.1, the concept is reduced to a counting process; in episode 1.2, the definition and the property are placed on the same level through questioning, and in episode 1.3, measuring is suggested as a proving process.

Episode 1.1. The teacher presents the area of a rectangle as a process of counting and through this he generalises (11 years old pupils).

T(eacher) …Count the (number of) boxes (square centimetres), how many boxes are there?

P.(upil) There are 12 boxes.

T. So, the area of the rectangle is 12 square centimetres. So, how can we find the area of a rectangle? What do we have to multiply?

P. ....

P. The area of the rectangle is: base by height and we measure it in square centimetres or square metres.

Thus, the area of a rectangle is presented as the result of an action (counting) instead of being defined in a way that will reveal its multiplicative nature. In other words, the definition of the concept is reduced to a context-specific manipulation process, thus ignoring its identifying and differentiating role.

Episode 1.2. The way the teacher elicits the definition and the properties is muddled (12 years old pupils).

T. ... thus, what do we have in the isosceles triangle?

P. Two sides equal?

T. Yes, and what else?

P. And two angles equal

T (… a little later) So, what do we call equilateral triangle?

P The one that has three equal angles and three equal sides.

It is clear that the teacher refuses to accept that one feature (either the two sides are equal or the two angles are equal) is enough to characterise an isosceles triangle and, consequently, that the properties of a triangle derive from its definition. Thus, definition and properties are placed on the same level through the teacher's questioning.

Episode 1.3. Pupils are asked to categorise various triangles according to the size of their angles, which they first measure (11 years old pupils).

T. Now, take the protractor and measure the angles and decide what kind of triangle it is.

P. In the first triangle there is a 90^o angle.

T. How did you find it?

P. It looks like it, it seems to be a right angle.

T. In mathematics, we can't claim something without being able to prove it. Take the protractor please, measure the angle and tell me whether it is in fact 90^o.

Here, the process of measuring is used to prove that a certain property is in effect. That is, measuring is upgraded to a proving process. Such an approach might lead to problems later, when pupils will need proof not through measurement but based on theorems and properties.

The results of study 1 give rise to an interesting question. How is the homogeneity in the treatment of the epistemological features of mathematics realised in the context of the two main branches of the school mathematics curriculum, that is, algebra and geometry, given that these two areas differ epistemologically and promote rather distinct patterns of thinking? School algebra is usually introduced as generalised arithmetic and is often described as "an epistemological transition from a procedural to a relational perspective" (Arzarello, 1998), requiring ability in the recognition of structures, the expression of symbolic relations and operations on these expressions through reasoning, which is sequential, logical and verbal. School geometry, on the other hand, places great emphasis on the visual aspects of the subject matter, thus leading children to deal with geometrical objects in a visual-perceptive rather than relational-analytical way (Hershkowits, Parzysz, & Van Dormolen, 1996). The latter is often seen as one of the main obstacles in children gaining access to the formal aspects of Euclidean geometry and, in particular, to its rigorous deductive thinking.

Study 2 attempted to investigate the way in which the epistemological features of mathematics are treated in the context of school algebra and geometry (Kaldrimidou, Sakonidis, & Tzekaki, 2000). The corresponding analysis of the data showed that this carries the same characteristics as those identified in study 1, only adapted to the special features of the two epistemologically different topics. In particular, definitions and theorems are often reduced to processes of manipulation in algebra and of visual recognition or drawing in geometry. Furthermore, the way mathematical knowledge is handled in the classroom appears to foster reconnoitring and morphological elements in algebra and handling/manipulative elements in geometry. These findings suggest that the management of the mathematical knowledge in each of the two contexts does not only prevent the differentiation of its epistemological elements, but, on the contrary, unifies them.

The following episodes are indicative of the ways in which the teachers in the sample dealt with the mathematical content in the two contexts. In the first episode, the mathematical knowledge is presented as a set of ready-made instructions of factual character, the emphasis being placed on the morphological elements of the transformations of the algebraic expressions (i.e., on purely symbolic manipulations). In the second episode, the definition is reduced to a manipulative set of step-by-step instructions on the actual drawing of the altitude of a triangle.

Episode 2.1. (algebra). The teacher starts by defining factorisation (14 years old pupils).

T. The factorisation of an algebraic expression consists of its transformation to a product of two or more other algebraic expressions.

A little later the mathematical method turns into a process:

T. We will study about ten cases, will see them one-by-one and we will learn practical rules ... so, if we are given this, we will do that, and so on.

By the end of the lesson, the two cases studied become rules:

T. Let me make one or two observations: we notice that when the powers of the same letter appear in all the terms of the polynomial, then the power of this letter with the smallest index comes out of the bracket. The second case concerned the grouping of the terms …. The common factors of each group come out of the bracket and what remains inside the bracket in each group is the same.

Episode 2.2. (geometry). The teacher starts by giving the definition of a height of a triangle. However, the definition is virtually 'destroyed' in the following, as the focus of the lesson moves on the procedure by which the height is to be drawn (13 years old pupils).

T. We first give the definition. What is the height of a triangle: we call height of a triangle the distance of a vertex from its opposite side.

A pupil repeats this. The teacher works on the drawn shape.

T. So, how are we going to place the ruler, here, watch ... The one side will go through the point (vertex) and the other (should be placed) on the side. We will do it practically, take the (right-angled) ruler.

Studies 3 and 4: Teachers' Practices in Dealing with Pupils' Work and Errors—Interventions

In trying to understand the complexity of the mathematics classroom, many studies have carried out an analysis of teaching episodes, focusing in particular on the way teachers intervene in order to support or guide pupils' work. This analysis has led to the identification of a number of elements, which appear to be very common in the mathematics classroom, and which have certain consequences for the mathematics generated, the pupils' attitude towards mathematics and their knowledge about mathematical knowledge. The relevant literature shows that all these types of interventions function as external indicators, which are often misinterpreted by the pupils, who tend to adapt them to their existing system of knowledge, this requiring less effort. As a consequence, the mathematical content of the task is often simplified and possibly distorted, and thus its cognitive value is reduced (e.g., Diezman, Watters, & English, 2001). The above suggests that it is very important for the mathematical knowledge elaborated within the classroom and for teachers' teaching practices to systematically identify *types* of critical teaching phases and their management by the teachers. To this end, we looked at teachers' interventions in two different and, at the same time, significant occasions of classroom activity.

(a) When difficulties emerge during the pupils' engagement with a task or the course of development does not follow the path intended by the teacher: on these occasions, teachers tend to intervene by, for example, offering premature or local character explanations (e.g., Margolinas, 1999); or addressing the competent students to secure the development of the lesson according to the initial plan. In study 3, we attempted to classify teachers' interventions according to the *degree of freedom* they provided (Kaldrimidou, Sakonidis, & Tzekaki, 2003). The analysis of the data showed that the dominant types of interventions are: focus on techniques, processes and representations; step-by step guidance, and demonstration of the solution to the problem. It is important to note that, on the whole, the type of intervention made by the teacher is related to the students' attitude and actions with respect to the situation or problem at hand. However, the examination of the teaching episodes revealed that the teacher often intervened for no apparent reason. In other words, s/he interfered independently of their action.

The two episodes below offer an insight into the ways in which teachers intervene in pupils' work. In both, the teacher tightly directs the pupils' thinking, allowing very little space for them to formulate ideas and complete their reasoning. Furthermore, in the first one, the teacher prevents any justification of the results on the basis of the properties of a triangle, whereas in the second the teacher slips into the required result, with no explanation.

Episode 3.1. The activity requires the calculation of the size of the angles of a right-angled and isosceles triangle (only the right angle is marked) (12 years old pupils).

T. Pay attention. This triangle has two characteristics. First, what type of triangle is this Nik, with respect to its angles?

P. Right-angled.

T. Right-angled. With respect to its sides, what type of triangle is this? Tania?

P. Isosceles.

T. Isosceles. Well done Tania. That is, this triangle is right-angled and isosceles. And we know one of the angles, the right angle, Michael?

P. Angles b and c …

T. Yes …

P. They are each 45^o.

T. But why?

P. Ehhh ... Because …

T. The triangle is …

P. The triangle is right-angled and isosceles.

Episode 3.2. Pupils are invited to conclude that the sum of the angles of a triangle is 180^o. They initially estimate the sum of the three angles of special cases of triangles and then measure to confirm their estimations (11 years old pupils).

T. George, what did you find for triangle B. Tell us.

P. 90^o.

T. Why do you say 90^o? All together, eh?

P. All together?

T. Doesn't the exercise ask you for the sum of the angles?

P. Because they are small.

T. The angles are small. Right. Tell us Chris.

P. 130^o madam.

T. About. Why my boy?

P. Madam, estimated by the eye.

T. By the eye, right. Charoula?

P. 180^o, madam.

T. Why 180^o Charoula?

T. Because, for the right angle, I say 90^o, for the other one, which is acute, because it is very small, I say 10^o and for the other one towards the right angle … but it is acute, I say 80^o.

T. About this, I don't know, it might be correct too.

Other pupils go on like this suggesting 60^o, 120^o, 185^o, 150^o, but the teacher does not ask for any more explanations, she simply says:

T. Did you estimate it by the eye?

(b) When dealing with pupils' errors and in using validation procedures in the mathematics classroom: both of these aspects of mathematics instruction are of great importance for the classroom construction of mathematical meaning, as they are related to the notion of transfer of control from the teacher to the pupils. This is a decisive factor in the development of autonomy and substantial thinking by the children. This is because it allows classrooms to become less judgmental and shifts responsibility for making sensible contributions to the children.

For the purposes of study 4, the transcripts of the mathematics lessons were analysed in relation to the teacher's treatment of the pupils' errors in two phases: before the error was made and after (Tzekaki, Kaldrimidou, & Sakonidis, 2002). The analysis of the episodes revealed that teachers maintain control over errors by warning and directing pupils or making the corrections themselves. That is, they seem to believe that errors are something to be avoided. In this context, they often do not pay attention to pupils' contributions, thus missing opportunities for a fruitful interaction in the construction of mathematical meaning. As a result, teachers invariably use morphological or procedural rather than conceptual elements for the elaboration of mathematical meaning. This practice allows them to keep their leading role intact in the construction of mathematical knowledge.

The following episodes underline this type of teaching practice.

Episode 4.1. The class is trying to simplify an algebraic expression (15 years old pupils).

T. In the worksheet I gave you, I have included a case where … there is a minus in front of the expression (describes). What will I do?

P. –2a

T. -a-1 or, if I don't want to change it immediately -(a+1). You should be very careful. This is where most of the errors appear. Solve as many equations as you can, starting from the simplest ones.

Episode 4.2. The pupils are trying to find the values of *a* in order for the denominator to be ≠ 0 (15 years old pupils).

P. $a^2-1 = 0 \Rightarrow a^2=1$

T. This is one of the ways. Are we sure that we will not lose the root? In the denominator? And what do we say then?

P. Will we say a=0?

T. Square root of 1. Thus, we get the 1. How will we get the –1?

P. We don't get the –1, because –1 times –1 equals plus …

T. We are fishing in unclear waters. Any safer way?

P. Shall we put plus?

T. No. The a^2-1 *can be written as* a^2-1^2*. Does this expression remind you of anything?*

Pupils. Difference of squares.

Episode 4.3. The pupils work with a problem of factorisation (14 years old).

P. Madam, in x^2-2x*, if we write x times x equals 2x? The x is cancelled and then x=2.*

T. Be careful! Which x's are going?… priority of operations… first we multiply.

P. Madam, we will do $x^2=2x$ *…* $x{\cdot}x=2x$

T. But you have a root! It is not allowed! All right? You lose a root. Don't do this kind of cancellation, because you lose roots. All right? However, when we take out the common factor, we don't lose the root.

Study 5: Linking the Management of the Epistemological Features with the Communicative Patterns in the Mathematics Classroom

A number of studies have emphasised the importance of the interactive patterns of teaching and learning in the acquisition and development of mathematical knowledge. However, as many researchers argue, there should not be a total shift of analytical attention from subject-matter structure to social-interactional structure. This is because there is then "a risk of destroying theoretical mathematical meaning by a reduction and a hypostasis of mathematical relations instead of inducing an enrichment of meaning by the interactive construction of new and more general relations" (Steinbring, 1998). An analysis of the interaction in the mathematics classroom that takes into serious consideration the epistemological as well as the social-interactional conditions helps to provide a better understanding of how the communicative patterns and routines emerge.

The previous studies suggested that there is a relationship between the mathematical content and the social structure of the classroom as determined by the norms of interaction and communication employed. In this final study, an attempt was made to examine the interaction in the mathematics classroom (Sakonidis, Tzekaki, & Kaldrimidou, 2001). For this purpose, the transcripts of the lessons were analysed by focusing on the interplay between the communicative patterns and the management of the mathematical knowledge employed by the teachers in two phases of the pupils' engagement with the activities: (i) at the completion of a certain activity in a unit of activities and (ii) at the completion of a whole unit and the generalisation of the results. The analysis indicated that there is a dialectic relation between the communicative pattern and the management of the mathematical content within the classroom, while the teachers' quick shift to different students does not allow any control over the flow of the meaning construction by individual pupils. This is supported by all the studies presented here, as they all disclose that the type of interaction that dominates the mathematics classroom is not negotiable, and shapes and at the same time is shaped by the way teachers organise and manage the targeted mathematical knowledge.

The episodes below are representative of the manner in which the communicative patterns interacted with the management of the mathematical content in the classrooms observed. With respect to the mathematical meaning, in the first extract, the teacher asks for an accurate solution, rejecting an approximation. In doing so, he limits the development of an interesting process of reasoning, and links the result with a rule and then with a formal procedure: we divide the numerator by the denominator. In the second episode, the fact that each shape is treated separately does not allow pupils to appreciate the purpose of creating definitions, that is, to identify and differentiate. For both teachers, this particular handling of the mathematical content is materialised through rather distinct communicative patterns, that is, by: (1) frequently posing questions, 'hunting for the correct answer', (2) breaking down the task and the pupils' responses, (3) allowing little space for the children to formulate ideas and complete their reasoning, and (4) rarely reasoning or asking for reasoning about a correct answer.

Episode 5.1. Pupils are engaged in an activity asking for the equivalent of 36/45 as a percentage. One pupil calculates as follows: 100÷45=2.22… and multiplies the result by 36 (=79.999…). But the teacher is looking for an accurate answer (12 years old pupils—phase (i) above).

P. We have found it, sir. We divided 36 by 45 and multiplied by 100.

T. How much did you find?

P. 0.8 times 100 equals 80.

T. (tries to generalise) Thus, when we know a ratio of two quantities …

P. Yes …

T. What operation should we perform in order to find the percentage?

P. Sir, division!

T. Division. What do we divide?

P. The numerators by the denominator.

T. Very nice.

Episode 5.2. The pupils are first invited to group together three kinds of shapes: parallelograms, rectangles and squares. Then, they have to identify the characteristic features of each group and formulate a definition for the corresponding shape (12 years old pupils—phase (ii) above).

T. What are the characteristics of a parallelogram? Then what does it say?…Which are the features of a rectangle?

A little later on:

P. The parallelogram has its opposite sides parallel and has 4 sides

T. Opposite sides parallel and 4 sides. But this is true for the rectangle too. Stefania?

P. In the parallelogram, all the opposite sides are parallel and equal

T. (Repeats). Do we have to add anything else? About the angles?

P. They are right angles.

T. Right angles?

P. Acute, obtuse maybe?

T. Two acute, two obtuse. Tell us Anna.

P. In the rectangle, all the angles are right angles and all the opposite sides…
T. Do we agree?
P. Yes (all together)!
T. Let's go for the last one, Katerina.
P. The square has 4 sides and 4 angles, that is, it is a quadrilateral and its sides are equal and parallel; its opposite sides and its angles are right angles.

Concluding Remarks

A number of studies have identified the need for a better understanding of the interaction between the management of the mathematical knowledge and the communicative patterns emerging in the classroom. This is because such an understanding will make it "possible to re-establish a sound interactive mathematical reasoning that has been destroyed" (Steinbring, 1997) by the latter.

The results of our studies suggest that there is an interplay between the epistemological organisation of the mathematical content and the organisation of the mathematics classroom. More specifically, the lack of differentiation among the elements of the mathematical content and its mixture with morphological, procedural and management elements of the classroom mathematical activity renders to the latter a dominant status in the activity. Teachers' interventions when students face difficulties as well as their management of the validation procedures and the errors made by the pupils, tend to focus predominately on these elements (seen as more effective indicators of production of mathematical knowledge), thus reinforcing them and consequently distorting the mathematical meaning. Hence, by dominating the mathematical activity, these elements become necessary to the students; that is, they acquire the status of *terms* for the everyday organisation of the mathematics classroom in relation to what is negotiated each time. In this respect, it is not the mathematical meaning aimed at that is developed within the classroom, but "a system of reciprocal obligation", which, however, is "specific to the 'content', the target mathematical knowledge", thus constituting what Brousseau (1997) calls *didactic contract*. This classroom reality reinforces and at the same time is reinforced by the presence of some distinctive communicative features that were found to persist across the lessons observed.

We argue that the results of the five studies reported here support the claim that there is a dialectic relation between the communicative patterns and the management of the mathematical content within the classroom. The mathematical and socio-cultural classroom norms are continuously generative of one another. This suggests that the two norms can only be examined in relation to one another. The latter questions the metaphor of mathematics as content, which "entails the notion that mathematics is placed in the container of the curriculum, which then serves as the primary vehicle for making it accessible to students … [characterising] what is traditionally called mathematical content in emergent terms" (Cobb, 2000, p. 73). Furthermore, it raises a number of important questions which, we believe, are worth studying, such as, for example: how do the two norms differ? How do they 'feed' each other? Which is more deterministic in the cycle of their co-existence, and how can this cycle be intervened upon? And more crucial of all: how do these norms exactly shape the construction of mathematical meaning by the pupils?

It is apparent that the existing research tradition that focuses on the mathematics classroom has a complex but somewhat fruitful task to pursue. Its fulfilment will provide significant insights into the development of students' mathematical learning in relation to the local social situations in which they participate and to the emergence of which they contribute. Furthermore, it will enable the effective support of teachers in developing informed teaching practices that foster pupils' intellectual development as well as their social autonomy in mathematics.

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Preservice Elementary School Teachers' Knowledge of Fractions

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The study reported here analyzed preservice elementary school teachers' knowledge about fractions in terms of the concepts and representation ability of fractions. The sample consisted of 115 preservice elementary school teachers in their third year of a 4-year teacher education program in South Korea. A questionnaire was used to investigate the preservice school teachers' knowledge about fractions. The findings suggest that preservice elementary school teachers tend to view fractions primarily as the part-whole relationships and measures. While they have a profound understanding of the meaning and representation of fractions as part-whole relationships and measures, they have significant difficulty with conceptualizing fractions as quotients, operators, and ratios. The result implies that a widely held assumption about elementary mathematics should be challenged.

Throughout the last decade, the notion of teaching practice in mathematics has changed greatly from emphasizing skill mastery and memorization of routine procedures toward focusing on meaningful understanding of mathematical concepts. This transition of teaching practice requires teachers to understand mathematical concepts in depth as well as to change their fundamental roles, from knowledge distributors to learning facilitators. According to reform documents (e.g., National Council of Teachers of Mathematics [NCTM], 2000), a conceptual understanding of fundamental mathematics is required for effective teaching. It is also well known that a teacher's pedagogical understanding of the content to be taught is an important knowledge base for effective mathematics teaching.

Fractions are one of the most difficult ideas in elementary mathematics, more abstract than any other domain. Fractions are mostly understood as part-whole relationships, or as measures, or as quotients. As implied above, the effective teaching of fractions requires teachers to have deep conceptual knowledge of the meaning of each fraction concept as well as pedagogical understandings, such as the representational knowledge required to communicate mathematical ideas. Meaningful understandings of fraction concepts are centered on the reform-oriented way of teaching practice, and representation is an effective tool for helping students to understand fundamental ideas of fractions.

The goal of the present study is to better understand preservice elementary school teachers' knowledge of fractions in terms of fraction meanings and representation ability. It is expected that our deeper understanding about what preservice elementary school teachers need regarding mathematics teaching practice will stimulate professional researchers and teacher educators to seek a better model of teaching practice for preservice and inservice teachers as well as for schools.

Background

A long and widely held assumption about elementary school mathematics has been rarely challenged by teacher educators, researchers, and policymakers in Korea, namely that if one can correctly get the answers, then one can teach mathematics. This message implies that anybody can teach elementary school mathematics because it is easy for anyone to get the right answers. Contrary to this wrong assumption held by some people, however, studies (e.g., Ma, 1999; Ball, 1990) and reform documents (e.g., NCTM, 1991,

2000) show that teaching elementary mathematics in a meaningful way focused on the development of students' thinking, reasoning, and problem solving is never easy, and elementary school teachers are required of a profound understanding of elementary mathematics for effective teaching.

Fractions are viewed as one of the most difficult areas in elementary mathematics for teaching and for learning, due to their complex nature. Among many contributors to our understanding about fraction concepts (e.g., Lamon, 2001; Kennedy & Tipps, 1988; Kieran, 1976, 1980), the views of Kieran have attracted one of the most attention from researchers. According to him, fractions are categorized into five subconstructs — part-whole relationships, measures, quotients, operators, and ratios. In order to fully understand fraction concepts, it is necessary to understand the concepts of each of five subconstructs as well as the interconnectedness between them. The interpretation of each of the five fraction concepts are as follows:

- Part-whole relationship: A fraction is used to indicate a quantity of equal sized parts of a unit.
- Measure: A fraction is used for indicating a distance of equally-divided lengths on the number line.
- Quotient: A fraction is used as the amount of each person's share in terms of equally-divided units of something.
- Operator: A fraction gives a functional rule by operating on the whole.
- Ratio: A fraction is used for describing the relationship between two quantities.

A fraction as part-whole relationship is the first to be introduced in the elementary curriculum and is used as the primary meaning for teaching and learning. According to Pitkethly and Hunting (1996), however, overemphasis on fractions as part-whole relationships tends to limit students' development of other fraction concepts.

Reviewing a plethora of studies about fractions to examine underlying features of fraction concepts, Pitkethly and Hunting found out that partitioning schemes using continuous or discrete models and identifying the unit are fundamental to the development of fraction concepts. This finding implies that partitioning is an effective scheme to strengthen the understanding of fraction concepts.

As was noted earlier, reform documents such as NCTM's *Principles and Standards for School Mathematics* (2000) require teachers to have representational abilities in order to organize and communicate mathematical ideas as well as to effectively help students understand mathematical concepts (e.g., Fennell & Rowan, 2001; Greeno & Hall, 1997). According to Shulman (1986), representation is one of the most effective forms of teachers' pedagogical content knowledge. Since his work, research about representation has been rapidly growing in the field of mathematics teaching and learning.

Various definitions of representation (e.g., NCTM, 2000; Pimm, 1995; Lesh, Post, & Behr, 1987; Brinker, 1996) can be found in the studies of mathematics education. For example, Brinker suggested a definition of representation in an objected-oriented way by referring to notations, pictures, objects, and structured materials such as fraction strips. On the other hand, Lesh and his colleagues defined representation in terms of the relationship between the creator's internal and external conceptualizations. Considering all these definitions, the term representation can be defined as referring to process and product to capture mathematical concepts or relationships by taking a variety of forms, including tables, diagrams, sketches, concrete materials, and words.

Methods

The study reported here focused on preservice elementary school teachers' knowledge of fractions. Thus, a total of 115 participants (30 male and 85 female) were selected from a four-year elementary school teacher education program at Gwangju National University of Education, in South Korea. They were all in the third year of the program and took one mathematics course and one elementary mathematics teaching methods course before participating to the present study. The data were collected in the fall semester of 2003.

The instrument used in this study was designed to examine preservice elementary school teachers' knowledge of fractions. The items covered the following five subconstructs of fraction concepts: part-whole relationships, measures, quotients, operators, and ratios. Subjects' knowledge was measured in terms of their understanding of fraction concepts and representation ability. For instance, preservice elementary school teachers' understanding of a fraction was measured by asking them to construct a word problem for the fraction 3/4 as a part-whole relationship. They were also asked to generate an appropriate representation corresponding to the fraction. Although a qualitative approach using interviews or observations has been frequently used to measure teacher knowledge in the field of mathematics education research, a questionnaire is an effective way to determine general trends for teachers' mathematical understanding.

In order to analyze preservice elementary school teachers' understanding of fraction concepts, their responses were coded in two ways: (0) incorrect answer and (1) correct answer. No response was coded as an incorrect answer. However, their ability to represent fractions was analyzed by coding their responses in three ways: (0) unable to generate a representation, (1) an inappropriate representation, and (2) an appropriate representation. Cronbach's alpha coefficient, the most widely used method for calculating test reliability, was adopted to compute the reliability of the present instrument. The result produced the coefficient of $\alpha=.69$, indicating that the test items of the present study have a considerable degree of reliability. A correlation analysis between preservice teachers' understanding of fraction concepts and their representation ability was calculated as part of the study.

Preservice Teachers' Understanding of Fraction Concepts

Table 1 shows that preservice elementary school teachers were most familiar with fractions as part-whole relationships and least familiar with fractions as operators. About 68.7% of them (79 out of 115 respondents) were able to successfully create word problems of a fraction 3/4 used to express a part-whole relationship. The example "3 parts out of the whole partitioned into 4 parts of equal size" is representative of the solicited responses. Similarly, about 62.6% of preservice elementary school teachers (72 out of 115 respondents) were able to create appropriate word problems for the fraction 3/4 as a measure.

As shown in the table, however, only about 43% of preservice elementary school teachers successfully devised word problems of a fraction as quotient and ratio. Furthermore, it was found that only about 23.5% of them (27 out of 115 respondents) were able to correctly explain the meaning of a fraction 3/4 used as an operator by creating a word problem.

These findings indicate that preservice elementary school teachers have significant difficulty in understanding the concept of fractions as operators. In effect, it was not surprising that preservice elementary school teachers lack understanding of fraction concepts as ratio, quotient, and operator, because the school curriculum and current teaching practices regarding fractions emphasize fractions as "parts out of the whole" rather than equally emphasizing the other views of fractions. Interestingly, the result of overemphasis on fractions as part-whole relationships is also exemplified in Korean students' development of fraction interpretations. According to Kwon (2003), for instance, it was revealed that more than 90% of sixth-grade students in Korea tend to interpret fractions primarily as parts out of the whole, but no responses were found for fractions as operators. This implies that students' understanding of fraction concepts is strongly connected to teachers' knowledge of fraction concepts and teaching practice. Thus, a teacher's limited understanding of fraction concepts may lead to limited opportunities for students to explore various notions of fraction concepts.

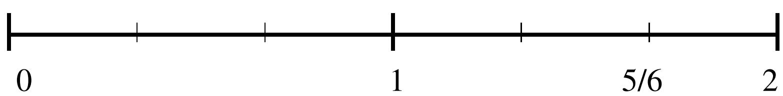
Table 1
Frequencies (%) of Preservice Elementary School Teachers' Knowledge of Fractions (N=115)

	Generating word problems	Representation		
		Unable to generate	Inappropriate	Appropriate
Part-whole	79 (68.7)	3 (2.6)	26 (22.6)	86 (74.8)
Measure	72 (62.6)	13 (11.3)	13 (11.3)	89 (77.4)
Quotient	49 (42.6)	25 (21.7)	48 (41.7)	42 (36.5)
Operator	27 (23.5)	43 (37.4)	58 (50.4)	14 (12.2)
Ratio	51 (44.3)	30 (26.1)	40 (34.8)	45 (39.1)

Preservice Teachers' Representation of Fractions

As was mentioned in the previous section, a teacher's representational knowledge is closely related to teaching practices in classrooms and to students' learning. Table 1 presents the frequencies and percentage of preservice elementary school teachers' responses to representation questions on each of the five subconstructs of fraction concepts. The results indicate that more than 74% of them were able to successfully generate appropriate representations of a fraction as part-whole relationships or measures. In doing so, continuous models were most frequently used to represent a fraction 3/4 as part-whole relationship or measure. For instance, fraction circles were the most frequently used tool for representing fractions as part-whole relationships, while number lines and rectangles were most preferred for representing fractions as measures. In a few responses, drawings were used to represent the real world context appropriate to the fraction 3/4.

When the fraction 3/4 is used as measure, it means three parts out of four equal parts of the unit. It is noteworthy that many preservice elementary school teachers have substantial difficulty in identifying the unit and distinguishing between the whole and the unit. For instance, when asked to mark the location of the fraction 5/6 between 0 and 2 on a number line, they tended to inappropriately point out the fraction between 1 and 2, as shown below.



This representative example of incorrect answers demonstrates a confusion of the concept of the unit with the concept of the whole. That is, they seemed to consider the fraction 5/6 is used to measure five parts out of six equal parts of the whole rather than as 5(1/6) of the unit. This finding implies that their overreliance on the concept of fractions as parts out of the whole creates difficulty in understanding the concept of the unit in fractions.

In addition, the results of this study shown in the table revealed that the percentage of preservice elementary school teachers' successful representations was significantly decreased for the other fraction concepts such as quotient, operator, and ratio. For instance, only about 12.2% of them (14 out of 115 respondents) were able to generate appropriate representations about the fraction 3/4 as an operator. Similarly, only 36.5% of the respondents were able to appropriately represent fractions as quotient using tables or number lines. Additionally, the percentages of respondents who were unable to represent at all were 21.7%, 26.1%, and 37.4% for fractions as quotient, ratio, and operator, respectively.

In sum, the findings of the present study indicate that the majority of preservice elementary school teachers understood the notions of a fraction as part-whole and measure, and they also successfully generated appropriate representations of such fraction subconstructs using mostly continuous models such as fraction circles and number lines. The results further showed that they have significantly limited knowledge about fraction concepts such as quotient, operator, and ratio as well as limited representations of such concepts. A correlation analysis was conducted to measure the relationship between the depth of preservice elementary school teachers' understanding of the meanings of fraction concepts and their representation ability. The result showed that with a correlation coefficient of r=.623, preservice elementary school teachers' understanding of the meanings of fraction concepts is significantly related to their representation ability.

Conclusions

The study reported here was carried out in an attempt to examine the depth of preservice elementary school teachers' understanding of and ability to represent fractions. In order to do so, they were asked to create appropriate word problems for a fraction 3/4 as five different meanings — part-whole, measure, quotient, operator, and ratio — and represent the fraction using pictorial models.

A fraction is primarily introduced in the context of part-whole relationships in the third grade, and students usually explore the fraction through activities such as paper-folding or partitioning of circles and rectangles into equal parts. The findings of this study revealed that preservice elementary school teachers are familiar with a fraction primarily as part-whole relationships and measures, consistent with the students' primary view indicated earlier. On the other hand, preservice elementary school teachers have significant difficulty with fractions as quotients, operators, and ratios. This result implies that preservice teachers' unbalanced view about fraction concepts may be influenced by educational contexts in Korea, probably overemphasizing the part-whole relationship of fractions over the other concepts in the curriculum or instructional practice.

Another goal of this study was to examine preservice elementary school teachers' ability to represent fractions. Educational researchers (e.g., Greeno & Hall, 1997; Shulman, 1986) in general and mathematics educators (e.g., Lamon, 2001; Goldin & Kaput, 1996; Pimm, 1995; Lesh, Post, & Behr, 1987) in particular maintained that teachers' knowledge of representation plays a significant role on mathematics teaching practice in classrooms and is necessary for helping students conceptually understand mathematical ideas. The result of this study revealed that more than 75% of preservice elementary school teachers were able to generate appropriate representations of a fraction as part-whole relationships or measures, while most of them had significant difficulty in generating representations of fractions as operators, quotients and ratios. This result means that preservice elementary school teachers are not fully ready for teaching elementary mathematics, even though they may find it easy to get the right answers.

In sum, the present study revealed that preservice elementary school teachers lack a profound understanding of fraction concepts, especially used as quotients, operators, and ratios. Overdependence on the part-whole model may limit students' opportunities to explore and develop other fraction concepts. Preservice elementary school teachers need to understand deeply the meanings and representations of fractions as quotients, operators, and ratios, as profoundly as their profound understanding about fractions as part-whole relationships and measures. Thus, the widely held assumption about elementary mathematics mentioned earlier should be changed. Teachers require deep understanding about what fractions really mean and how fractions are represented with models in order to teach mathematics in a way consistent with reform documents.

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Official Pedagogic Identities from South African Policy—Some Implications for Specialist Mathematics Teacher Education Practice

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In South Africa the National Curriculum Statements for FET Mathematics (DoE, 2003) together with the Norms and Standards for Educators (DoE, 2000a) are key policy documents that provide the official basis for mathematics education reform and for the construction of new pedagogic identities. In this paper I use a framework based on the work of Bernstein (1996, 2000) to theorise the construction of pedagogic identities. I use this to build on Graven's (2002) description of the new *official* pedagogic identity of the SA mathematics teacher, and on Adler et al. (2002) and others to raise questions related to teacher knowledge and the challenges of developing specialist mathematics teacher identities through initial teacher education programmes.

The past decade has been characterised by major transformations in South African society. There has been a concerted effort by the state to radically transform the Apartheid educational terrain through new policies and practices. A major *political* project has been to radically transform the pedagogic identities of teachers working within the system and to produce new teachers who meet these transformation ideals.

A major concern of education reform is to change "the bias and focus of official knowledge" and to construct new *pedagogic identities* in teachers and learners. The new pedagogic identity emerges as reflections of differing discursive bids "to construct in teachers and students a particular moral disposition, motivation and aspiration, embedded in particular performances and practices" (Bernstein, 2000, p. 65).

New policy statements overtly give details of the *kind* of teacher and learner envisaged by the new curriculum:

> … (T)eachers and other educators are key contributors to the transformation of education in South Africa. The National Curriculum Statement Grades 10-12 ... visualise teachers who are qualified, competent, dedicated and caring. They will be able to fulfil the various roles outlined in the Norms and Standards for Educators.

And

> The kind of learner ... is one who will be imbued with the values and act in the interests of a society based on respect for democracy, equality, human dignity and social justice as promoted in the Constitution. … (L)earners emerging from the Further Education and Training band must … have access to, and succeed in, lifelong education and training of good quality; demonstrate an ability to think logically and analytically, as well as holistically and laterally; and be able to transfer skills from familiar to unfamiliar situations. (DoE, 2003, p. 5)

These quotes, from the introduction to the National Curriculum Statement for FET[16] Mathematics (NCSM), give a symbolic picture of 'ideal' teachers and learners. They point to the vision of the kind of *moral disposition, motivation and aspiration* desired in teachers

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[16] South Africa schooling is divided into 'bands'. Early Childhood Education (ECE)—pre-school, General Education and Training (GET)—grades 1 to 9, and Further Education and Training (FET)—grades 10 to 12.

and learners by the SA state and more generally by SA society. The role of teachers as agents of transformation for a new democratic order is clearly articulated. The NCSM goes on to describe some of the *particular performances and practices* in which these should be embedded, and indicates both the nature of mathematical knowledge to be acquired and how it should be acquired and assessed.

Other policy, the Norms and Standards for Educators (NSE, DoE, 2000a), describes what it means to be a 'competent professional educator' in South Africa. It provides a vision of a professional teacher who is able to integrate a complex set of seven teacher roles with social, economic and moral responsibility. The roles include being: mediators of learning; interpreters and designers of learning programmes and materials; leaders, administrators and managers; scholars, researchers and lifelong learners; community members, citizens and pastors; assessors; and subject specialists. The NSE describes in generic terms the 'applied and integrated competences' that constitute the roles. These are: foundational competence (knowing that/what), practical competence (knowing how); reflexive competence (knowing why), integrated so that teachers know *what* to do, *why* it should be done, *when* to do it, and *how* to do it in the moment of practice.

The Criteria for Recognition and Evaluation of Qualifications (Criteria) (DoE, 2000b), is a further policy, which compliments the NSE. The NSE has a largely symbolic function presenting a holistic picture of an ideal teacher towards which teacher education curricula should aim. The Criteria plays a largely regulative function making it mandatory for higher education institutions involved in teacher education to design curricula in line with the NSE. From the perspective of the Department of Education (DoE), these norms, standards and criteria indicate to all providers (public and private) the kinds of teacher qualifications and learning programmes that the DoE will consider for employment. And for the public providers, the kinds of programmes and qualifications the DoE will consider for funding (Parker, 2003).

The NSE together with the NCSM projects a *symbolic image* of what is expected of mathematics teachers in the new reformed system. This is an *official* image of a desired pedagogic identity, a policy image, rather than a constructed reality based in practice. The *competent professional mathematics teacher* in post-apartheid South Africa is characterised through these images and is expected to be produced through curriculum reform in teacher education, as regulated through the Criteria. Teacher education is thus charged with a major challenge: to produce new teachers in this new image through newly designed pre-service and in-service teacher qualifications, and so, to institutionalise the 'bias and focus' of official knowledge.

How do mathematics South African teacher education providers respond to this transforming context and to the challenges presented by these new policies? What positions do they take in response to the policy, and how do they design and organise the mathematics teacher education curriculum so as to produce new specialist mathematics teachers for this new social and political context? These are the main questions that frame the major research project from which this paper emerges. In order to answer these questions is was necessary to first investigate the institutional and policy changes that occurred in relation to teacher education during the first 10 years of the new post-apartheid order. This laid the foundation for investigating how various institutional providers of mathematics education have responded to these changes, what knowledge resources they have selected, how they have organised these in their curricula and what pedagogic identities they have attempted to institutionalise through their programmes.

In the sections that follow I outline the context of teacher education reform in SA. I go on to briefly theorise the notion of 'pedagogic identity' and provide an analysis the *official* pedagogic identity of FET mathematics teacher in SA as projected by the policy. I discuss this in the light of debates in the field around knowledge for specialist mathematics teaching and teacher learning. Tensions between different demands produce challenges for mathematics teacher educators in relation to the way in which they could construct their curricula. How they select and privilege knowledge and practices for teacher learning will have consequences for the construction of a specialised identity of 'mathematics teacher' in and for SA within this new context.

The Context of Teacher Education in SA

Teacher education has undergone rapid transformation that has included a delocation and relocation of pedagogic practices from colleges of education regulated and controlled by the state, to relatively autonomous Universities and Technikons located in the higher education sector. This movement has created a space for mathematics teacher educators/researchers and mathematicians to play a major reform role by designing new curricula (criteria) for the development of new *mathematics* teacher identities.

In the terms of the NSE the 'specialist role' is marked out as the "the overarching role into which the other roles are integrated, and in which competence is ultimately assessed" (DoE, 2000a, p. 12). In terms of initial qualifications for FET mathematics teachers, there is no prescription of what ought to be taught, how it ought to be taught, or what *"the disciplinary basis of content knowledge, methodology and relevant pedagogic theory"* (Op. cit., p. 28) is in substantive terms. It is left up to the teacher educational professionals to produce the criteria for the specialisation. The policy sees FET teaching as a *specialist domain* and specifies the possibility of providing single subject (discipline-based) qualifications. This produces the possibility of focussed qualifications designed to integrate highly specialised knowledge for teaching into the learning programme.

There are two ways to qualify as a FET mathematics teacher in SA: a 3-year general formative degree with at least 2 years study in mathematics, followed by a professional certificate in education (PGCE), or a new undergraduate Bachelor of Education (B.Ed) which integrates the academic and professional components into a 4-year degree. I am interested in the possibilities inherent within the field for the development of *initial* mathematics teacher identities through a specialist B.Ed programme, particularly in the potential for different forms of specialised curricula to produce different forms of 'specialist consciousness' (Bernstein,1996, 2000) in mathematics teachers.

In South Africa, there are multiple dimensions to this task. As Adler (2004) points out, we work in a "socio-cultural and political context deeply scarred by apartheid education" (p. 6). In the field of mathematics the unequal distribution of knowledge and 'ability' is starker than in most areas of the school curriculum, and is a product of unequal opportunities under apartheid. The National Strategy for mathematics and science (DoE, 2001, p. 12) highlights the dismal performance of black African candidates in mathematics. In the interests of transformation it is necessary to create access routes into mathematics teaching for students who would not normally 'make the grade' for entry into university mathematics courses. This is a major challenge for teacher educators: it is not only necessary to develop an identity as 'mathematics teacher', it will also be important to develop an identity as 'able mathematics learner'.

Theorising Pedagogic Identities: Official and Local

Theoretically, pedagogic identities are 'forms of consciousness', and any particular educational reform represents an approach to regulating and managing moral, cultural and economic change, which are expected to become the lived experiences of teachers and students, through the *shaping of consciousness* (Bernstein, 2000).

For Bernstein, the power (classification) and control (framing)[17] relations of any pedagogic practice regulate the acquisition of pedagogic identity. The selections of knowledge(s), performances and practices and their evaluation rules (criteria for recognition and realisation[18]) relay a particular social order and way (mode) of knowing and being, whether explicitly or tacitly. The acquisition of the specialised consciousness produces particular *orientations to meaning*—ways of recognising and realising what is constituted as the 'legitimate text'. This comes "to have the force of the natural order and the identities that it constructs are taken as real, as authentic, as integral, as the source of integrity" (Bernstein, 1996, p. 21). Educational reforms require changes in the recognition and realisation rules of the pedagogic practice and therefore can be seen as "the outcome of the struggle to produce and institutionalise particular identities" (Bernstein, 2000, p. 66).

For Bernstein (2000) local identities are social identities, constructed through social location. These vary with age, gender, social class, occupational field and economic and symbolic control. They are not necessarily stable positions and shifts can be expected depending on maintaining the discursive/ economic base of the identity. This fits with Castells (1997) concept of identity as a source of individual meaning and experience that should be distinguished from social 'roles'. Roles are defined by norms structured by institutions and organisations of society, whereas identities are sources of meaning for the actor, constructed through a process of individualisation. Identities organise meaning and roles organise functions. Meaning is the symbolic identification by social actors of the purpose of their actions.

This helps point to the difference between an *official* pedagogic identity and a *local* pedagogic identity of a teacher. The official pedagogic identity is constructed through descriptions of what 'ought to be' based on particular projections by institutions of the roles, knowledge codes and social modes individuals ought to take up (official knowledge). Local pedagogic identity is constructed sociologically in local educational and historical contexts. Thus while official teacher identities can be designed on the basis of 'teacher roles', local teacher identities cannot—teacher identities emerge, enabled or constrained, within the pedagogic context (Graven, 2002).

In this understanding local pedagogic identities are not individual (cognitive) attributes, neither are they simply constructed politically or as a result of a curriculum prescription, they are constructed through an interplay of the 'voice-message' system (Bernstein, 1996), an interplay between official and local knowledge and practices within an educational community. Thus the 'legitimate' text (e.g. what is accepted as 'good mathematics teaching practice') is constructed through a relay between specialists in the field of teacher

[17] Classification and framing are key concepts for Bernstein (1990, 1996, 2000). Classification "provides us with our voice and the means of its recognition" and framing is "the means of acquiring the legitimate message". Classification is a product of power and framing of control.

[18] According to Bernstein (1990, p. 15) "(r)ecognition rules create the means of distinguishing between and so *recognising* the speciality that constitutes a context, and realisation rules … regulate the creation and production of specialised relationships internal to that context."

education, novice teachers, and experienced teachers within the social contexts of educational practice. Teacher identity is therefore embedded in the social practices of an education community within 'a particular social order' and develops in this context through relationships "of reciprocal recognition, support, mutual legitimisation and finally through negotiated collective purpose" (Bernstein, 1996, p. 73).

According to Bernstein (1996, 2000) individual pedagogic identities are constructed both inwardly and outwardly. The introjected identity faces inwardly and is most often related to the acquisition of stable inner loyalties related to esoteric forms of thinking and doing, for example, working in principled ways with disciplinary knowledge, or developing a therapeutic identity related notions of child development and internal, or sacred, religious and cultural values. The projected identity faces outwardly and is most often related to external demands from the state and the market for producing particular kinds of citizens, and for regulating and controlling them. The challenge for teacher educators is to design programmes that enable the construction of introjected identities leading to 'good mathematics' and 'good mathematics teaching'. This needs to happen within the economic constraints and competitive environment of the higher education sector, and should be balanced with projected identities that meet some of the transformational ideals of the state: particularly the need to provide access to powerful mathematics to a wider range of South African students.

What is considered 'good mathematics' and 'good mathematics teaching' practice within these contexts becomes a major issue: who defines what this means, on what basis is that decision made, and how is access to the criteria (recognition and realisation rules) for these new notions of mathematics and mathematics teaching made possible? Any notion of 'good practice' that a particular institution attempts to institute will have an ideological basis, and the particular selections of knowledge contents and practices together with how these are made available to students, can be analysed to identify it. Whether this is an ideology that is based on and driven by political and social concerns, academic and intellectual concerns, or practical and professional concerns, or some combination of these, is of interest and will have consequences for the kind of specialisation of consciousness that may be made possible within the educational context. In a context of the poverty of mathematics education alluded to earlier, this becomes a crucial concern. Improving access to meaningful relationships with powerful forms of mathematics within the schooling system will to a large extent be dependent on producing teachers who have acquired this identity, as interested and able mathematics learners themselves.

The experiences student teachers have, both in the teacher education lecture theatre and out in practice will influence their specialisation of consciousness. Whether their understanding of the nature of mathematics, their relationship with the subject matter, and what they consider and construct as 'good' mathematics teaching practices, is substantially changed from prior, and probably internalised, notions forged during their 12 years of schooling and determined by the apartheid educational order or not, becomes a central question underpinning the research project. In order to investigate how teacher educators in the various institutions have responded to policy and what ideology lies behind the image of 'good practice' they project from their institutions (as embedded within the organisation of their curricula), it was necessary to analyse the official identities projected from the mathematics curriculum policy.

The *Official* Pedagogic Identity of Specialist Mathematics Teachers Projected from SA Policy

Policy documents can be analysed to identify the particular 'bias and focus' of official knowledge and to examine the official pedagogic identities they project, and therefore to unpack what it might mean to produce the *kind of teacher* expected. This could be critically reflected on in terms of research in the mathematics education field to produce a local resource for the construction of curricula for specialist mathematics teachers. A clear picture of the projected official pedagogic identity requires a detailed document analysis, I have insufficient space here to provide details of this document analysis[19], and thus simply sketch of some of the characteristics of the policy image based on such an analysis. The analysis required working through all four chapters of the NCSM, sentence by sentence, categorising these using a framework based on Bernstein's concepts discussed earlier, and building on work done by Graven (2002).

Graven's (2002) analysis of the official pedagogic identity projected from the SA policy base, focuses on senior phase general education teachers (grades 7-9), and effectively illuminates some of the main differences in the 'outgoing' roles of teachers and their future 'incoming' roles as designed within the new education system. She shows that there is a movement in thinking about teaching and learning within SA education from a performance-based to a competence-based pedagogy[20], and from a collection to an integrated knowledge code[21]. She uses this together with an analysis of specific curriculum statements for the grade 7 to 9 mathematics 'learning area' to identify four different *orientations to mathematics,* and from this four *mathematical roles* teachers are expected to fulfil, each with its own mathematical demands. These orientations to mathematics are summarised as: mathematics for critical democratic citizenship; mathematics as relevant and applicable to aspects of everyday life and local contexts; mathematics for its beauty and intrinsic value, mathematics as a way of communicating in, thinking about and viewing the world; and, mathematics as conventions and skills to master in order to gain access to further studies.

My analysis of the new NCSM (Parker, 2006) shows that while there are some differences much of Graven's (2002) analysis still holds for the FET. The logic of competence (Bernstein, 1996) is clearly visible, particularly in the first chapter of the statement. A shift in approach to mathematics teaching is visible—a socio-constructivist, learner-centred, discussion-based approach is advocated. This is clearly articulated through

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[19] See Parker (2006) for a discussion of the detailed analysis of the NCSM

[20] She draws on Bernstein (1996) distinction between two pedagogic models underpinning a curriculum: competence based and performance based. In general competence models are directed at what the student knows and can do at the end of the learning process, whereas performance models focus on specific learning contents and texts. See Bernstein (1996, pp. 58-63) for a useful comparison in relation to time, space and discourse; orientation to evaluation; pedagogic control; pedagogic text; pedagogic autonomy; pedagogic economy.

[21] See Bernstein, 1977, 'On the Curriculum'. According to Bernstein there are two broad types of curriculum: Collection and integrated, although these can be thought of as a continuum rather than a straight dichotomy. In a collection type the contents stand in a closed relation to each other (bounded and insulated from one another) – here the learner has to 'collect' a group of favoured contents in order to satisfy some criteria of evaluation and classification of knowledge contents is strong. In an integrated type the contents stand in an open relation to one another (blurred boundaries and hybrid) – here the learner follows a course structured around some overarching 'big idea', and classification is weakened.

the reference to the roles of a teacher described in the NSE and the *kind* of teacher and learner advocated by the curriculum (as mentioned in the introduction to this paper). These role descriptions demand significant changes from teachers in relation to their orientation to knowledge and learning, and in their conception of what it means to teach. In particular, the teacher is projected as a *learning mediator*: she no longer teaches given content knowledge, she facilitates learning. She is responsible for *interpreting and designing learning programmes* to meet the needs of her learners—the teacher is expected to interpret the broad outcome descriptions and assessment standards in the new curriculum statements and select contents and learning activities to provide learners with appropriate experiences to achieve the outcomes. The new roles thus place high demands on teachers. Teachers do not teach: they mediate learning through the skilful development and use of learning materials. The control of the pedagogic space is displaced from the teacher towards the text (activity/learning material) and the learner is required to take responsibility for his/her own learning (individually and in groups). This represents a move from directly teaching given texts towards the management of knowledge, learning and learning spaces. Thus there is a shift in the locus of classroom control and a visible flattening of hierarchical relations in the classroom. In other words, a movement towards what Bernstein (1996) described as invisible pedagogy which he associated with a competence based curriculum.

This is in contrast to the markedly different practices still existing in schools under the old curriculum, where teachers follow a content laden syllabus prescribed by the department of education and the curriculum is strongly externally controlled (framed) through a high stakes matriculation examination which focuses on an orientation to *received* knowledge[22]. The locus of control is with the teacher and the classroom relations are more hierarchical and authoritarian—in Bernstein's (1996) terms, a visible pedagogy is in place which can be associated with a performance based curriculum.

The NCSM document indicates a commitment to integration in general terms as one of the underlying principles of the curriculum:

> Integration is achieved within and across subjects and fields of learning. The integration of knowledge and skills across subjects and terrains is crucial for achieving applied competence … and … seeks to promote an integrated learning of theory, practice and reflection. (Op. cit., p. 3)

However, a close look at the assessment standards and contents shows that the real emphasis on integration is *within* mathematics rather than across fields of learning. For example the idea of 'function' is a key integrating principle. This marks out a significant change in the organisation of the contents of the FET curriculum from that discussed by Graven (2002) or that exists within the existing curriculum. Mathematics remains fairly strongly classified in relation to contents outside of the field of mathematical sciences, but there is a weakening of classification values within the field itself. Instead of 'topics', such as algebra, trigonometry, geometry and calculus, that were well insulated from one another in the old curriculum and organised vertically, the contents of the NCSM are organised in terms of four learning outcomes—Number and Number Relationships; Functions and Algebra; Space, Shape and Measurement, and Data Handling and Probability—and are connected horizontally through mathematical processes such as "making conjectures, proving assertions and modelling situations" (Op. cit, p. 10).

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[22] See Boaler (2002) for a useful discussion on connected and received knowledge and the relationships with mathematics that are implied by each.

Conceptual progression in the disciplines underpinning the subject Mathematics as defined in NCSM is emphasised and the more overtly political and controversial radical integration aspects of the original GET version discussed by Graven (2002) are de-emphasised. There is a focus on application but in contexts where it is appropriate to the core disciplines that form the 'subject'. This curriculum broadens the focus of school mathematics learning from entry into a single discipline (pure mathematical topics) into a region (the mathematical sciences: mathematics, applied mathematics and mathematical statistics). There is a focus on access to the discourse of abstract mathematical knowledge, its structure and processes for entry into further studies in the mathematical sciences. Each of the components of the mathematical sciences is relatively strongly insulated within the NCMS, that is, there is a principle of internal classification which enables clear distinctions to be made, for example between statistics and mathematics, and between mathematics and applied mathematics. Statistics is most strongly insulated appearing in the document under a single outcome: Data Handling and Probability, which is an entirely new area in the FET curriculum. Other previously insulated topics in mathematics are spread across the other three learning outcomes and integrated horizontally in terms of mathematical structures, conventions and processes.

Thus in the new curriculum for FET mathematics, there are significant shifts in the specialised contents and processes to be taught and in the underlying philosophy of mathematics projected. Mathematics is seen as a fallibilistic discipline (Ernest, 1991), and mathematics learning is seen as relational and meaningful in its own right, and useful and meaningful to life. The NCSM provides a definition of mathematics that projects an image of mathematics as *practice,* a "human activity practised by all cultures" that enables creative and logical reasoning. It sees mathematical knowledge as constructed by "observing patterns, with rigorous logical thinking, … lead(ing) to theories of abstract relations" (DoE, 2003, p. 9). It is thus a systematic way of seeing the world and thinking about the world using structured abstract principles. Further it is "developed and contested over time through both language and symbols and by social interaction and is thus open to change" (Op. cit., p. 9). Mathematical problem solving is seen as a key element which "enables us to understand the world and make use of that understanding in our daily lives" (Op. cit., p. 9). The idea of empowerment as a purpose of mathematics learning is visible: access to mathematical knowledge empowers learners "to make sense of society" by enabling learners to "respond responsibly and sensibly to personal and broader societal concerns" and to engage "responsibly with quantitative arguments relating to local, national and global issues" (DoE, 2003, p. 10).

This is a broad conception in which mathematics is characterised as a "discipline in its own right and pursues the establishment of knowledge without necessarily requiring applications in real life" (Op. cit., p. 9). At the same time, it is also specifically emphasised that mathematics is more than a cannon of specialised knowledge contents, "competence in mathematical process skills such as investigating, generalising, and proving is more important than the acquisition of content for its own sake" (Op. cit., p. 9). While there is a focus on application of mathematics, the idea of unproblematic transferability of everyday knowledge into mathematics so prominent in the GET curriculum, is absent—the focus is on the "establishment of *proper* connections between Mathematics as a discipline and the application of Mathematics in the real world" (Op. cit., p. 10, emphasis added). Mathematical modelling is seen as the means to analysing and describing the world mathematically. Other proper connections are in relation to the use of mathematical tools for problem solving in other subject areas, such as physical, social and management sciences.

Thus there is a focus on mathematics as a *discipline*, a *practice* and a *tool*—it is a specialised knowledge form with its own unique conventions, symbolism and structure; it is a specialised practice involving specialist processes of thinking, reasoning, proving; and it is a powerful tool for problem solving in a variety of contexts including mathematical (for example, abstract problem solving) and nonmathematical (for example, as applied in issues of public health, finance, or other subject areas such as the physical sciences). Furthermore, Mathematics has a *history*—it is viewed as socially constructed and therefore as a fallible discipline.

In terms of the pedagogic discourse to be realised at the classroom level the NCSM implies new relationships between teachers and learners and between these actors and the subject matter to be taught—changes in both the instructional and the regulative discourse (the what and how)—both in general terms and in very specific terms in relation to what is seen as legitimate mathematical knowledge (concepts) and ways of knowing it (habits of mind and the regulatory order for its learning).

This does not seem to be a reform curriculum that is based on 'generic' knowledge and a 'watering down' of mathematics, rather it seems it is a curriculum that is very concerned with mathematics and mathematical ways of being and seeing—but these are not images that are necessarily common in the SA context. The new FET mathematics teacher needs to be competent in these extended curriculum areas—she needs to develop a number of specialised pedagogic identities, each related to a specialist knowledge discourse: an identity as mathematician; applied mathematician, statistician and mathematical historian. Access to the grammar of mathematics, applied mathematics and statistics as distinct knowledge discourses, knowledge about their historical development and ways of coming into being, and the ability to apply these meaningfully to problem solving situations, are a key. Teacher education is faced with this complex task—a need to provide curricula for this access, and create paths for the acquisition of these discourses for teachers who, in their own schooling had very different experiences of mathematics. But even this is not enough—they also need to develop practices for teaching these discourses as distinct from learning them. That is, in addition to acquiring the criteria (recognition and realisation rules) for these specialised forms of mathematical consciousness, the new teacher needs to develop a specialised pedagogy in relation to each "for the complex task of transforming this knowledge into appropriate opportunities for learning in school" (Adler et al. 2002, p. 151).

Whereas the earlier curriculum was very much product oriented working on the basis of 'received' knowledge (as discussed by Boaler, 2002)—a hierarchy of concepts, facts and skills expressed as definitions, products and methods to be learnt and practiced—this curriculum is not. It is more practice oriented and focused on producing "connected" knowledge". It focuses on the practices of mathematics (e.g. investigating, making conjectures, justifying, generalising etc.) as well as the skills (e.g. factorising) and the products (e.g., 'laws' of exponents'); and on making meaning though problem solving contexts. The implication of this curriculum is that teachers' mathematical identities should be constructed as 'connected', they should have 'productive dispositions' (Kilpatrick, et al., 2001) towards mathematics and be able to engage in a 'dance of agency' (Pickering as used by Boaler, 2002).

These changes in the curriculum represent major shifts for most prospective mathematics teachers whose mathematical identities constructed under an 'old' (outgoing but still existing) education system (Graven 2002). Teachers are required to implement these new ideals in their classroom practice. This means that they are required to develop new images of 'good practice' for mathematics teaching (recognition rules), and new pedagogic identities (forms of consciousness) that enable them to carry out these practices (realisation rules). Teacher educators will need to construct curricula for producing these outcomes.

While the curriculum statements can project images of ideal mathematics teachers, these intended identities will not necessarily be acquired. What happens in practice will depend on what occurs in real educational contexts and how the student teachers respond to these. The design of teacher education curricula can only work at the level of projecting official identities, but these can influence the emergence of new teacher identities through the relations they set up with the particular knowledge discourses and practices they make available. *What* resources are used as a basis for the specialisation of the consciousness and *how* these are made available to the student teachers will be a crucial issue. Acquisition of the recognition and realisation rules for a specific practice (say learning mathematics or teaching mathematics) will depend on the evaluation rules of the pedagogic discourse—the criteria of what is seen to be the 'legitimate text'. So a different specialised consciousness could be acquired depending on the selection and organisation of knowledge contents: what is recognised as legitimate knowledge and practice, and the pedagogic modes of its transmission.

Specialising the Consciousness of a Mathematics Teacher: Resources, Discourses and Criteria for Recognition and Realisation

In my wider research project the empirical focus is on identifying the knowledge resources and discourses that teacher educators select for their specialist mathematics teacher education programmes and the way these are organised, co-ordinated and made available to new teachers within their educational contexts. The major focus is on the production of their criteria for the recognition and realisation of 'good mathematics' and 'good mathematics teaching' practices within their teacher education programmes.

In the context of designing initial 4-year education programmes, teacher educators in SA should take care. There is a danger: much of the work in the literature relates to in-service work, or initial teaching where the teacher has previously developed an identity as 'able learner of mathematics'. In the SA context this needs to be part of the initial education programme, particularly in the light of the generally low level of personal mathematical competences developed in our prospective teachers through their prior schooling experiences (Parker, 2004), and the high demands of the new curriculum. In the wider research field, learning mathematics (becoming a mathematician) is often conflated with learning to teach mathematics (becoming a teacher of mathematics) and practising as a mathematics teacher (becoming a mathematics teacher). For example, Ball and Bass's (2000) criticism of the 'fragmented curriculum' of teacher education programmes in terms of the difference between working as a mathematician (compressing knowledge), which they seem to want teacher education programmes to discard, and working as a teacher (decompressing knowledge) which they want to privilege. Another example is Ensor's (2000) work which is concerned with teacher's mathematics education (and teacher education) practices and not teachers learning mathematics. Prior mathematical competence is taken as given.

In this paper I do not have space to elaborate on any findings from my wider research; however I do propose some tentative conclusions for a model based on my initial analysis of research in the field. I suggest that practising *mathematics teaching* (learning a professional practice) and *practising mathematics* (learning mathematics) are two distinctly different types of activity related to distinct knowledge discourses (Bernstein, 2000). I propose that initial mathematics teachers require *both*, particularly in times of reform where *new* mathematical learning identities *and* teaching identities need to be formed. Although these are connected discourses, I would suggest they should *not* be learnt at the same time and in the same space, since they work in opposite directions (as Ball and Bass (2000) so clearly show with their discussion on compressing and decompressing mathematical knowledge). I also identify a third distinct discourse, created in the growing research domain of mathematics education, which focuses on developing knowledge about teaching and learning mathematics (learning mathematics education).

Thus there are at least three different mathematically related pedagogic identities that a novice teacher should develop through any teacher education programme. An identity as a student of mathematics (becoming an able mathematical learner, thinker and actor); an identity as a student of mathematics education (becoming someone interested in learning from research in the field); and an identity as a mathematics teacher (becoming someone who can utilise their knowledge to help learners develop productive mathematical identities and be motivated to learn the discipline at higher levels). Each of these identities is a product of access to a different knowledge discourse, and in each case recognition and realisation rules for what comes to be seen as the 'legitimate' discourse and its practices need to be developed. Knowledge resources and practices need to be selected and organised in the curriculum for these purposes. A key debate and issue of contention in the empirical field is centred on the extent to which these should be integrated or not, and who should take responsibility for developing them (mathematicians/mathematics education specialists/teachers).

Thus I suggest that there are at least three specialist (mathematically related) knowledge discourses that initial teachers need to acquire—each with its own ways of thinking and doing, and different organisational structures (vertical and horizontal) and grammar (Bernstein, 2000). These should be co-ordinated in the teacher education curriculum to bring a 'notion of best teaching *and* learning practice into practice' (an adaptation of Ensor's (2000) language). Each discourse requires a different kind of specialisation, probably best developed at different times and in different spaces, and finally co-ordinated in the practices of the classroom alongside a competent teacher. In this way, distinctions can be made, boundaries between the different discourses can be set up and transgressed and they can be used as knowledge resources to be recruited in practice. I suggest that the curriculum designed for the construction of each of these identities should be based on knowledge produced in the growing domain of mathematics education research, and not simply on the basis of interpretations of what is 'good' from policy or local teaching experiences and resources.

I do not have space here to elaborate on the possible modalities for the acquisition of each of these identities, to discuss the different types of discourses nor to theorise what type of specialised consciousness different modalities might produce. That is part of my wider research project, and is left for later dissemination. However what is clear to me is that each one requires specialised mathematical work and *not* generic practices, and each one needs to be *designed*, with careful consideration given to the criteria for the selection of the privileged reservoir for recognising the practice and repertoire for realising the practice (Ensor, 2000).

Conclusion

What does it mean to know mathematics, to teach mathematics and to develop mathematical and other forms of knowledge and practice for teaching? This is a key question for mathematics teacher educators to ask and extremely difficult to answer in any straightforward manner. However, the answers we give to this question will be crucial for designing curricula for our student teachers to acquire the criteria for the realisations of the specialisation—effective specialist FET mathematics teacher.

In the context of curriculum reform, teacher educators with an interest in producing specialist mathematics teachers have a responsibility to contest for space and time in the 4 year curriculum. To argue for the specialised focus, to compete for resources to project their particular 'bias and focus' into the official pedagogic identity projected from their institutions. A responsibility to research and produce criteria for novice teachers to navigate the acquisition of the recognition and realisation rules for specialist mathematical pedagogic identities. This requires mathematics teacher educators to develop criteria for what constitutes 'best practice' in mathematics and mathematics teaching: a clear notion of what *kind* of knowledge(s) and practice(s) mathematics teachers should acquire to be in a position to put this 'best practice into practice', and *how* these should be acquired and co-ordinated in the teacher education programme.

The modalities of practice and knowledge discourses selected and co-ordinated in the 4 year degree curriculum do matter, and may have profound effects on the construction of new specialist mathematics teacher identities for and in SA.

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Acknowledgements

This paper is based on work supported by the National Research Foundation (SA) under the Thuthuka (Researchers in Training) programme, Grant number 2052952—Mathematics Teacher Education. Any opinion, findings and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Research Foundation.

Mathematics Student Teaching in Japan: Where's the Management?

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This study followed three Japanese preservice teachers during a 4-week mathematics teaching experience in a Japanese junior high school during which each student teacher taught three lessons. Conversations with their cooperating teachers included talking about how to teach mathematics and how students would respond to various tasks. Unlike their counterparts in the United States, however, they never talked about classroom management issues. Although students at this Japanese junior high school were generally well behaved, management problems did exist but were never discussed. When do student teachers learn to deal with these classroom management issues? During their first year of teaching, they are closely mentored by other teachers in the school and have opportunities to discuss any problems that arise in their own classrooms.

In the early 1990s, the Third International Mathematics and Science Study (TIMSS) was conducted to measure the achievement of 4^{th}, 8^{th}, and 12^{th} grade students in various countries around the world. In conjunction with the achievement portion of the research, a video study of mathematics teachers in Germany, Japan and the United States was also conducted, beginning in 1993. The reported research discussed a clear distinction between the teaching that takes place in Japan and that of the United States. These results, along with others, beg the question: "How do Japanese teachers learn to teach the way they do?" In particular, how might student teaching experiences in these two countries contribute to differences in methods of teaching?

Although student teaching is a critical component of the teacher education process (Wilson, Floden, & Ferrini-Mundy, 2002), research on student teacher supervision is "sporadic" and much less common than research on inservice supervision. (Glickman & Bey, 1990). The subset of studies that looks specifically at the dialogue that occurs in the cooperating teacher (CT)/student teacher (ST) dyads is even smaller. According to O'Neal and Edwards (1983), conversations between STs and CTs in the United States focus on classroom events and activities, specific teaching events and the methods and materials of teaching. Tabachnick, Popkewitz, and Zeichner (1979) found that classroom management, procedural issues, and directions were the primary foci of CTs in their interactions with STs, and that what was taught and the purposes for teaching it were seldom addressed. "Analysis and reflection on teaching are not common; the substantive issues of conferences tend to focus on teaching techniques, classroom management, and pupil characteristics" (Guyton & McIntyre, 1990, p. 525).

Borko and Mayfield (1995) conducted a detailed study of the conferences held between secondary mathematics STs and both CTs and university supervisors. They identified four domains of teacher knowledge that were addressed in the conferences they observed: pedagogy, students, math-specific pedagogy, and mathematics. They note that general pedagogical issues were discussed in eight of the nine conferences and account for most of the specific suggestions offered to STs. Students were discussed in all nine conferences, but primarily in ways that were concerned with lesson flow rather than with student understanding. Mathematical pedagogy was also discussed in each of the nine conferences, but mainly at the level of general strategies rather than specific representations for particular mathematical concepts. Mathematics was rarely discussed, and then only in a superficial manner, in any of the conferences. Classroom management was discussed in only three of the nine conferences and thus, was not identified as one of the four domains of frequent conversation. It should be noted, however, that all of the conversations in this study were recorded at the end of the student teaching experience when many classroom management issues would have been resolved.

Peterson and Williams (2001) conducted a study of the dialogue and mentoring that occurs during student teaching practicum in the western United States. Eight ST/CT pairs were selected for the study and all 16 participants were interviewed twice, once in the middle of the 14-week student teaching experience and once at the end. The conversations that took place between the ST and CT during the student teaching experience were also recorded and analyzed. The analysis found discussion themes such as subject matter knowledge, pedagogical content knowledge, classroom management, activities, individual students, and the relative difficulty of the mathematics they taught. In the conversations, most pairs spent the majority of the time (as high as 77%) discussing classroom management. Some pairs spent little time (as low as 1%) discussing the mathematics that they were teaching.

Morey, Nakazawa, and Colvin (1997) conducted a study of critical incidents reported by elementary STs in Japan and the United States. They found that both groups of teachers were concerned with discipline but each saw this issue in a different light. The following statement summarizes this conclusion.

> The content analysis did reveal differences in the frequency of certain themes within topics rather than a markedly different set of topics. Of particular interest were differences between the Japanese focus on relationships with students as essential to promote learning and the American concern for classroom management and individual student discipline as a condition for learning. (p. 210)

In the studies reported here that focus on the dialogue between STs and their CTs, classroom management is a common conversational topic. There is also a lack of emphasis, in many instances, on the analysis of teaching that occurs. The research question addressed in this paper is, "Do mathematics CTs in Japan have the same focus on classroom management in their discussions with STs as their counterparts in the United States?"

Method

This study was conducted at the junior high school affiliated with a university in southern Japan. In the fall of 2003, there was a 4-week student teaching session for STs whose primary focus was teaching at the secondary level. During this student teaching session, there were 7 mathematics STs and 3 mathematics CTs. Each of the 7 STs taught only 3 lessons (one with each CT) during the 4 weeks of student teaching. Since the structure of the Japanese schools has the students in the same classroom all day long and a homeroom teacher assigned to that classroom, each of the STs was also assigned to a homeroom class. They participated in all of the homeroom class activities including morning and afternoon announcements, lunch, afternoon cleaning, and field day practice. The group of STs assigned to a specific homeroom also had the responsibility to plan and teach a moral education lesson. Because of this assignment, the STs had interactions with the homeroom teacher as well as the mathematics CTs. Three STs were selected to participate in the study based on the convenience of teaching schedules.

With this structure of the student teaching experience, the following types of video taped data were gathered: 1) Initial and Final interviews with all CTs; 2) Initial interviews with all STs; 3) Final interviews with the 3 selected STs; 4) All conversations between the 3 selected STs and the CTs; 5) All lessons taught by the 3 selected STs (9 total); 6) The corresponding *hanseikai* (reflection meeting) that was held after each lesson was taught.

The author was present in the school every day during the student teaching experience and did all of the data gathering described previously. Because he is fluent in Japanese, he conducted the interviews himself but did rely on a native speaker of Japanese to assist in translating the research questions from English. Additionally he observed each of the 3 CTs teach mathematics lessons on a daily basis over a 6-week time frame surrounding student teaching. Field notes were taken on these observations as well as on the video data gathering episodes.

A detailed analysis of the conversations between the CTs and STs is pending but based on the field notes, there were no conversations about classroom management. Thus, this study focuses on participants' responses to interview questions about classroom management. The results of this analysis were then compared to the data gathered in a previous study by Peterson and Williams (2001).

Interview questions for the CTs were as follows: 1) Does your method of teaching have an influence on the behavior and attitude of the students?; 2) Does the behavior and attitude of the students have an influence on your method of teaching?; 3) What is the most important thing that the STs should learn through student teaching?; and 4) What are the strengths and weaknesses of the individual STs? The STs were asked the following types of questions: 1) What is your way of seeing or thoughts about the subject of mathematics?; and 2) Regarding classroom management and discipline, what kinds of things did you and your CTs talk about? Most of the interview questions for this study were adapted from questions used in Peterson and Williams (2001). Because of the differences in the settings of student teaching in the two countries, however, there was not a one-to-one correspondence between the interview questions asked of either the CTs or the STs in the two countries. For example, the initial interview in Japan took place before the student teaching experience began so they were asked the question, "With regard to student teaching, what are you most nervous and concerned about?" In the United States, on the other hand, the initial interview took place after 6-7 weeks of student teaching so they were asked, "What do you feel your weaknesses (strengths) as a teacher are?"

Results

In reporting the data pseudonyms are used for all subjects. The STs from Japan will be referred to as ST-Keiko, ST-Hideki or ST-Yoshi and the CTs from Japan will be referred to as CT-Honda, CT-Sasaki, or CT-Tanaka. The STs from the United States will be referred to as Anne, Blake, Connie, Dawn, Jennifer, Peter, Sunny, and Tara. Since the STs and CTs in the United States were paired exclusively together, the CTs are referred to as Ms. A worked with Anne, Mr. B worked with Blake, etc.

Teaching Methods Influence on Student Behavior

When the CTs in Japan were asked if they felt their method of teaching affected the behavior of the students, the word *furumai* was used for behavior or conduct in addition to the word *taido* which means attitude. The first two CTs that were interviewed focused on the student's attitude toward the math and made little reference to classroom behavior. Thus, the word *taido* was not used in the third interview so it would not bias the CT toward students' attitudes. However, the third CT still gave a response that was void of any reference to classroom management. In response to this question, CT-Tanaka talked about "paying attention to students facial expression and adjusting the teaching accordingly." CT-Honda was "even willing to allow some whispering that was unrelated to mathematics as long as the mathematics was understood by everyone." The main focus of all three responses was a goal of "teaching mathematics because I want students to like mathematics" (CT-Honda).

In response to similar questions about teaching affecting the behavior of the students, 6 out of 8 CTs in the U.S. study referred to some aspect of discipline, control, or classroom management. Ms. A commented, "If it's [an activity] not well planned out then it can be chaos and no learning takes place." Mr. B said, "When you've got ninth graders still in pre-algebra, you have a room full of behavior problems," implying that when older students are still in lower mathematics classes, they usually have not had much success in mathematics and so there are more behavior problems. Many of the U.S. teachers related their comments about classroom management to their attempts to do activities with the students. The students' behavior while doing the activities would determine whether such activities could be used again. Ms. S summarized this idea:

> If I can't have a fun activity and have them, even though they are noisier, have them in a controlled situation, it doesn't help to do the activity because they don't get anything out of it. Then I choose something where then I can keep them controlled and in their seats.

Important Issues in Student Teaching

During the initial interviews of the Japanese CTs, two common themes emerged: 1) the lack of reference to classroom management issues; and 2) an emphasis on communicating to the STs the fun of mathematics and a true sense of the joy and difficulty of teachers' work.

When the Japanese CTs were asked what they thought the most important thing for the STs to learn was, they all focused on the human element of teaching and developing an understanding of the profession. CT-Sasaki talked about how much she loved being with the students and how important it was for teachers to possess that "human like attribute." CT-Honda stressed the importance of considering the human element of the students when preparing a lecture, having a desire for students to understand and "to enjoy learning with students." CT-Tanaka said that he wanted STs to "feel that teaching is fun. It is a hard job but worth it." In contrast to the responses from the US teachers, none of the CTs made any reference to managing a class or controlling students (Peterson & Williams, 2001). Rather the responses to these questions suggested the importance of the human element that they seemed to place on teaching. They felt that developing a relationship with the students and "wanting them to understand" was very important. This emphasis on a human relationship with the students was consistent with the inherent desire to have students understand the mathematics and see the fun in it.

In a similar vein, the Japanese STs' responses to the initial interview questions had no reference to student behavior or classroom management. When asked, prior to their student teaching, what they were nervous about, they responded that they were concerned about teaching a good lesson.

Student Teacher Strengths and Weaknesses

The STs from the United States were interviewed midway through their 14 week student teaching experience and were asked what they felt their strengths and weaknesses were. Seven out of the eight STs made reference to their ability to manage a classroom or manage and control the students. Dawn said, "I am a wimp. I'm horrible at discipline. I have a hard time with classroom management." Blake said that his strengths were "probably, right now, management, class control." In his final interview, Blake made a comment when talking about overcoming his weaknesses that directly addressed the question asked of the Japanese STs. He said, "One of my big fears early on was 'how do I deal with 25 kids?'" This comment was in direct contrast to anything stated by the Japanese STs about their concerns prior to student teaching.

In the final interviews, both the Japanese CTs and the U.S. CTs were asked to discuss the strengths and weaknesses of the STs with whom they had worked. Seven out of the 8 U.S. CTs made mention of classroom management as either a strength or a weakness. They described these strengths and weaknesses with comments like "his big weakness (that he overcame) was learning to be aware of what was going on in the classroom" (Mr. B). Another CT, Mr. P, commented that his ST had developed strength in "how to discipline and ways to take care of discipline." Mr. D described Dawn's new strengths as follows:"Probably the most noticeable is that she's become more assertive, and maybe more confident in taking charge of her class and controlling her class."

On the other hand, when the Japanese CTs were asked to describe the strengths and/or weaknesses of their STs, none of them made any mention of classroom management. Instead they talked about personality characteristics, saying "she was positive and cheerful" or "He is sweet." They also talked about the ST's preparation of their lessons and their willingness to accept guidance. CT-Sasaki said the following about ST-Yoshi:

> At least, he understood he couldn't teach well without considering the student's feelings. After the first lesson, I told him to prepare better and his response was something like 'I can teach well without preparing it.' But he prepared much more for his last lesson than he did for his first.

CT-Sasaki described ST-Keiko as "a person who could accept others' suggestions gratefully and it really helped her gradually grow."

Absence of Classroom Management Conversations

An analysis of the field notes indicate that during the conversations between the three Japanese STs and the three CTs as lessons were being prepared, statements that focused on anticipating poor student behavior and how one might deal with it were not heard. The majority of the conversations centered on what the students would be able to do mathematically or how they might think about or respond to the wording of a certain mathematical question. In the reflection meetings that followed the lessons, there were still no conversations about classroom management.

Field notes also indicate that some student misbehaviors similar to those seen in the United States were observed. For example, one of the CTs would have to occasionally remind the students to stop their excessive whispering or talking. When she wanted to talk, she waited for them to be quiet and called 5 students' names out to get their attention before proceeding. When a student was sharing a solution, she asked other students by name to pay attention. These incidents were observed in some of the regular day to day lessons taught by the CTs. Although issues of classroom management do exist in the lessons taught by the CTs and occasionally in the STs' lessons, there were no documented instances where CTs discussed student misbehavior or classroom management with the STs.

Where is Classroom Management Discussed and Learned?

Because of the absence of discussion of classroom management by the Japanese CTs in the initial interviews and during the lesson preparation and reflection conversations with the STs, the final interviews in Japan focused on the question "Where will these STs learn to deal with the classroom management issues of student misbehavior?" Asking this question, however, required that the meaning of classroom management be clarified by describing specific student misbehaviors observed during mathematics lessons.

When ST-Hideki was asked about classroom management in his final interview, the following dialogue ensued:

> Interviewer: The next question is about classroom management. It is about discipline. What kinds of things have you and your instructors talked about What sticks out in your mind?
>
> ST-Hideki: Do you mean the student greeting?
>
> Interviewer: I am not asking you about the student greeting. For example, if a student doesn't listen to a teachers' lecture and the teacher tells him/her to be quiet. Did you talk about this kind of thing?
>
> ST-Hideki: I don't think we talked about it. I talked about it with a homeroom teacher though. The homeroom teacher told me that I could tell the students anything because this was my student teaching. She also told me that I am not God so if I make mistakes I should apologize to a student. I should just do what I think is right. What I actually did was when the school was conducting a field day practice, I talked to the students who did not practice seriously or were complaining.
>
> Interviewer: OK. Where do you think you can learn to deal with this kind of thing? You and your cooperating teachers did not talk about students who didn't participate in a mathematics lesson.
>
> ST-Hideki: We did not talk about this, but we took a survey after each of my 3 math lessons to get students' ideas about my teaching. Some students wrote on the survey after my 2nd lesson that I did not pay attention to the students who were chatting during a lecture. The 3 instructors did not directly tell me about it, but I felt I should have paid more attention to chatting during a lesson since students mentioned it to me.

ST-Hideki agreed that he had not talked about classroom management with the CTs even when there may have been cause to do so. Neither of the other two Japanese STs made any mention of talking about classroom management issues with their mathematics CTs. In contrast, when U.S. STs were asked a similar question in their final interview, every one of them talked about classroom management issues that they had discussed with their CTs. Peter said that they talked about "how to handle kids that are talking and especially when the whole class is kind of rowdy." Connie said:

> We've talked a lot about that [classroom management], more so than the mathematics, because the math you can usually find in the books. We talked about seating charts and how that affects the atmosphere in the class. We also talked about letting the class decide on the rules, how that affects management, and also about enforcing the rules.

In the initial interviews with the Japanese CTs, any questions about classroom management were responded to with an emphasis on student attitude toward mathematics. In the final interview with the STs, the term "classroom management" was described with specific examples as seen in the preceding dialogue. Similarly, the phrasing of classroom management questions in the CT final interviews was carefully worded to clarify any misconceptions about what the research meant by classroom management.

The direct translation of classroom management into Japanese is *kyoshitsu unei*. When U.S. teachers talk about classroom management they are primarily referring to managing student behavior. However, when Japanese teachers hear *kyoshitsu unei* they think of the administrative aspects of a homeroom teacher. Using other Japanese words for management such as discipline seems to further reinforce the image of classroom management being about the administrative duties and teaching that takes place in the homeroom.

Because of the difficulty in translation, the researcher clarified what he meant by classroom management in the final interview with the Japanese CTs. He did this by describing the specific student misbehaviors that were observed in the mathematics lessons and attributing them to "classroom management." He was then able to ask questions about how new teachers learn these "classroom management" skills. When asked if they talked about classroom management to the STs, the CTs indicated that they did not. The CTs also acknowledged that the behavior of the students at this university affiliated junior high school was better than what the STs would experience as new teachers in public schools. The following dialogue between the researcher and CT-Tanaka sheds light on where STs learn to handle student misbehavior:

Interviewer: The last two questions are about classroom management. If you see students' bad behavior, for example, and one student says something inappropriate to another student during class or they don't participate in the class or their conversation really interrupts the whole class. If this problem occurs, how would you solve the problem? Do you solve it by yourself or solve it with the homeroom teacher?

CT-Tanaka There are many levels. For example, suppose there is a fire. If the fire is put out when it is small, then there is no problem. But after this fire gets bigger, then I have no way to put it out by myself. Class is the same. If the problem is solved when it is small, I can handle it. Every student has the possibility to chat in a class even though they are not especially bad students if a lecture is boring. So if a class is boring and it caused chatting, then I have to do something during this class period. Otherwise there will be many other fires coming out from other places. We need to instruct students when the fire is small. If the fire extends to the whole class then I need to get the homeroom teacher's help and work with him/her. Students have the right to study. If a student interrupts another student who want to study then I will kick this student out of the classroom. Then I will talk to him/her in person. Otherwise I feel sorry for students who have the desire to study.

Interviewer: Yeah, but you did not talk much about it with the student teacher, did you?

CT-Tanaka: That is true.

Interviewer: So where can they learn about it?

CT-Tanaka: Teachers?

Interviewer: Student teachers don't have many experiences with classroom management, especially at this school. But most student teachers will teach at public schools where they will face many of these problems. So where can they learn about it?

CT-Tanaka: Well, presently the first year teachers, I mean new teachers, will have instructors with them every day for a year to be taught. Usually these instructors are someone who just retired from their junior high school president position. So if something happens on the first year, these instructors will teach them well. New teachers actually will learn many things after becoming a teacher.

CT-Tanaka clearly addresses when and where new Japanese teachers learn about classroom management, namely during their first year of teaching as they are carefully mentored by a more experienced teacher in the school.

Discussion

In the final interviews, the comments by CT-Tanaka and ST-Hideki confirmed that there was no discussion about classroom management between the mathematics CTs and STs. This was in direct contrast to the nature of the conversations observed between U.S. CTs and STs in the Peterson and Williams (2001) study as well as the other studies mentioned earlier. The causes for this difference, however, can only be discussed as hypotheses and will require further study to verify. Some of the possible causes of the different emphases on classroom management in the two countries are 1) the different structure of student teaching, 2) the behavior of the students in general or 3) beliefs about learning to teach.

Structure of Student Teaching

The STs in the United States take on most of the responsibilities of the classroom including many administrative ones such as taking roll and grading homework. Jennifer's comment describes her response to dealing with some of those responsibilities: "At first I felt overwhelmed with grading papers, entering them in the grade book, entering them in the computer, and that has nothing to do with your teaching." After a period of time, the STs in the United States teach all of the classes and perform all of the duties of a regular teacher. Thus a ST's lesson is not a special event and the students behave more like they would with the CT.

The settings of the ST lessons in Japan, however, made discussions of classroom management less needed. The STs only teach 3 lessons over the course of the 4 weeks of student teaching and each lesson is observed by the CT and 4-7 other STs. Thus ST's lessons are not special events for which the students behave similar to how they would with the CT.

Behavior of the Students

Another factor that may influence the lack of classroom management conversations in Japan is the general behavior of the students. Because this Japanese junior high school is affiliated with a university, the students had to take an exam to be admitted. The teachers are also considered to be very good. Thus the students were better behaved than what is found in typical Japanese and U. S. classrooms. However, the mathematics lessons were not necessarily quiet. Since most Japanese mathematics lessons are problem based with students working in groups, teachers are tolerant of high noise levels.

Although problems with student behaviors are less frequent in Japan, particularly during student teaching, they do exist. As Morey, Nakazawa, and Colvin (1997) point out, Japanese STs are concerned about it. CT-Tanaka's comments indicate that teachers must develop skills to deal with student misbehaviors. New Japanese teachers learn some classroom management skills as they relate to the homeroom from the homeroom teacher during student teaching. The problems that may occur during a mathematics lesson, however, go unaddressed during student teaching.

Learning to Teach

Although the causes mentioned in the previous paragraphs are real possibilities, the best candidate for further research is what appears to be a fundamental difference in beliefs about how one learns to teach. This study shows that there is not a direct translation for the English term of classroom management into Japanese. This vocabulary disconnect highlights the difference in emphasis on this aspect of teaching. On the other hand, the Japanese term, *hatsumon,* which means "*asking a key question* that provokes students' thinking" (Shimizu, 1999, p. 109) has no direct translation into English. This word focuses on the importance for a Japanese teacher to have a clearly articulated question or goal for each lesson. Having no comparable idea in English is consistent with Mr. B's belief that learning to teach is easy. In his final interview he said:

> In the junior high level, the whole name of the game is classroom management. … Teaching strategies and different lessons are easy to learn and you can pick a lot of that up from watching other teachers or workshops.

These language differences suggest that the lack of conversation about classroom management during the Japanese student teaching experience is not because there is nothing to talk about, but rather because beliefs about learning to teach are centered in the conceptual development of the lesson.

In summary, STs in the United States spend the majority of their time learning about classroom management. Japanese mathematics STs, however, spend their time during student teaching learning how to prepare, teach and reflect upon their lessons; discussions of dealing with student behavior are left relegated to the first year of teaching. It is clear that student teaching has a different focus and purpose in each country.

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Mathematics Teachers' Professional Development and Identity in a Distance Education Setting

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This paper discusses the influence of an in-service distance education course in the construction of mathematics teachers' professional identity, especially regarding their views and practices of reflection and collaboration and their relation with information and communication technology. The course was based on open-learning pedagogy and focused on conducting exploratory and investigative work in the mathematics classroom. Evaluation results show that the perspectives and involvement of the participant teachers depend very much on their previous professional experience and relationship with the Internet. Teachers that use e-mail for collaborative work found this a very stimulating experience whereas those with less professional involvement had some difficulty in assuming the roles and values required for this kind of activity.

What influence might an in-service distance education course have on teachers' professional development and on the construction of their professional identity? This paper addresses this question based on an in-service course with the format of a "study circle", structured according to open-learning pedagogy (Collis & Moonen, 2001), and focused on conducting exploratory and investigative work in the mathematics classroom (Mason, 1991; Ponte, 2001; Ponte, Oliveira, & Brocardo, 2003; Ruthven, 2001; Skovsmose, 2000; Wood, 1994). An evaluation was undertaken to ascertain the reactions of the participants to the structure and functioning of this in-service activity and its influence on teachers' professional development and identity. Special attention was paid to how participants regard reflection and collaboration and relate to information and communication technology. Specifically, this paper discusses the main issues bearing on the emergence of new aspects of teachers' professional identity in this distance teacher education setting, and considers their potential implications for research and practice.

Teachers' Professional Development and Identity

During their careers, mathematics teachers change and develop professionally. The professional development of a teacher may be regarded as a growth process of his/her competencies concerning mathematics teaching practices and other professional practices, and in the self-control of his/her activity as an educator and as an active participant of the school organization. In this perspective, professional development concerns issues related to mathematics education but also includes issues related to every other aspect of educational activity and personal as well as relational aspects regarding other teachers and members of the school, local, and educational communities.

Therefore, professional development is much more than acquiring bits and pieces of fragmented knowledge (Popkewitz, 1992). It includes learning more mathematics or mathematics education theories and becoming able to use them in teaching practice but it is much more than that. In fact, professional development may be regarded as a complex process in which the teacher is involved as a person in his/her professional, cultural and social context and in close relationship with other teachers. It includes all the learning experiences—natural and planned—of a teacher that yield him/her direct or indirect benefits that support the quality of his/her work with pupils and with other participants in educational institutions (Day, 1999).

For a mathematics teacher, the development of a professional identity involves assuming the fundamental roles, norms and values of the teaching profession. The key features of this identity are highly problematic since teachers are not a homogeneous body but encompass several professional subcultures (Feiman-Nemser & Floden, 1986). Key aspects of the culture of the mathematics teacher include the way he/she regards his/her professional role as a transmitter of knowledge or a promoter of pupils' development interaction norms with pupils and colleagues, participation in professional activities such as projects, meetings and informal groups, and the teacher's own stance regarding his/her development. The way teachers relate to other teachers, working mostly individually or collaborating in important issues, is an important indicator of their professional culture (Hargreaves, 1996).

Furthermore, professional identities have a dynamic nature, evolving along with the changes that occur in society and in the nature of occupations. For example, in recent years, in several countries, the structure of the teacher's activity has undergone important changes involving new curriculum goals and professional responsibilities. Also, as happens in many other fields, information and communication technology provides new opportunities for teachers' work—using such technology in classes, preparing lessons and classroom materials, writing reports, carrying out administrative work, and sharing information and experiences with other teachers.

Professional identity is one aspect of social identity. For Berger and Luckman (1973), in objective terms, a social identity can be regarded as belonging to a certain world, and can only be understood, in subjective terms, together with that world. One's social identity is made of multiple aspects, including language, culture, social group, political and religious beliefs, and the role assumed in the social division of labour. The most important activity systems in the construction of the identity are communication, which structures interaction among individuals, and instrumental activity, related to the working processes and their underlying technical and organizational means (Dubar, 1997).

In the perspective of symbolic interactionism (Blumer, 1969), the individual is not just a passive element of a group, who internalises its norms and values, but is also an agent who assumes a useful and recognized role in that group. Therefore, we may speak of a dialectic between an "I" identified and recognized by the other as a member of the group and an "I" that assumes an active role and participates in the permanent process of reconstruction of the community. For Dubar (1997), the harmonious integration of these two sides of the "I" is the key to the consolidation of one's social identity.

The social identity is permanently reconstituted in the socialization process. That is, the identity is not given, but continuously constructed and reconstructed in conditions of permanent uncertainty. So, social identities arise as dynamic entities and not as "objective data" or "subjective feelings". As Dubar states, "social identity is no more than the result, simultaneously stable and provisional, individual and collective, subjective and objective, biographic and structural, of the several socialization processes that, together, individuals and institutions construct" (1997, p. 105). In his view, with the notion of social identity we can make a deeper analysis of professional issues than with the classical notions of group, class and category (of macro-sociology) and role and status (of micro-sociology), as it "introduces a subjective, lived, psychic dimension in the kernel of sociological analysis" (p. 105).

Modern psychological and sociological theorists speak of decentring, displacement, or fragmentation of the subject, according to which individuals assume multiple identities (Bruner, 1990; Hall, 1999). This is a phenomenon that is increasingly occurring in our society—we have not one but several identities, sometimes contradictory and unresolved. The processes of identification, through which we project ourselves in our cultural identities, have become more provisional, variable and problematic.

One of the factors that contribute to this change in the nature of social identities is information and communication technology, specially the Internet. This is a multisided new media that allows a wide range of personal experiences. First, it is an immense assembly of resources with information about events, news, documents, papers, lesson plans, *software,* and so on. Second, it also enables the publication of our own productions—papers, classroom plans, software, video clips, PowerPoint presentations, etc.—making them available to a wider public. But, third, more than an instrument of gathering information and disseminating educational products, the Internet provides the possibility of virtual interaction among people, including teachers, pupils, parents, future teachers, teacher educators, scientists, professionals, politicians, and many other social agents. The participants in these interactions form groups and networks that may be regarded as virtual communities, since their interactions occur through the cyberspace. The Internet therefore constitutes a new cognitive and social ecosystem in which individuals may embark on a process of adapting and restructuring their relational and cognitive activity, with possible consequences in the ways they view the world and they regard themselves. The extent to which this may happen and the specific features it may assume are issues to be explored, notably insofar as they affect mathematics teachers. In this paper, the main concern is how a distance education setting, based on a flexible pedagogy, stressing collaboration and reflection (both oral between pairs of teachers and written, writing papers and e-mail messages and contributing to a forum) assists the development of a teacher culture that places a higher value on professional interactions and partnerships as well as on the use of information and communication technologies.

Methodology

Objectives, Format and Participants

This paper examines a distance education in-service study circle that was attended by 36 teachers, 34 of whom completed it successfully. A most prominent feature of this study circle was that teachers were required to register and work in pairs. The aim of this in-service activity was to offer teachers some theoretical ideas and practical experience regarding a current curriculum orientation—working with mathematical explorations and investigations—and to contribute towards their professional development, providing opportunities for reflecting on their own practice, using ICT, and developing a culture of collaboration. This course lasted for six months and it was divided in three segments: (i) dynamics of the mathematics classroom; (ii) investigations in mathematics and in professional practice; (iii) one experiment with investigations in the classroom. The participants were mathematics teachers in grades 5 to 12 from different regions of Portugal (two from the rural North, eight from Porto; six from the Centre; thirteen from the Lisbon area, four from the city of Lisbon; one from the Alentejo) and two from Brazil.

The setting designed for this course includes a Web environment, through which various materials are provided. Teachers had to read mathematics education papers, conduct searches on the Internet and complete several tasks, some of which involved observation and reporting about their classrooms. For each segment, there is a study guide and several papers, some of which are required, others optional. These papers were to be read and discussed by each teacher with his/her partner and possibly with the teacher educator, and also with other participants in a mailing list. Some papers were written for this course and others were drawn from the professional and academic literature; all of them were in Portuguese (original versions or translations). Examples of required papers are Fonseca, Brunheira and Ponte (1999), Poincaré (1996), Ponte, Boavida, Graça, and Abrantes (1997) and Skovsmose (2000).

The tasks were open and diversified. In task 1, the teachers had to comment on one of the required papers; in task 2, they had to describe and analyse a classroom situation that they had experienced; in task 3, they had to select and analyse a Web site relevant to mathematics investigations; in task 4, they had to study a problem from the history of mathematics; and, in task 5, they had to design a mathematical investigation, use it in their classroom, and reflect on this experience. The open nature of these tasks enabled the participating teachers to carry them out according to their interests and concerns. The emphasis on writing, a multirepresentational and integrative process, favours the restructuring of meaning, therefore constituting a key activity in promoting reflection (Zabalza, 1994).

Dynamics and Roles

This in-service course had three sections (two on Numbers/Functions and one on Geometry), organized according to the teachers' preferences. The first session was an introduction to the activities and working procedures and was carried out face to face at the university allowing participants and teacher educators to get to know each other. The last (double) session was the participants' presentation and discussion of their work and a reflection about the study circle.

- There were several different kinds of interactions among the participants: Teacher educators and teachers interact face to face (in the first and last sessions);
- Teachers interact with their partner teacher, as they work collaboratively;
- Teachers interact with the system, downloading materials and looking for information on the Web site and elsewhere;
- Teachers interact with teacher educators, via email and the Web site, sending tasks, answering questions, and reporting their progress;
- Teachers and teacher educators interact in a discussion list.

Besides the participating teachers and teacher educators, the course involved a coordinating team, overseeing the whole system, a technician who took care of the Web environment, and a team of external evaluators.

Regarding his/her participation in the study circle, each teacher was evaluated taking into account three main aspects: carrying out the tasks, participating in the discussion list, and self and group evaluations.

Program Evaluation

The program also had an internal ongoing evaluation and an external evaluation. One focus of interest was the setting and the materials used; another focus, which is the main concern of this paper, was its effects on the participants. For the external evaluation, data collection methods included a questionnaire, interviews, observation, and document analysis. As their final task, the participants had to answer an open-response anonymous questionnaire with questions concerning the activity of the course and self-assessment. The documents provided in the course and the assignments produced by the participants were another source of data. The teacher educators' meetings were observed and the messages exchanged by participants and teacher educators as well as those sent to the discussion list were also taken into account. To study in detail the experiences of six teachers who participated in the course, there were also in-depth case studies of three groups, based on face-to-face interviews. In these interviews teachers were asked to reflect on their experiences throughout the course—readings, assignments, collaboration with their partner, exchanges with the teacher educators and other participants. The three groups of teachers were chosen so that there were (i) teachers with many years of experience as well as novice teachers, (ii) teachers from different regions of the country, and (iii) teachers with different levels of involvement in the course. In each case, data analysis followed a detailed schema of categories addressing the issues of interest concerning the activity and setting of the course and the participants' experiences. This paper draws mostly on the results from the questionnaire and two of these case studies.

Results

General Evaluation

As indicated by the answers to the questionnaire, the distance education course was generally successful in attaining the set goals. Teachers became more aware of the issues involved in carrying out investigations in their mathematics classes, had opportunities to work closely and face to face with the colleagues with whom they developed a close collaboration and to discuss at distance with their teacher educators and other course participants. Furthermore, in their responses most of the teachers found the structure of the course, the materials provided, and the tasks proposed useful and adequate for their professional development. These are some of the answers to the questionnaire that underline this:

> My commitment to the study circle was strong and fruitful. The discussions towards the preparation of each assignment were rich and involving.

> This distance format made my commitment to this study circle greater than in any other in-service course that I attended during the school year.

In the teachers' responses to the questionnaire, there are hints pointing to the development of new aspects of a professional identity. In particular, these concern the attitudes of the participating teachers regarding reflection and regarding their relationship with information and communication technology. In fact, about half of the teachers indicate that, in one way or another, they had opportunities to reflect on their classroom practice and professional practice, as illustrated by the following responses:

> I reflected, I was questioned, I was commented on, and I was evaluated about my views regarding mathematical investigations … The study circle represented a reflection about our own conscientiousness; we stopped to reflect about our professional life … I feel that the study circle led me to assume my own consciousness.

> We felt the need to reflect about our own practice. We had already carried out several investigation activities with our pupils (more or less guided) but we had not stopped to reflect in an organized way about such classes.

This last teacher talks directly about "consciousness'", which reflects a deep awareness of self and directly evokes the issue of identity. Only four teachers in the whole group showed this kind of self-awareness. The first reply speaks of reflecting in an organized way to improve practice. Carrying this out in a consistent manner may also bring about changes in the teacher's identity in the long run.

The teachers also indicate several reasons that promoted such a reflective stance. These included (i) working with a partner in team work, which, as one said, "yielded good opportunities to reflect about our activity"; (ii) the material provided, since "in the papers we also found hints to reflect about/upon"; and (iii) the questions posed by the teacher educators, as they "helped to develop my reflecting and analysing ability". As a result of such reflection, two of the teachers indicated that they changed some of their conceptions regarding mathematics teaching:

> I was led to reflect about issues already addressed in my master's course … that I ended up seeing in a different way.

> I changed the meanings that I ascribed to several things that I had got from the *ProfMats* [national meetings of mathematics teachers], publications, NCTM Standards.

In their responses to the questionnaire, most of the teachers who had poor skills in using information and communication technology indicated that they had developed them and others recognized that this was a useful way to find materials to use in their teaching practice:

> I learned how to use the Internet.

> [I got to know] a lot of information from the Internet (essential in this kind of work) that I was not aware of before.

> I also learned that it is complicated to do searches on the Internet … Some perseverance is necessary to find good material.

Overall, these results point towards the development of a positive stance regarding reflection and collaboration, and also towards a better relationship or use of the Internet. Given the open-ended nature of the questions, these results just show a general trend. However, they do suggest that there was a wide variation of working experiences in this course. That is why case studies were conducted and we turn to them now.

Case 1

One of the groups included two teachers, Isaura and Anabela. They worked in different schools but had met a couple of years before, when Anabela was a student teacher and Isaura was her school supervisor. We may regard this as a heterogeneous group, as Isaura was much more experienced than Anabela. They were very interested in getting "practical" ideas, that they could take to their classes and regarded with little interest all that they considered "theoretical". They viewed the discussion list that was provided in the course as a place where theoretical discussions could be carried out and they had little participation in it. Isaura and Anabela showed interest in interrogating certain aspects of their practice, namely those more closely related to the mathematical content to be taught. They carried out productive work during the course, although they had a rather difficult moment when the teacher educator stated they had to reformulate one assignment.

These two teachers enjoyed the fact that they could work in a very flexible way and did not have to physically displace themselves to attend meetings in a teacher education institution. However, in the interview, they indicate some uneasiness because of the absence of face-to-face interactions:

> At the beginning I missed the personal contact between us and the teacher educators and the other groups … (Anabela)

> This is a weakness. We are used to [direct] interpersonal relationships, to discussing face to face. And we are not used to doing this at a distance … It was difficult to be working and not to have immediate feedback from the other side. (Isaura)

The activity of the course required a lot of writing (messages to send to the teacher educator and to the discussion list and tasks that had to be delivered at certain times). However, these teachers did not seem comfortable about expressing themselves in writing. Isaura recognized this explicitly: "We find it difficult to express our opinion in front of a computer". The tone of their messages was formal, just saying what was strictly necessary. This is in sharp contrast with the warm way they relate to other people in face-to-face settings, as we witnessed in the meetings carried out at the university and in the interviews.

These two teachers find it very positive that in this course they do not work in isolation but in pairs. In their view, working at a distance could become painful: "One person is completely isolated" (Isaura). From their perspective, pairs must be made up of people who like to work together: "I like if it is with someone who I like to work with" (Anabela). That is, they see working with a partner mostly as a motivation to overcome the difficult parts of the course.

These two teachers do not report major problems in dealing with information and communication technology. Isaura claimed to be experienced in using this technology, especially "to search things" in the Internet. Anabela admitted she had little experience, but indicated that in the study circle "she could do everything that was required". However, in the tasks proposed, these teachers had some trouble in interpreting what was asked of them. The teacher educators considered that their work did not correspond to the course's expectations and asked for reformulations. Isaura and Anabela felt quite uneasy with this demand and wondered whether they should drop the course. Assuming that the next tasks would be more interesting, they decided to continue. There were clearly communicational difficulties between these teachers and the teacher educators and different levels of expectation regarding the quality of the work that had to be done. The fact that all this negotiation had to be carried out by e-mail only seemed to make matters more complicated.

In short, these two teachers showed interest in questioning some aspects of their professional practice, namely those closely related to the topics that they had to teach. However, they showed little interest in analysing aspects related to the curriculum or the classroom dynamics, that they found too "theoretical" and, therefore, with little relevance to their practice. They were able to use information and communication technology to gather information and interact with the teacher educators but felt rather uneasy about this form of communication and reported they missed the face-to-face interactions of usual teacher education courses. That is, these two teachers represent a case of very little adherence to new forms of communication and professional interaction and therefore of developing their identity, as proposed in this teacher education setting.

Case 2

Before they were involved in this distance teacher education course, Julia and Maria, together with two other colleagues, already constituted an informal working group carrying out several professional activities. They used to meet face to face once a week and constantly exchanged e-mail messages among themselves (according to Maria, about ten daily messages). They recognize that participation in this course led them to write much more than they usually did. Julia, in the interview, stated that this was quite positive:

> There is one aspect … that I have felt for a long time. Many times we have very interesting experiences and we discuss issues, and so on, and we always postpone writing, passing them onto paper, recording. This distance education setting requires that we write.

At first, these teachers reacted negatively to the limit in the number of words that they could write in their assignments. But, with time, they recognized that this limitation led them to a deeper reflection about what they wrote: "That may be an aspect that created discipline in us, in the sense of reflecting again, improving, and fixing the essential things and also helping us to focus on objectives that would otherwise remain somewhat vague" (Julia).

For Julia and Maria, writing does not seem to be a natural means of expression but they recognize its importance in their professional practice. As Julia said in the interview, their dynamic of collaborative work became stronger with this course, which they regard as making a significant contribution towards their professional development:

> Both as a pair and individually, we assumed our professional personality, and that is deeply gratifying … Working in group helps to share the competencies of each person and helps to overcome personal inhibitions because of the friendly and responsible commitment of each one towards herself and her partner.

Julia and Maria show great interest in reflecting about their practice and in carrying out activities stemming from such reflection. They look for ideas arising from their experience, from the work of other teachers, and from professional and educational literature. They read the papers suggested in the course with interest and used them to analyse what went on in their classes.

These teachers acknowledge that this distance teacher education setting led them to write much more than they were used to, as shown above in the quote of Julia. For them, writing does not appear to be a natural act, perhaps more because of the diversity of demands that they feel all the time than because of lack of fluency in writing. However, they recognize the importance of writing for their professional practice.

They show great commitment towards taking action in their classrooms but are also greatly concerned with what happens in their professional community, recognizing that it will take a long process so that it may grow in activity and in social and educational influence. As Julia stated in the interview:

> For us, communicating at a distance (searching the Internet, communicating by e-mail, and so on) was already a routine activity. But it was interesting to experience in the study circle the creation of patterns of communication between us … and the teacher educator. … We felt that it is necessary

> and urgent to create among teachers a communication culture such as this one, in which people feel free to participate, within rules of politeness, without hurting each other.

These teachers already had a strong professional experience and had previously developed many professional projects. They were used to working collaboratively and to making intensive use of the Internet. In this course, supported by technological media, they were able to assume a productive division of tasks and had the experience of reflecting and writing about professional issues. As a result of this process, they refer to having become more confident about curriculum and professional issues and developed a stronger sense of their professional culture. Julia and Maria, who are able to take advantage of information and communication technology in a very flexible way, constitute indeed a very unique group, showing strong professional commitment.

Conclusion

Teachers involved in this study circle experienced a process of professional development in their knowledge regarding mathematical investigations and in their ability to carry such activities in the classroom. Many of them developed a more reflective stance and had a positive experience of professional collaboration. Some of them also developed their inclination to use information and communication technology for professional purposes. Such evolution may be regarded as pointing to changes in their professional identity.

The professional culture of teachers tends to be strongly individualistic (Feiman-Nemser, 1986; Hargreaves, 1996). However, as this experience has shown, information and communication technology, used in a formal way in distance education courses and in an informal way in professional exchanges, may provide an opportunity for teachers to develop a dimension of virtual interactions with other teachers. In some cases, these interactions may become strong enough to speak of virtual communities. We may also have groups of teachers for whom face-to-face and virtual interactions are interrelated in a strong and fruitful way, as in the case of Julia and Maria reported in this paper.

Writing is not a natural activity for many mathematics teachers. This is also a feature of their professional identity. They think of themselves as being at ease with using numbers and symbols fluently but not written language. However, this kind of communication was an important feature of this in-service activity. Teachers had to read mathematics education papers, turn in written assignments, communicate by e-mail with the teacher educators and participate in a (written) discussion list. For most of them, this was not a normal way of expressing themselves and some felt rather uneasy about it. But there is ample evidence that the emphasis placed on writing helped teachers to think about professional issues in a deeper way and helped them to develop a more reflective stance. Julia and Maria even recognise that this form of expression may become an important feature of the professional culture of mathematics teachers.

The teachers who participated in this study circle were different in many regards. For example, whereas Anabela was just learning how to use information and communication technology; Isaura could do searches in the Internet but did not use it for communication purposes; Julia and Maria were already quite experienced and intensive users. The distance education study circle appeared to help all these teachers to explore new ways of using this technology—downloading papers, exchanging documents, discussing issues by e-mail, and participating in discussion lists. Of course, a deeper involvement in communicating and interacting through electronic media is not a process that develops in just one step, much less during a single in-service experience. However, the course format, stressing reflection on practice, connection to theoretical ideas, writing about professional issues, collaboration among teachers, and interactions of teachers and teacher educators, put an emphasis on professional values that may be regarded as significant in developing new aspects of these mathematics teachers' professional identity.

These two pairs of teachers—Anabela and Isaura, on the one hand, and Julia and Maria, on the other—showed to have a rather different relationship with information and communication technologies but also, and more importantly, in their professional stance. Anabela and Isaura were interested in getting some more ideas for their classroom practice. Julia and Maria wanted to explore the possibilities of distance education for their professional development and wanted to reflect on classroom issues but also on the curriculum and on wider professional issues. The teacher education course tried to match the interests of each group, according to curriculum and professional orientations and concerns.

This experience shows the potential for teachers' development of open learning distance teacher education stressing collaboration and writing and opens up interesting issues for the design of in-service activities and resources. It also raises questions for further investigation regarding the tendencies and constraints of teachers changing professional identities supported by virtual learning communities. Other course formats can be designed, with similar or different features, and we can think of more informal ways of using the Internet to support interactions, written reflections, collaborations and exchanges among teachers as well as interactions and collaboration involving teachers and teacher educators. It would be interesting to know what these media and these forms of work may bring to mathematics teachers who want to develop professionally and what that may mean in terms of developing new sides of their professional identities, perhaps in a stronger relationship with writing and with a greater appreciation for reflecting and collaborating practices.

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Future Teachers' Perceptions of their Mathematics Education Program

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On the basis of the analysis of classroom lessons, individual interviews were constructed with the intention to probe in detail the rationales and background influences that underpin the teaching actions of future secondary level mathematics teachers. This report, part of a larger study (Proulx, 2003), underlines the varied perceptions of future mathematics teachers of their teacher education program. This variety of perspective presents and opens a range of issues in regard to the structure, the development, and the possible objectives of mathematics teacher education programs. The problematic of fixed objectives will be discussed in relation to the concept of 'objectives to attain' *versus* 'objectives to work on'.

This report is based on a larger study (Proulx, 2003) which was concerned with the rationales that underpin the practices of prospective high school mathematics teachers in relation to their mathematical oral explanations. The specific focus here is on the elaboration of the future teachers' perceptions of their mathematics education program as a background influence that played an important role in their personal construction of teacher knowledge: What are those perceptions? How do these future teachers enact them in their practices?

Some Theoretical Concepts Concerning the Impact of Teacher Education Programs

In an extensive review of forty years of research into learning how to teach (not specific to mathematics), Kagan (1992) explains that teacher education programs have little influence or impact on the beliefs and images already developed by future teachers. To that, Bauersfeld (1994) adds that when the new teacher is confronted with conflicting situations, habits and ways of doing[23] that are strongly rooted in his or her personal experiences as a student will emerge: he or she will privilege the former methods and, in that privileging, reproduce the traditional school model. Bednarz, Gattuso, and Mary (1995) summarize Kagan's and Bauersfeld's ideas in stating that the stability of those previous representations stay unchanged and follow the teachers into their classroom teaching practices.

However few the effects, Ball (1988) explains that when prospective mathematics teachers practice teaching, some tend to model their approach on observations of their university instructors, whereas others focus on specific events that were significant for them and helped them understand the mathematical concepts in their mathematics education courses. Those ideas prompted me to think about the concepts of 'didactical copying' and 'didactical re-production' (Proulx, 2003). The former refers to an uncritical re-use of concepts as shown in courses. Didactical copiers do not have rationales to explain why they make use of specific concepts, except for the fact that the university instructor

[23] Bauersfeld (1994) uses the sociological concept of *habitus* developed by Pierre Bourdieu to describe those old habits.

told them and showed them it was important and 'good'. 'Didactical re-production'[24] refers to a more critical attitude toward the models, ideas and actions highlighted by the instructors while appropriating and transforming them in his or her own way. Didactical re-producers have reasons and rationales to explain why they are acting in a certain way.

Brief Clarification of the Methodology

The results I report here stemmed out of the methodology of the larger project, but were obviously not results that I aimed for or designed into the study at the beginning[25]. I will report briefly on the methodology used in the large study to give an idea of the approach taken. However, I provide specific details insofar as they pertain to the sub-study.

The five student teachers I worked with were at the end of their second year of the 4-year program at the Université du Québec à Montréal (UQÀM), and they had just finished their second practicum (out of four) in the schools the semester before[26]. In UQÀM, the program's structure moves away from the prominant model that is centered on a training in the specific discipline followed by training in psycho-pedagogy and then a practicum. Courses in the subject matter and in pedagogy and learning to teach are intertwined in a program where mathematics courses and mathematics education courses are taught by mathematics educators. The intentions of the 4-year program are focused on the practice of teaching mathematics rather than learning to teach mathematics—a matter of learning-in-action, rather than learning-about-action (see Bednarz & Proulx, 2005, for details on the philosophy underpinning the program and Bednarz, 2001, and Bednarz, Gattuso, & Mary, 1995, for an extensive review of the activities of the program at UQÀM).

Five participant volunteers were chosen on the basis of the subject they were teaching in relation to the program. Since they were in their second year of the program, they had taken some courses in didactics/mathematics education (called "didactique des mathématiques" in French[27]), for example, 'Didactics of Mathematics 1 and Laboratory', 'Proportional Reasoning and Associated Concepts' and 'Didactics of Algebra'. I then looked at the topics taught by the future teachers and decided to work with five prospective teachers in total: two of whom had studied those topics in previous courses (Nathan and Raphaela); two of whom had been introduced briefly to those topics in previous courses (Mike and Tony); and one of whom had not yet studied the topics (Sam). The second year in the program was picked for practical reasons, that is, it coincided with my research schedule.

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[24] Désautels (2000) stress an important difference between 'reproduction' and 're-production'. 'Re-production' is not referring to a mechanical process where the agents of a situation are seen as determined puppets, 're-production' highlights the fact that cognitive notions are re-produced by agents while also being transformed by them. In fact, the concept of 'didactical copying' would mostly refer to 'reproduction' (in one word).

[25] It is with discomfort but for syntax reasons that I use the word 'I' everywhere in this article, instead of 'us' or 'we'. This study was realized under the supervision of my masters' thesis advisors, Dr Nadine Bednarz and Dr Carolyn Kieran.

[26] Except for one of them, Raphaela, who was in her 3rd year and did not follow the "traditional" path.

[27] The term 'didactics' is taken here in its French sense which differs from its sometimes deprecating English connotation of an "art or science of teaching, implying in particular a set of problematics, concepts, and methodologies for use in a given educational situation" (Larochelle, Bednarz, & Garrison, 1998, p. 271). 'Didactics' is intended as a field of study that concentrates on the analysis of teaching practices and educational situations to better understand them and to render them explicit.

To gather data, each future teacher gave me three videotaped classroom lessons taken at different moments in their practica. Based on a first analysis of the videotaped classroom lessons given by each participant, individual semi-structured interviews were constructed to better understand the rationales that underpinned the actions and explanations given by the future teachers in his or her classroom. The interviews consisted of two distinct parts: The first part was specific to their lessons and to aspects or events that occurred in their classrooms; the second part was on a general level, and was the same for each future teacher. It consisted of questions about their background influences and their ways of seeing teaching. The decision to use semi-structured interviews was based on the belief that by triggering a discussion on some events (regardless of which events), many concepts would emerge concerning the rationales underpinning their practices.

Rather than entering the discussion with pre-determined and standard questions, all of the questions grounded in the observations of participants' teaching practices. All those questions were aimed at delving into details about their teaching intentions and ideas; and to develop a sense of the rationales that underpinned their teaching practices. For example, I was interested to question the prospective teachers on their classroom routines and pattern, their goals, their oral explanations, their opinions or remarks made while teaching, their teaching strategies and so on. Here is an example of a question I asked to Raphaela concerning her classroom routines (she taught algebraic operations in the 9th grade):

> I have noticed that, in your three lessons, you always proceed in the same way: mental arithmetic, usage of rectangular areas, and then asking students to take note of the rule to solve algebraic operations.
>
> a) Why did you work in that way? What were your reasons?
>
> b) Where did these ideas come from? What made you think of all this?
>
> c) Could you have ordered things differently? How and why?

The second part of the interviews had common questions that invited the future teachers to talk about their past experiences as high school students and as university students, and of the possible influence of their supervising teachers, their students, and textbook use. Here is the example of the question I asked concerning their experience in the teacher education program and the possible influences it had on them:

> Do you consider that your university education has helped you, or played a role, in the way that you taught [name of the topic taught]? If yes, how and why? If no, why? Do you have particular or specific examples where it helped you?

The interviews consisted of approximately 15 questions each, and lasted about an hour. Each interview was audio-taped and transcribed.

Results

The individual interview process enabled me to shed some light on the participants' personal interpretations and perceptions of their mathematics teacher education program.The answers given to all questions enabled me to get a sense of their personal interpretation and perceptions of their teacher education program as a whole. These results emerge from the interview without any initial intent to focus (or get information) on the perceptions of the prospective teachers of their teacher education program.

Since the central aim of the study was to obtain information on teaching rationales, I created categories/themes to organize the possible teaching intentions and influences that were orienting the participants' teaching. Throughout all the categories or themes, I was able to find a common thread that enabled me to have a deeper understanding of their interpretations and perceptions of their program.

Based on my own interpretation of the responses that were given in the interview I will highlight, for each of the five future teachers, the principal characteristics of their personal 'readings' and interpretations of their teacher education program.

Tony 'the technician': The Teacher Education Program as a 'potential for resources'

"I tried some technical little things."

Tony's first comments on his teacher education program were very negative[28]. He explained that he did not find his courses to be relevant in the beginning, but that he discovered in his practicum that many aspects studied were very useful and could help him to teach effectively: "At first, I did find it un-useful. Until my practicum, I was saying to myself 'we are wasting our time'!".

Tony made use, in his teaching practicum, of some elements that were addressed in his teaching education program—manipulatives, visual materials, or specific type of problems. He explained that he *picked* particular elements from his program and used them at specific moments in his teaching—especially when he had to introduce a new algebraic notion, where he felt he could work on a more concrete level to introduce the algebra. For example, concerning the idea of choosing geometric figures instead of algebraic letters to represent the distributive law[29], he told us that: "Someone did that [in one of my course] and I found it fun so I tried it".

In a sense, like a technician, Tony went on to pick some tools that would help, in his view, his teaching in the classroom; he saw his teacher education program as an opportunity to acquire specific tools on mathematics teaching. In brief, in Tony's case, his teacher education program appeared to serve as a source of resources to help him at some moments, to teach more effectively.

Mike 'the mimic': The Teacher Education Program as a 'didactic authority'

"I had to…"

Mike had a particular relation to authority – be it the authority invested in a teacher, a textbook, some specific rules, or elsewhere. He never really questioned those authorities and used them as his core arguments. Put differently, he obeyed them. A typical remark he used was that he did it in that way because his teacher educator said it was good. Mike seemed to accept without any doubt the remarks or affirmations of his mathematics educators; he applied them without question. He respected the statements of authority figures and considered them as experts in the field.

[28] Detailed analysis of Tony is found in Proulx (2004a).

[29] It is important to notice that nowhere in the paper am I making a judgment on the qualities of the choices taken. The purpose is simply to highlight examples to help the reader make sense of what I am implying.

The interview shows in a clear way that he was not able to explain his teaching actions – his rationales were structured around the fact that some experts have told him so. This was an excellent example of 'didactical copying'. He did not know why he did what he did, he just did it because someone had told him to do so – and most of all because he *had* to. As he explained: "we have been *told* often to do so…"; "they showed us that *it was* important, so I *had* to put an emphasis on this"; "it *was* what I had to show"; "WXY [professor] *said* to…", and so on. This was Mike's way of justifying his personal choices. Any sense of personally construed rationale or reflection on his actions were almost absent in his discourse.

So, his teacher educators had an authoritative status in Mike's opinion; the mathematics educators played the role of experts for him. The program provided a 'didactic authority' and a beacon to be followed: It is unquestioned and unquestionable, something to be mimicked.

Sam 'the self-assured teacher': The Teacher Education Program as a 'confirmation of his personal and professional identity'

"At least ¾ of what we covered in class I was already doing naturally in my teaching"

Sam told me in the interview that he was aware and was already implicitly using most of the principles that were introduced in his mathematics teacher education program. For Sam, his teacher education program had made explicit some elements in which he personally recognized himself as a teacher and, in the same way, allowed him to confirm his personal and professional teacher identity regarding his own personal principles of teaching mathematics. Sam noted that he had given individual private courses for the last four years, thus he had experience in teaching this topic and felt that he knew what seemed to work, what did not, and what should be focused on more directly: "I already knew the whole section by heart in my head, I knew what it was".

Unlike Mike, Sam had a very strong and explicit rationale for his teaching actions: "I am personally convinced of these things so I apply them". He reported that he knew what he was doing and why: He could explain the purposes of his actions, he was comfortable with his choices and did what he thought was important. For example, he justified the reasons why[30] he was giving counter-examples: "It is the idea of a cognitive conflict. By giving a counter-example, the student realizes on the spot that it is not working, so he forgets about it and I only have to recall the real property"; why he was asking them to give their answers in a fraction-like manner: "because they are not good in mental arithmetic", "I wanted to show that […] it could go faster like this", "in the trigonometric circle […] sine of 1.36 is harder than to recall the sine of $\pi/6$ and the like", "it goes faster than using decimals if we did not have a calculator", "there is also the idea of precision"; or when he explained that he was focusing on the algebraic resolution step-by-step of a problem in analytic geometry because his students were so poor in algebraic solving – "they have difficulties isolating a variable" – that he needed to take some extra time to make sure, by going one step at a time, so that everybody was able to solve it.

Sam had a strong teacher identity, and this identity guided his choices and actions as a teacher. He was self-assured and confident regarding the choices he made. However, I want

[30] Again, here, I do not intend to judge the reasons given by Sam. The main point is to show how assertive he was in his way of supporting his claims and actions.

to stress that having a strong teacher identity is not necessary the same thing as being a good teacher, nor that deliberate and intentional actions are necessarily the basis of sound pedagogy (Gore & Zeichner, 1991). In my opinion, many of his choices and actions in the classroom could be contested.

Sam's teacher education program had rendered explicit some elements that he had personally recognized in himself as a teacher – and at the same time provided confirmation of his personal and professional teacher identity.

Raphaela 'the reflective practitioner': The Teacher Education Program as a 'philosophy of teaching'

> My didactics courses helped me to put certain things into words, the way to present notions, the way to explain, the verbalization in general. […] What are the difficulties of the students? What should I work on more? How will I present the notion so that it is as accessible as possible for the student? These are the points that you look for in the 'conceptual analysis' document[31].

Raphaela's[32] attitude toward her teacher education program is completely different from the others: She operated at a more conceptual level than a pragmatic one. The conceptual principles that Raphaela derived from her teacher education program concerned the construction of meaning to the mathematics and of development of comprehension and reasoning. Raphaela was not focussed on specific issues or aspects in her teaching; on a meta-level, she stressed the importance of creating mathematical meaning and understanding for the students. To do that, she noted that it was important to have students verbalize and explain their solutions, to offer many diverse and flexible interpretations, to create smooth transitions between the notions, to work toward new understandings from already known notions, to adapt teaching, to endeavour to better understand where students were, and so on. She took these underpinnings of her mathematics education program as a kind of a guiding light in her teaching.

Of note was the fact that Raphaela was unusually reflective in her stance. On 12 occasions during the interview, she took unusually long pauses to reflect and think about the issues at stake. She never responded without first trying to make sense of what happened. She was able to explain why—both in terms of her background and her intentions—she acted as she did and how she might improve on those teaching actions that needed to be changed. After a while, the interaction felt more like a conversation than an interview. For example, at one point, when I questioned her on the reasons for having concluded her lesson by explaining the algebraic rule, she replied: "I wonder how we could conclude differently?...I don't know…Do you have an idea?" I theorize that for Raphaela the interview was educative and informative around her teaching and her learning. In a way, she was doing the job of a researcher concerning her own practices: She was analyzing and making conclusions and remarks on her teaching, she was reflecting on her practice, as Griffiths and Tann (1992) explained:

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[31] The conceptual analysis documents are reference documents, produced by people that are in the context of mathematics teacher education – students or teacher educators – describing in detail the concepts studied in regard to the important notions and understandings to work on, the diverse difficulties and obstacles that can emerge for the child and in the teaching, and some interesting paths concerning the teaching of the concept (see Bednarz, Gattuso, & Mary, 1995).

[32] Detailed analysis of Raphaela is found in Proulx (2004b).

> Central to the spirit of reflective practice is reflection on the personal and professional concerns of the individual student teacher. The reflective practitioner reflects on his or her own practice. The theories which are used are taken on wholeheartedly, and criticised open-mindedly. (p. 2)

Nathan 'the natural teacher': The Teacher Education Program as a 'teaching model'

> I tried to take something from all the professors who made an impression on my thinking and to combine their ideas into something that works.

Nathan might be described as a teacher who reflects *in* the action of his teaching. In other words, his actions and decisions are made on the spot and not planned in advance. I hypothesise that he did not always know the reasons for his actions (Proulx, 2003). For example, he noted the importance of student explanation: "It is an occasion to let them talk a little, to give them a chance to express themselves because some of them have only recently learned French."

Nathan is what I call a 'natural teacher' who possesses intuitive instincts for the teaching 'in action' and a capacity to analyze, make decisions and ask questions: an exemplary intuitive teacher in action. An example of this practice was evident in his constant "revoicing" (Forman & Ansell, 2001) of students' answers, as he explains in the interview: "Often I complete what the students have said. And if there are words [...] that the students use that are not always the right term or it is not really exact, then I retake what the students have said." His explanations are not planned in advance, they emerge or occur in the happenings of the classroom.

Knowing that, it is quite logical that what Nathan got from his teacher education program is better understood in terms of immediate practical action than explicit guiding principles. By watching and analyzing one university professor in particular, whom he repeatedly named in the interview, Nathan had interiorized a specific teaching model. The actions of this specific mathematics educator were considered important and interesting for him. In the action of seeing an educator teaching he was able to analyse the practices of the educator (and not exclusively or explicitly the notions worked on) and learn how to teach. Nathan learned how to teach by *watching* the educator's actions and not by *listening* to what was taught about teaching. With or by this constant 'in-the-action' analysis, he construed and incorporated his own teaching model. In a sense, his teacher education program provided him with a kind of an exemplary model to follow and adopt.

A Perspective that Serves as a Guiding Light

In addition to the participants' various perspectives regarding their teacher education program, I also found an important coherence in their interpretations of other aspects that played roles in their teaching (e.g., textbooks, supervisor, final exams, previous high school teachers, colleagues, personal experiences as teachers or students). For example, Tony took bits and pieces of the textbook and used these ideas at specific times in his teaching. Mike would listen carefully and do exactly what his supervising teacher told him to do. He reproduced under authority, including the authority of the textbook or the final exams. Sam would take the comments of his supervising teacher concerning his teaching as a support and confirmation that what he was doing was suitable and should be done. Raphaela took the comments of her university supervisor or supervising teacher as guidelines and opportunities to reflect. Nathan would talk about his former high school teachers as models on whom he based his own actions.

What I call their 'guiding light' is helpful in understanding their interpretations of the influences in a general way, including their teacher education program. This guiding light appears to provide a filter through which they interpret and 'read' the many influences and experiences that they encounter and live as learners.

Implications

The different perceptions of the mathematics teacher education program that were highlighted above to indicate the wide range of ways that such a program can be appropriated by its students. I have only highlighted five different ways. Each student teacher appropriated the program in a different way, and so its role in their teaching lives varied dramatically from one to the other—irrespective of the intended orientations that the program could have had.

A question arises: With this variety of profiles, how can we structure a teacher education program that works for everybody and that helps each teacher evolve in their professional lives? I would argue that educators need to be sensitized to the broad diversity of possible interpretations that are uniquely constructed by individual teachers. This may be something we all already knew, but there is not much evidence that we are acting on this knowledge. This important issue cannot be overlooked by mathematics educators as we construct, plan and teach our mathematics teacher education programs.

To this, we can also add the fact that the literature is not clear in regard to the possible outcomes of the mathematics teacher education programs. Some studies show definitive effect, some show none, some are nuanced in relation to copying and re-production, and so on. The results here show a wide diversity of enactments, ranging from a philosophy that transcends all teaching acts to un-reflective didactical reproduction. How, then, might we think about the goals or intents of a mathematics teacher education program if future teachers are so different and their perceptions and enactments.

Re-Thinking the Objectives of a Mathematics Teacher Education Program

However divergent the possible 'effects' of mathematics teacher education programs, this is not to say that it is unproductive to educate teachers, nor that we cannot have specific goals in a mathematics teacher education programs. A fruitful path might be located in the way we treat the notion of *objectives*.

The English word *objective* is linked to the French *objectif*. According to *Le Robert-Dictionnaire historique de la langue française*, objectif comes from the Latin *objectivus* which means 'something that constitutes an idea'—a representation of the mind—but *not* an independent or predetermined reality[33]. Informed by this etymology and oriented by the research results, I would offer a redefinition of objective. Objectives could be looked at as starting points to develop from, instead of looking at them as end points to obtain.

For this, I have tried to coin a tentative distinction between '*objectives to attain*' and '*objectives to work on*', from which I would theorize that instead of pre-specifying a goal or an objective at the end and narrowing our actions in long-term planning by focussing

[33] See *Le Robert-Dictionnaire historique de la langue française* for an account of how the concept's current meaning evolved.

tightly on it—what Bauersfeld (cited in Voigt, 1985) calls the "funnel approach"—objectives and goals could be framed in terms of *expanding the space of the possible* (Davis, 2004). This would then change the focus, away from final products to converge onto or conform with (the objective itself), and toward an idea of evolving within those very objectives. A shift in the underlying imagery is suggested here, from linear trajectories (A to B) toward ideas of emergence, expansion and non-directionality more aptly illustrated by the instance of an erupting volcano.

> This […] prompts a redescription of lessons plans as 'thought experiments' rather than 'itineraries' or 'trajectories'—as exercises in anticipation, not prespecification. So framed, a lesson plan is distinct from a lesson structure, the latter of which can only be realized in the event of teaching. (Davis, 2004, p. 182)

> Oriented by complexivist and ecological discourses, teaching and learning seem to be more about expanding the space of the possible, about creating the conditions for the emergent of the as-yet unimagined rather than about perpetuating entrenched habits of interpretation. Teaching and learning are not about convergence onto a pre-existent truth, but about divergence—about broadening what is knowable, doable, and beable. The emphasis is not on what *is*, but on what might be brought forth. Learning thus comes to be understood as a recursively elaborative process of opening up new spaces of possibility by exploring current spaces. (Davis, 2004, p. 184; emphasis in the original)

Davis's (2004) thesis of expanding the space of the possible moves us away from ideas of conformity and convergence onto a specific state to be or a specific way to teach. Even if the ideas of striving toward a generative model of teaching— be it, inquiry based teaching, discovery learning, problem-solving, etc.—can be legitimized, the research results highlight that it seems utopian to think that we can 'control' the outcomes of our mathematics teacher education programs, that is, that conformity can occur.

Conclusion

These research results prompted me to question our understanding of the possible outcomes of a mathematics teacher education program in regard to its objectives. The diversity of interpretations inherent in the results enabled me to move away from ideas of conformity or intention of a cause-effect outcome on the future teachers—the project of achieving the "perfect image" of a mathematics teacher that is critiqued by Breen (1999).

This obvious evidence of un-controllability prompted me to theorize differently about what an *objective* might be or represent. The idea of *opening up the space of the possible* (versus closing or funnelling the possible outcomes) is seen as a fruitful frame that could inform our understanding of how a mathematics teacher education program can be structured. It calls for an openness to new possibilities and a re-creation of them, instead of an un-reflected reproduction of what is already known.

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Developing Trusting Relations in the In-Service Education of Elementary Mathematics Teachers

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Research on professional development has highlighted the importance of communities and school-based work for promoting teachers' professional growth. However, despite discussions on school cultures and learning communities in the literature, not much has been said about how to build trust in a community as it develops. Trust seems to be taken for granted in professional development projects. This paper presents issues relating to trust in project SIPS (Support and Ideas for Planning and Sharing in Mathematics Education), a school-based professional development initiative aimed at helping teachers improve the quality of their mathematics instruction by building a mathematics education community within their school. The paper focuses on data from the first year of SIPS and discusses factors that helped build teachers' trust in the mathematics educators.

By the end of the 1990s, many reviews of the literature on professional development attempted to summarize the growth the field experienced in that decade, pointing to new directions. In particular, Wilson and Berne (1999) reviewed research on successful professional development to verify whether certain truisms of the field were supported by research. They concluded that all of the successful projects they analyzed "involved communities of learners that are redefining teaching practice" (p. 194). These projects also privileged "teachers' interactions with one another" (p.195). In an effort to define new research-based essentials for professional development, Hawley and Valli (1999) reviewed research in cognitive psychology, school reform, and professional development. They searched for research implications for new professional development initiatives. They proposed a model for professional development in which an important component was having initiatives that are school based and integral to school operations. As these reviews indicate, research on professional development in the last decade has highlighted the importance of communities and school-based work for promoting teacher change and professional growth.

In their analysis of successful professional development projects, Wilson and Berne (1999) noted that each project struggled with how to build community and how to build trust among participants in professional development initiatives. However, despite discussions on school cultures and learning communities in the professional development research literature, not much is said about trust and how to build trust in a community as it develops. Trust seems to be taken for granted in many reports about professional development projects.

This paper presents issues relating to trust in a professional development project that underscored the importance of working with school communities as the unit of change in mathematics education reform. Project SIPS (Support and Ideas for Planning and Sharing in Mathematics Education) was a three-year, school-based professional development initiative to help teachers improve the quality of their mathematics instruction by building a mathematics education community within their school. SIPS begun in 2000. In its first year, one of the main goals of the project was to begin building what we called a mathematics education community at the school. In this community, we wanted to support teachers in reflecting about their mathematics instruction and provide them with ways to think and talk about student learning. Our goal was to build a supportive community where teachers were more willing to modify their mathematics instruction based on their students' mathematical needs. As we pursued that goal, we began to realize that developing trust among participants—in particular, developing trust among school-based and university-based educators—was crucial for the success of SIPS.

This paper uses on data from the first year of SIPS and responds to the research question: What factors contributed to the development of trust among university-based and school-based educators in the process of building a mathematics education community in an elementary school? More specifically, the paper addresses one aspect of trust, namely, building teachers' trust in the mathematics educators. This research focus emerged from the aim to build a mathematics education community and was not explicitly articulated at the onset of the SIPS professional development project. We interviewed, observed, and listened to the SIPS project teachers who were collaborating to build a mathematics education learning community in their school. Teachers' reflections helped us identify trust as an important aspect of the professional development process, and led us to refine the research question that guided our analysis of factors contributing to the development of trust.

Teacher Learning in a Mathematics Education Community

From the beginning of SIPS, a fundamental assumption of the mathematics educators was that teachers are constantly trying to implement what they see as the best possible teaching. A second assumption was that in order to offer *all* children a mathematics education that is aligned with current societal demands, teachers need to experience new ideas for teaching mathematics. Teachers need to deepen their content knowledge of the subject, examine their attitude towards mathematics, and expand their pedagogical resources. Third, mathematics educators assumed that teachers do not typically modify their practices by themselves—they need professional collaboration and a supportive community in order to learn and make changes. Thus, the SIPS mathematics education community was conceived as a dynamic and interactive environment to support teachers' learning.

Summarizing research on learning, Bransford, Brown, and Cocking (2000) propose four integrated perspectives on learning environments that are aligned with current knowledge about how people learn: the learner-centered, the knowledge-centered, the assessment-centered, and the community-centered perspectives. The authors use the term learner-centered to refer to "environments that pay careful attention to the knowledge, skills, attitudes, and beliefs that learners bring to the educational setting" (p. 133). The knowledge-centered perspective draws attention to the "well-organized bodies of knowledge that support planning and strategic thinking" (p. 136), which learners need to acquire to better function in society. Becoming knowledgeable implies that learners will go beyond what they bring with them to the learning environment. Assessment is also an important perspective in effective learning environments because learners need ample opportunities for feedback and revision as their knowledge develops. Finally, with the community-centered perspective, Bransford and colleagues highlight the importance of "norms for people learning from one another and continually attempting to improve" (p. 144).

Bransford, Brown, and Cocking (2000) stated that what is known about learning applies to teachers as well as their students. Therefore, learning environments for teachers should consider these four perspectives. In relation to working with teachers in a community-centered environment, the authors explain that "an important approach to enhancing teacher learning is to develop communities of practice, an approach that involves collaborative peer relationships and teachers' participation in educational research and practice" (p.197). The two major themes that emerged from studies that examined teacher collaboration were "the importance of shared experiences and discourse around texts and data about student learning and a necessity for shared decisions" (p. 199).

The value of teachers working within communities, sharing experiences and decisions, has been an on-going theme in the professional development literature of the 1990s. Teachers who work together as colleagues find themselves better prepared to teach (Little, 1990) and in schools where faculty comes together in a professional community, teachers strengthen their pedagogical preparation (Louis, Kruse & Marks, 1996). Thus, similarly to what was indicated by research on learning, research on professional development points to the idea that when teachers have the opportunity to work with each other in a professional community, learning increases.

In mathematics education, professional development projects are beginning to capitalize on the importance of communities for teacher learning and growth. For example, Stein and Brown (1997) proposed that professional developers should consider mathematics teachers' development processes from a sociocultural view of learning. Their work was based on the analysis of a project that focused improvement efforts in mathematics on the school mathematics programs instead of on individual teachers. In the project, the entire school mathematics faculty, administrators, and mathematics educators worked together. Teacher learning relied heavily on the collaboration among teachers and between teachers and other partners. In a similar way, Franke and Kazemi (2001) also took a sociocultural perspective to design a professional development program in which teachers had the opportunity to come together to "create a community in which they could learn together about the teaching and learning of mathematics:" (p. 56). Their goal was for teachers to share and challenge each other, increasing their understanding about the development of children's mathematics.

Considering the importance of communities for teacher learning, project SIPS was designed to foster collaboration within a school-based mathematics education community. From the beginning, mathematics educators worked to facilitate sharing and exchange of knowledge within the SIPS community. In this attempt, all four perspectives of effective learning environments (Bransford, Brown & Cocking, 2000) were considered. Mathematics educators took teachers' current knowledge into consideration when planning activities for the group (learner-centered); SIPS meetings had a strong focus on discussion of mathematics and children's mathematical knowledge (knowledge-centered); and teachers received ongoing feedback about their ideas concerning mathematics and mathematics teaching (assessment-centered). However, the learning environment aspect of SIPS that received most attention was the community-centered aspect. As the description of the activities conducted within SIPS will show, teachers had many opportunities within the project to exchange ideas and share their teaching practices with colleagues—within and across grade levels.

Building a community is neither a simple nor a short-term process. In particular, building trust among community members is a complex issue to tackle. Webster's Ninth New Collegiate Dictionary defines trust as "assured reliance on the character, ability, strength, or truth of someone or something; one in which confidence is placed." Tschannen-Moran and Hoy (1998) pointed out that the concept of vulnerability is common across various definitions of trust, and that "where there is no vulnerability, there is no need for trust" (p. 337). Working on building trust means that potential for vulnerability exists. In professional development settings, teachers who are learning and changing their practices always have the potential to be in a vulnerable position; they are vulnerable to their peers' opinions and their administrators' expectations.

Tschannen-Moran (2001) indicated that there is a link between level of collaboration in a school and level of trust. Teachers' collaboration with the principal and trust in the principal, their collaboration with colleagues and trust in colleagues, and their collaboration with parents and trust in parents are connected. Brewster and Railsback (2003) also indicated that while trust alone does not guarantee that a school is successful, schools with little or no trust have almost no chance of being successful. In the context of professional development initiatives that involve a partnership between university-based and school based-educators, these ideas imply that teachers collaboration with university-based educators is probably linked to teachers' trust in the educators, and if teachers do not trust the educators, the professional development project has little chance of succeeding.

Thus, the notions of reliance and vulnerability are particularly important when university-based and school-based educators participate together in a professional learning community. For example, when describing the development of a partnership between university and schools, Jones, Yonezawa, Ballesteros, and Mehan (2002) stated: "the first *two years* in the formation of our collaborative approach to partnerships primarily involved establishing trusting relationships with our colleagues in partnership schools" (p. 6, emphasis added). The authors continued to explain that the university educators had to "convince" local educators of their commitment. Jones and colleagues concluded that "establishing trusting and supportive relationships with schools is vital for the success of any school-university partnership" (p. 7).

In our project, teachers indicated that for the SIPS mathematics education community to become a learning environment, trust needed to be established. We found that investigating the notion of trust in this particular context helped us better understand how the community operated and evolved. Therefore, in this paper we explore factors that supported the establishment of the community and the development of trust within project SIPS.

Background Information

At Adams Elementary School (pseudonym), 90% of the children qualify for free or reduced lunch. In its school district, Adams has the highest percentage of Hispanic children (39% in 2003), although the school population is mostly African American (51% in 2003). In an initial SIPS background survey we found that Adams' teachers rarely participated in mathematics-related professional development initiatives. Of the twenty-two teachers who returned the background survey, twenty (91%) said they had not completed any in-service program or a graduate course in the last five years in which recent research on children's learning of mathematics was discussed. Instead, most of their professional development experiences were in the areas of reading and technology. Therefore, with teachers' input and recommendations, SIPS was designed to provide teachers professional development activities to increase their mathematical content and pedagogical knowledge while building a mathematics education community among the school staff and the mathematics educators.

Activities during the First Year of SIPS

During the first year of SIPS, teachers participated in a variety of professional development activities, the most important being the SIPS worksessions and the mathematics faculty meetings. SIPS worksessions took place at the school during school hours. Teachers worked with the mathematics educators within grade-level groups. Each group met for a half-day activity every other month and substitute teachers were hired to allow for teacher participation. Each half-day worksession addressed research on children's learning of those mathematics topics selected by teachers as critical to the grade-level. For example, one 2nd grade worksession focused on place value and subtraction. During the worksessions, teachers were introduced to activities and ideas for teaching mathematics, explored their knowledge of and teaching strategies for the mathematical topic in focus, and planned lessons to implement in their classrooms. The after-school mathematics faculty meetings were attended by the whole school staff and, whenever possible, by school administrators. These meetings were devoted to building and maintaining a mathematics education community within the school. During these meetings, teachers had the opportunity to share what they were doing in their mathematics classrooms with their colleagues.

SIPS Research

As a research project, SIPS shares the overall goal of understanding "the complex world of lived experience from the point of view of those who lived it" (Schwandt, 1994, p. 118). Understanding this world means interpreting it, and as inquirers, SIPS researchers have attempted to "elucidate the process of meaning construction and clarify what and how meanings are embodied in the language and actions of social actors" (p.118). Thus, in its research component, SIPS is interested in unveiling teachers' perceptions about the development of trust within the mathematics education community.

A plethora of data was collected during the first year of SIPS, including videotapes of all monthly faculty meetings, teachers' written reflections after worksessions and faculty meetings, and mathematics educators' field notes. This paper focuses on interview data collected at the end of the first year. These interviews, conducted by an external evaluator, were designed to allow teachers to freely voice their opinions and make suggestions for changes in the project. They were conducted in focus groups of three or four teachers, organized mainly by grade level (seven groups for prekindergarten to grade 5). The semi-structured interviews lasted approximately 45 minutes and were all transcribed.

Through content analysis of the interview transcripts, we searched for patterns in the teachers' discussion of SIPS and for recurring words and themes that expressed teachers' appreciation of and engagement with the project. We looked for instances in which the teachers talked about "relying" on the mathematics educators' character, knowledge, and actions. We also looked for instances in which the teachers talked about developing confidence and engagement with the project, its leaders, the activities carried out, and the community they were developing. We were interested in finding aspects of the project that helped teachers feel valued, less vulnerable and more willing to collaborate with mathematics educators in activities carried out within the community.

As we examined the interviews, we coded all instances in which teachers indicated trust in the project. We also examined the transcripts in search of indicators that might be interpreted as examples of teachers' lack of confidence in the leadership, goals, or activities associated with the SIPS project. After completing this initial coding, we looked within interviews and across the seven interviews to bring up issues that were important to teachers, trying to represent an overall view of all the teachers. These issues were clustered into three main categories that the teachers talked about: one that referred to the mathematics educators themselves and their participation in the SIPS community; one that referred to the organization of SIPS and what the project was offering teachers; one that discussed the ways in which the school and the university were working together in the context of SIPS.

Through this analysis process, we found teachers talking about their reliance on mathematics educators' character, ability, and knowledge. They identified factors that, from the teachers' perspectives, helped them develop confidence in the project, in the implemented activities, and in the community they were forming. We found that the notion of trust effectively captured the ideas and perspectives being raised by the teachers.

Factors that Helped Build Trust

Three main aspects of SIPS emerged from the teacher interviews as factors that helped build trust during the development of a mathematics education community involving both elementary school teachers and university-based mathematics educators. The factors were the mathematics educators, the organization of the project, and the school-university relation, with specific characteristics of these factors being highlighted by the teachers. Teachers repeatedly mentioned specific characteristics that were important to them and to their participation in the SIPS community.

Characteristics of the Mathematics Educators

In evaluating SIPS, all groups interviewed spoke of the professional conduct of the mathematics educators and commented on aspects of what the mathematics educators did and said that helped teachers feel valued and comfortable within the project. The availability of the mathematics educators as well as their attitudes toward the teachers and the school were highly appreciated by the teachers. As one teacher explained, "they did not come in and say, 'We are going to help you with these.' They came in and said, 'What do you need help with?' And I mean, that made a difference." In particular, teachers repeatedly mentioned particular characteristics of the mathematics educators as professionals: flexibility, how they valued teachers' knowledge and experience, and their knowledge about classroom and school realities. These three professional characteristics of the mathematics educators were important for teachers to feel valued and comfortable about exploring their mathematics practice.

Characteristics of the Project

In addition to characteristics of the mathematics educators, project characteristics related to SIPS organization were highly valued by the teachers. Two factors —providing teachers with time and resources as well as giving teachers practical ideas to take to their classrooms— helped teachers appreciate the project, For example, we used various trade books, journal articles, and teacher guidebooks to give teachers new ideas for their teaching while at the same time engaging them in discussions about the value of using the resources to help students think about mathematical ideas and concepts. These resources and discussions further developed the trust teachers conferred to project SIPS.

Characteristics of the School-University Relation

A few activities that were not initially planned by SIPS became very important in the project. They were the 100th day of school celebration, the school math night, collaborating with pre-service elementary teachers (undergraduate students at the university), and the on-site work of the graduate research assistant during the second semester of the first year of SIPS. These activities were highly valued by the teachers because of the way they integrated school and university lives. For example, SIPS meeting times were used to plan activities for the 100th day of school and resources from the university (such as children's literature books and manipulatives) were made available for teachers to use during that day. All these events were mentioned several times in the various group interviews. Thus, integration was an important characteristic of the school-university relation. Bringing together two different spheres of action (school and university) was important for SIPS teachers.

Working on Building Trust

The characteristics of the mathematics educators (flexibility, valuing teachers' knowledge and experience, and knowledge about classroom and school realities), of the project (provided teachers with time and resources and gave teachers practical ideas to take to their classrooms) and of the school-university relation (integration) allowed teachers to feel respected and to appreciate SIPS. Teachers felt their knowledge and realities were taken into account during SIPS activities, and their needs were fulfilled by the project. Teachers also saw the project grow as it developed, including other ideas that came from the integration of school-based and university-based activities. Within this scenario, teachers felt less vulnerable and university-based and school-based educators developed trusting relations.

Trust and Professional Development

In searching for an understanding of how Project SIPS developed trusting relations between university-based and school-based educators, it is useful to consider Noddings' (2001) discussion of caring relations in education. From this perspective, caring involves a carer and a cared-for in a relation through which both grow. The carers are attentive and receptive to the needs of cared-fors. Carers experience motivational displacement when their "energies flow toward the projects of the other" (p.100). Cared-fors complete the relations by recognizing the care and responding with growth. Both the assessment of the needs of others and the values of the carers play a role in establishing goals for caring relations, as these relations should strive for an on-going drive towards competence. Although SIPS researchers were not originally considering Noddings' care theory as they developed and worked on SIPS, the characteristics raised by the teachers as important in the trust-building process of the project can be all placed within the notion that educators participating in SIPS enacted caring relations.

Noddings uses caring relations to talk about teachers and students. We think care theory aptly applies to relations between teacher educators and teachers; a relation in which the teacher educators take the initial role of carers, respecting and valuing teachers, as well as pushing teachers towards competence in teaching. In SIPS, teachers reported that they felt they were involved in caring relations, which allowed the development of trust in the community.

It is important to note that, for Noddings (2001), the caring teacher (educator) is not one who possesses certain stable, desirable traits but rather one who can establish relations of care in a wide variety of situations. Thus, the characteristics of the mathematics educators, the project, and the school-university relation raised in this paper are more important as instances of care than as "fixed qualities" for all professional development initiatives to embrace. Above all, trust developed in SIPS because mathematics educators and teachers worked to initiate and maintain caring relations. The characteristics of the professional development initiative raised in this paper illustrate one instance in which caring can be manifested in professional development that aims at building a trusting mathematics education community.

As we continue to investigate school-based programs of professional development in mathematics education, there is a need to critically examine the ways in which caring and trust are built among participants as a foundation for community building. Since community-building has been identified by Wilson and Berne (1999) and others as an important element in successful professional development, it is clear that we need a more carefully nuanced understanding of the ways communities are built and maintained. The research within the SIPS mathematics professional development project points to several important aspects to be further investigated. Noddings' care theory, which helped us make sense of the development of trusting relations in the context of the SIPS mathematics professional development project, could help conceptualize and inform other professional development projects that set out with a goal of creating learning communities in schools. Research questions to be further investigated include: How do educators in different school contexts (secondary and elementary schools, urban and rural settings, for example) enact trusting and caring relations in the context of building professional learning communities? What are the threats or barriers to and the supports for the development of trusting and caring relations in different professional development settings? What power relations exist between university-based and school-based educators and how might these relations support or hinder the development of a learning community? And finally, how are trusting and caring relations in professional development sustained and how do they contribute to changes in teachers' practices and increases in students' learning of mathematics?

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Standards for Excellence, Sustainable Assessment and the Development of Teacher Identity

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The Australian Association of Mathematics Teachers has recently developed *Standards for Excellence in Teaching Mathematics in Australian Schools*. These *Standards* outline what teachers believe are the characteristics of highly accomplished teachers of mathematics, and provide both a framework against which teachers can be assessed and for teachers' on-going professional learning. Boud (2000) argues that assessment should be sustainable in that it equips students with the skills and attitudes that will enable them to meet and monitor their own future learning. This paper describes how the *Standards for Excellence* were used to develop an assessment methodology in the context of teacher education that has the potential to develop a powerful and robust sense of teacher identity for exit students.

Theoretical Framework

Standards for Excellence

The Australian Association of Mathematics Teachers (AAMT) *Standards for Excellence in Teaching Mathematics in Australian Schools* (AAMT, 2002) were developed over a period of three years as a Strategic Partnerships with Industry Research Grant, in which Monash University was the research partner and AAMT was the industry partner. The methodology involved extensive consultation with teacher focus groups, with input and advice from the broader mathematics and mathematics education community. The development of the *Standards for Excellence* is set within a national and international context in which professional standards have become an increasingly important element in describing and promoting excellent teaching (Commonwealth of Australia, 2003; Ingvarson, 1995; NCTM, 1991; Ramsey, 2000).

The AAMT *Standards for Excellence* outlines three domains in which excellence in teaching mathematics is evident: professional knowledge, professional practice and professional attributes. Professional knowledge includes knowledge of mathematics, of students and of how children learn mathematics. Professional practice includes creating an effective learning environment, planning for learning, teaching in action, and incorporating appropriate assessment in teaching. Professional attributes include personal attributes such as enthusiasm for teaching mathematics, a commitment to personal professional development and adopting community responsibilities such as promoting mathematics.

The *Standards for Excellence* are intended to serve at least two major purposes: enabling a transparent and defensible method of accrediting teachers of mathematics as highly accomplished teachers, and providing a framework for effective professional development. Popkewitz (1987, p. 23) questions the use of standards, claiming that standardization can serve as a ritual of differentiation and homogeneity, thus reducing the potential for diversity and the development of individual identity. However, rather than providing a recipe for what constitutes excellent teaching, the AAMT *Standards for Excellence* encourage diversity rather than advantaging a particular style of teaching.

Statements such as "Excellent teachers of mathematics plan for coherently organised learning experiences that have the flexibility to allow for spontaneous, self-directed learning" force teachers to wrestle with ways to enact this in their own classroom, and hence to develop their own identity. Thus the *Standards for Excellence* provide a description of one high-level step along a teacher's professional journey, and a vision of teacher identity at this point.

Sustainable Assessment

Boud (2000) describes the existing paradigms of formative and summative assessment, arguing that summative assessment does not equip students well for the processes of effective learning in a learning society, and that we need to develop a new focus on formative assessment. He argues for a new paradigm termed "sustainable assessment", that has the potential to equip students as life-long learners. As the AAMT *Standards for Excellence* makes clear, life-long learning is a key attribute of highly accomplished teachers of mathematics. However this life-long learning is not seen as a uni-directional process; rather it is seen as a process of questioning, reflecting and criticising.

Boud argues that assessment always does "double duty", in that it both judges achievement and transmits what we value; that it is assessment both for learning and for certification; that it has a focus on the immediate but that it also equips for life-long learning; and that it attends to both content and process domains. He suggests that sustainable assessment attends to these dichotomies, and that it enables students to evaluate their on-going learning and development without being dependent on formal, external feedback mechanisms. He sees sustainable assessment as an integral part of this life-long learning.

Teacher Identity

Teaching is a complex profession. As recognised by teachers themselves in developing the AAMT *Standards for Excellence*, excellent teaching is dependent upon knowledge, action and beliefs. These three aspects of teaching excellence do not exist in isolation; each influences and depends upon the others, and they are intricately woven to form the complex fabric of teaching. It is the teacher's motivations for, and feelings about, the complexity of teaching that I call teacher identity.

Twenty years ago Shulman (1986) discussed the distinctive kinds of knowledges necessary to be a teacher, identifying pedagogical content knowledge as a key aspect of excellent teaching. More recently he has articulated a taxonomy of learning, culminating in commitment and identity, which are realised as values. He argues that an educated person's "commitments always leave open a window for sceptical scrutiny, for imagining how it might be otherwise" (Shulman, 2002).

Mayer (1999) distinguishes between role and identity in self-formation as a teacher, suggesting that core beliefs constitute one's teaching identity. Students' reflective journals indicated that teaching personalities were privileged over pedagogical and subject knowledge, and that pre-service teachers often felt that what they were learning in their University studies, and what they were asked to do in schools during the practicum, were contradictory to their personal feelings about what it meant to be a teacher. Drake, Spillane, and Hufferd-Ackles (2001) describe teachers' identity as their sense of self as well as their knowledge, beliefs and orientations to work. They describe the many influences on primary teachers' sense of identity, in particular some of their feelings of failure as students in school mathematics and their struggle to make sense of and incorporate new ways of teaching.

Building on Lave and Wenger's (1991) influential study of five apprenticeship learning situations, Adler (1998) emphasises that knowledge about teaching is tied to the context of teaching, that it is dynamic and that it is "simultaneously personal and social". She suggests that this knowledge is not acquired in the academic study of teaching, but that it evolves through "legitimate peripheral participation in a community of practice" (Lave & Wenger 1991), of which pre-service education is one ingredient. For Adler this knowledge is tied to pre-service teachers' identities, and is built through discourse and through making the hidden assumptions of teaching transparent. Mayer (1999) also stressed the need for pre-service teachers' personal theories to be made explicit, deconstructed, and problematised through reflection and discourse.

Thus it would appear to be essential to construct learning and assessment opportunities in pre-service teacher education that promote the formation of habits of mind that enable pre-service teachers to link theory and practice (Ebby, 2000), through reflecting on their own teaching in a framework that makes explicit not "how to be" an excellent teacher of mathematics, but "what it is to be" an excellent teacher of mathematics. In Boud's (2000) terms, assessment in pre-service teacher education must be sustainable.

The Portfolio and Interview

Journal writing (Artzt, 1999; Brown, 2001), case studies (Hammermas, Darling-Hammond, & Shulman, 2001), professional conversations (Britt, Irwin, & Ritchie, 2001, Thornton & Blain, 2002), and the preparation and presentation of structured portfolios (Frid & Sparrow, 2003) are all recognised as valuable tools to promote pre-service teachers' capacity to be reflective practitioners. While this study did not specifically use these tools, it incorporated similar ideas by asking pre-service teachers to write and talk about their experiences in a school, to link these to relevant readings and to see themselves as active researchers of their own teaching in the context of the *Standards for Excellence* described above.

I teach a course Secondary Teaching Studies (Mathematics) to students at the University of Canberra, Australia. This one-semester course forms part of either a 1-year Graduate Diploma in Education or of the final year of a 4-year Bachelor of Education degree. Students enrolled in this course hope to teach mathematics to secondary students, aged 11 to 18, in the following year. For most of these students this course of 36 hours is the only one in which they look specifically at how students learn mathematics, at mathematics curriculum, and at different approaches to teaching mathematics. All students also undertake a 4-week period of Professional Experience, during which time they work full-time in a school under the guidance of an experienced teacher of mathematics.

Assessment for this course typically involves three assignments: an exercise in micro-teaching, the development of a set of detailed lesson plans, and the accumulation and presentation of a portfolio of activities, resources, lesson plans and reflections during the semester and particularly during the period of Professional Experience. While these assessment tasks have immediate and obvious practical value, it is debatable to what extent they meet the criteria of sustainable assessment, or to what extent they promote the development of teacher identity, as described above. Yet for these students, this is their only pre-service experience in mathematics education, hence it is critical that they are well positioned to become life-long learners of the art and craft of teaching mathematics.

It is noteworthy that many, but not all, of the students involved in this class were mature-aged students, who already had varied life experiences and a strong sense of personal identity. Two of the students had left extremely well-paid careers to become teachers, others had experience as parents and community leaders. These students had a strong sense of why they wanted to become teachers and what they hoped to achieve. In general they "wanted to make a difference". They were also very aware of their own experiences as students in mathematics classes, and while they had been successful, they felt that their school experiences had not engaged them, and had not promoted the development of deep mathematical understanding. In the words of one student: "I don't think I will make a very good maths teacher, because I have just begun to realise that I don't really understand anything I learned at school—I was just good at it."

As the lecturer of the course I had been concerned for some time that the portfolio presented by students tended to be little more than an unfocused collection of resources, journal articles and lesson plans with only brief annotations, and no apparent coherence. While it told me something about the pre-service teachers' capacity to collect resources, it told me little about their capacity to thoughtfully weave these resources into the complex web of teaching mathematics, nor to make sense of their teaching experiences in the light of what they had read and discussed in their academic studies.

In an attempt to make the portfolio assessment more focused, I decided to reframe it in line with the AAMT *Standards for Excellence* described above, and to add a 20-minute individual interview, during which time pre-service teachers were asked to explain their rationale for including parts of the portfolio, and to evaluate their knowledge of, practice of, and beliefs about, teaching. Each pre-service teacher was asked to answer three questions, segments of which were:

1. The AAMT *Standards for Excellence in the Teaching of Mathematics in Australian Schools* list three aspects of being an excellent teacher: professional knowledge, professional practice and professional attributes. From your own Professional Experience describe a situation where one of these aspects was evident. Use your portfolio to provide concrete evidence to support your answer.
2. With reference to the readings discussed during the semester, describe the characteristics of a classroom in which high levels of engagement with mathematical ideas are likely to be present. Refer to a class that you taught during Professional Experience and describe how you attempted to create and/or sustain such an environment. Use your portfolio as evidence.
3. Discuss one of the quotations below. Refer to readings during the semester, a class you taught during Professional Experience, and your portfolio to support your answer.

   "Of course setting is advantageous for instruction. It's just not advantageous to the students in the lower classes." Eileen Kott, teacher in Florida. In *Mathematics Education Dialogues*. November 1998 (p. 12) (A sample of one of the quotations)

Each student was then asked to bring their portfolio to an interview, to answer the three questions above, and to refer to their portfolio as evidence. Two mathematics educators interviewed the students, made notes during the interview, referred to the portfolio for any further clarification, and provided feedback within 30 minutes of the completion of the interview. Students were informed that the interview process was an experiment, and that it was being used as an attempt to make the portfolio more focused. Each student also agreed to have the interview taped for future reference.

The Interviews

As might be expected in any assessment task, there was a wide range of student responses and levels of performance. A few students were unprepared, had done little reading, and did not focus their answers or portfolio. At their best, however, the interviews were remarkable. They showed a capacity to be reflective of their own teaching, to be critical and constructive and to ask informed questions of the status quo. They provided a vivid and tangible image of pre-service teachers developing a very strong sense of teacher identity.

John focused on professional knowledge in his discussion of the *Standards for Excellence*. He drew parallels between a constructivist approach to teaching and his background in human communication theory. He noted that a key principle of communication was that "the receiver makes the message", and concluded that it was thus the teacher's role to know his students, their culture and their idiom well enough to enable each student to make the message in a productive way:

> By the third week (of Professional Experience) I was much better able to recognise the diverse requirements of the students in the class. The girl who did not listen felt she understood most of the topic and was bored. The boy at the front was being continually distracted by his girlfriend who sat next to him. The girl at the back had developed lots of go-slow tactics to hide the fact that she did not understand most of the topic. The boy in the middle needed more challenging problems to keep his interest (…). I began to make progress with most of these students but I have a lot to learn before I can manage appropriate learning opportunities for most people in the class most of the time. (John in interview)

John saw learning as problematic and dependent on a range of factors beyond transmission of information. He was able to incorporate what he had observed in practice with what he had read and discussed in his academic studies, and to incorporate his prior knowledge and experience. His sense of teacher identity would thus include a strong appreciation of diversity.

Malcolm reflected upon a singing observation sheet he had seen used in an early childhood setting. The teacher observed how each child sang, using prompts such as whether the child was opening her mouth, or moving her lips. Malcolm put a "productive mathematics behaviours" (Corkill, 1999) checklist on his list of things to do, so that he would be able to more effectively monitor changes in students' behaviours. This was at least partly in response to his observations that many students came to class unprepared both in terms of having the appropriate physical resources for learning and a productive frame of mind for learning. Like John, Malcolm saw knowledge of students as critical for effective learning, and recognised that he would need to take practical steps to continually develop that knowledge.

Both Malcolm and John articulated aspects of professional knowledge described in the *Standards for Excellence*, recognised their own limitations in relation to that knowledge and saw the need to engage in life-long learning of mathematics and the teaching of mathematics.

In her response to the second question, Linda chose to focus on the characteristics of classrooms with high levels of engagement. She described how teachers at the school at which she was teaching told her to "never have discussions, and always give short, sharp comments”. She felt that such advice was contrary to a classroom environment in which high levels of student engagement would be evident. She noted that her attempts to be creative and to engage students in solving problems “did not really pay off” in the school where she was teaching.

Linda was particularly interested in looking at mathematics learning in context. She described a journal article (Nicol, 2002) she had read in which pre-service teachers had visited workplaces, but often been unable to recognise the mathematics being used. She felt that this lack of capacity to see and appreciate mathematics in a workplace context militated against creating an engaging and relevant environment for students. She described one class in which students who had a history of failure in mathematics were given “real-life maths, not that stuff you get in other classes.” Yet the real-life maths was restricted to questions such as “How many days are there in May?”, or “If I spent $1.50 from a $10 note, how much change would I get?”. Linda was wrestling with the very complex issue of what relevance really means in a mathematics classroom, and recognised her own lack of knowledge of mathematics beyond the school classroom. Linda’s response showed that she had engaged with the dimension “Planning for Learning” described in the *Standards for Excellence* and had grappled with what it meant to provide students with opportunities to apply mathematics “beyond the school setting”.

In thinking about an important issue in mathematics education (Question 3), Melissa reflected on her experiences with, and reading about, setting students based on their perceived ability levels in mathematics. She discussed the pros and cons, noting that setting students into ability groups made life easier for the teacher, but asked whether the students were really being provided with differentiated learning opportunities, or whether they were just being given more (or less) of the same at a faster or slower pace.

Melissa described how, in teaching fractions to a year 7 class, her supervising teacher had asked her to split the class into three groups based on results in a pre-test. On reflection she felt that, while they had worked diligently through the work assigned, the most advanced students had not been challenged in any significant way, and that, in general, the lowest achieving students remained the lowest achievers. However one student who had been placed in the lowest achieving group was able to complete the post-test with only one error. This was exciting for both the student and his teacher, who had not expected such a result.

Melissa commented on the immense volume of literature on ability grouping, and asked why the practice continued to be widespread when there was significant evidence of negative social impact and limited academic impact. She expressed her disappointment that teachers at the school where she was teaching used the expression “Zoo” class to describe the lowest achieving group, saying that the grouping practice tended to concentrate students with behavioural problems into the one group. However she also recognised that, for one student in her year 7 class, being given work at a level with which he felt comfortable had completely changed his attitude towards mathematics, and she wondered if such a change would have taken place had the students been taught as a whole class. Melissa concluded by saying “I haven’t got an answer, I’m still sitting on the fence”. Popkewitz (1987, p. 4) questions how methods courses in teacher education “legitimate or make problematic the content and form of schooling”. The interview process allowed Melissa to acknowledge and value her uncertainty about the most effective way of structuring mathematics classes.

As noted by my co-interviewer these, and most of the other students in the group, had thought deeply about their teaching, about what they had read and talked about in their academic studies, and about how it related to their practical experience. They did not provide glib answers, but saw knowledge of teaching as developing through reflection over a long period of time. Melissa noted that "by putting it all together (for the interview) it's touched on layers of other issues". Thus the interview process forced the students to be focused and specific in relation to their own development as teachers, to the articulation of their beliefs and practice, and to their current state of professionalism within the framework of the AAMT *Standards for Excellence*. In this way it fulfilled the criteria of sustainable assessment, enacted the *Standards for Excellence* in a powerful way, and contributed to the development of a strong sense of teacher identity.

Student and Teacher Reflections

My co-interviewer commented upon the maturity of the students, and on how articulate they had been. She was impressed by their "willingness to expose and consider their weaknesses in an interview." She noted that this task had assessed higher order thinking skills such as critical reflection, and had put into practice much of the rhetoric of the teacher education course. She felt that the interview process and portfolio preparation had modelled professional excellence as described in the AAMT *Standards for Excellence* in a very powerful way, by respecting the pre-service teachers' background, knowledge and experiences, and their right to reserve judgement where they had not yet arrived at a firm opinion. She felt that, in this way, the process had been unusual in its value to the students.

The interviews provided strong evidence of developing teacher identity, in particular characteristics such as scepticism, the capacity to reflect on experience to link theory and practice, and a sense of self as a learner. The pre-service teachers' core beliefs about teaching, and about themselves as teachers, were challenged. They recognised their existing professional knowledge and highlighted their shortcomings; they evaluated their own and their supervising teachers' practice honestly and critically; they revealed a developing sense of what they valued in learning. The interviews provided clear evidence of the pre-service teachers' recognition of the importance of life-long learning as a critical component of professionalism and teacher identity.

However the most surprising outcome was the sense of community generated through the process. The pre-service teachers e-mailed each other after the interview to discuss their feelings about the task. This was an entirely self-motivated undertaking. I had not asked them to share their reflections and had expected that, like every other assessment task I had ever set, students would just be glad that it was over. On learning of this email exchange, I requested a copy with names removed, and the students were happy to provide their reflections:

> When the audience is sitting in front of you, there is more chance that you can adjust your presentation if they appear bored, confused or incredulous..
>
> Because the interview is so short and the time can disappear so quickly it is very important to be organised and be clear about the main messages in your presentation (just like in a lesson).
>
> I think it is a little dangerous to try and assess people on a 15-20 minute interview, as it tends to favour those who are articulate rather than (necessarily) those who have reflected deeply. Of course every assessment will have its own bias (essays, after all, will favour those who write well), but I think the danger of assessing style rather than substance are greater in [a] short interview scenario.

> Probably the most I got out of the whole process was how analysing, reading articles and reflecting continued to challenge me about my teaching. Many of the articles I read had direct relevance to what I had been teaching and raised lots of questions, and provided some answers, in teaching these topics. While I was preparing for an assessment item, I think I got more out of the exercise than the mark Steve gave me.

These reflections stand in stark contrast to the concerns of Popkewitz (1987, p. 17) that reflection is usually tied to utilitarian issues of finding resources, managing classes and organising time. These pre-service teachers saw the exercise as an important part of their on-going development as teachers of mathematics. They saw themselves as part of a community, and were keen to share their experiences and thoughts with others. Unprompted, they thoughtfully evaluated the validity of the interview process and made links with assessment practices beyond their current course. In this sense the portfolio and interview did "double duty" by focusing on both the immediate and the future, by transmitting what is valued as well as making judgements, and by giving students the reflective skills to attend to their on-going development as excellent teachers of mathematics.

Conclusions

The AAMT *Standards for Excellence* provide a framework through which teacher identity can be developed and evaluated. While pre-service teachers cannot be expected to show highly accomplished practice, as described by the *Standards for Excellence*, the *Standards* can provide a vision of what it means to be an excellent teacher. The portfolio and interview assessment task described above enabled students to describe their own experiences in the light of the *Standards for Excellence*. In the process it would appear that this assessment task met many of the criteria for sustainable assessment described by Boud (2000). In particular, the students' unprompted reflections provided clear evidence that they were able to evaluate their on-going learning and development without being dependent on formal, external feedback mechanisms. The portfolio and interview assessment served both the immediate purpose of evaluating current knowledge and the long-term purpose of giving students a framework for their life-long journey as teachers of mathematics.

Of course this assessment task did not stand alone. It was part of a course that included extensive instruction, discussion, reading and reflection. However it appeared to pull together students' experiences in a very powerful and revealing way. The extent to which the developing sense of identity exhibited by these students grows and develops through their careers as teachers remains to be seen, and could profitably be the subject of further research. The AAMT *Standards for Excellence for Teaching Mathematics in Australian Schools* provides an ideal framework by which such a longitudinal study of teachers' identity could be conducted.

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Student Teacher-Training in Science and Mathematics: Experiences from a Combined Teacher-Training and Engineering Programme

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This paper reports on experiences from a new combined engineering and teacher-training programme, run by The Royal Institute of Technology (KTH) and The Stockholm Institute of Education (LHS) in cooperation. The paper provides a brief description of the structure of the programme, where integration of subject matter, didactics and engineering skills is a key concept. From analysis of the profile and the expectations of the student group it seems that this programme to a very large extent has attracted students that wish to work as teachers in science and mathematics, but would have gone for a traditional engineering education in the absence of this new opportunity.

In Sweden, as in many other countries, the declining interest for mathematics, science and technology among young people is a concern, particularly for the educational sector. Recruitment to teacher-training programmes and university courses in these subjects as well as to engineering education is weaker than one would wish. This is highlighted in a recent Swedish official report (SOU, 2004, p. 97). Within the next ten years large groups of today's teachers will have retired, and there is thus a need to increase recruitment to teacher-training. An investigation by the Swedish National Agency for Higher Education finds that only 14% of prospective upper secondary school teachers choose to specialize in mathematics and science, in contrast with an estimated need of approximately 33% (Högskoleverket, 2004).

In 2002 the Swedish government commissioned The Royal Institute of Technology (KTH) and The Stockholm Institute of Education (LHS) to jointly develop new ways to educate prospective teachers in mathematics, science and technology. As a result of this, a new combined engineering and teacher-training programme, the so-called CL-programme (from the Swedish name *Civilingenjör & Lärare*, Engineer & Teacher) was developed for the academic year 2002-2003. A main purpose of this programme is to recruit new groups of students to teacher-training in mathematics, science and technology.

Purpose of the Program

The programme finds its identity in the intersection of pedagogical and engineering competences:

- Mathematics is combined with one more subject: chemistry, physics or computer science.
- The programme is for five full years, leading to a double diploma, in engineering and teaching.
- The international name of the diploma is Master of Science in Engineering and of Education, Degree Programme in Mathematics and Physics (or Mathematics and Chemistry or Mathematics and Computer Science and Information Technology).
- Didactics, pedagogy and practical training are integrated in the programme from day one.

- The programme is designed as a whole, rather than as separate blocks of subject matter, pedagogy and didactics each moulded in a different department.
- Practical training involves training in upper secondary schools and also in science centres.
- A large part of the courses are developed especially for this programme; in particular this is true for the courses in mathematics and mathematics education.
- The students follow courses for traditional engineering programmes in physics and engineering physics, chemistry and chemical engineering or computer science and information technology, depending on their choice of second subject.
- The master thesis should be in the area Technology and Learning, combining science with technological and pedagogical issues. The thesis work should relate to practical experiences in some kind of a pedagogical environment, for example an upper secondary school or a science centre.
- As engineers the students should, after finishing their studies, have a solid and broad basic competence in their field with an edge in technology and learning, thus being particularly well trained for in-house education and technical customer support as well as development tasks in knowledge industry. Technology is today developing with an increasing speed, and there is a great need for engineers with abilities in teaching and learning. This conclusion is supported by a report from the Royal Swedish Academy of Engineering Sciences, where for example skills in communication are pointed out as an example of new competences needed for the engineers of tomorrow (Kungl. Ingenjörsvetenskapsakademien 2004)
- As teachers in upper secondary school their special profile should be a broad understanding of the subject matter, special skills in problem solving and good knowledge of applied science, resulting in a good ability to make the subjects come alive. They should also be a good resource in course devolvement. One important point is that they hopefully will come to play a prominent role in inspiring more students in upper secondary school to engage in academic studies in mathematics, technology and science, and to serve as link between upper secondary school and engineering studies.

It should be noted that, at least in Sweden, there has always been a flow back and forth between the engineer category and math/science-teacher category. The CL-programme wants to educate for this double identity right from the start.

A Short Description of the Programme

As mentioned above, the programme is for five years. This is slightly more than for conventional engineering programmes or teacher-training programmes, which run for four and half years.

In providing a short description of the programme I refer to the Swedish credit point system, in which 40 credits correspond to one full year of studies. The CL-programme consists of a total of 200 credits. Table 1 provided approximate portions assigned to the different subjects.

Table 1
Overview of the Curriculum in the CL-Program

Subjects	Credits
Pedagogy, psychology, sociology and general didactics	30
Mathematics (including 10 credits of subject didactics)	60
Physics and engineering physics/Chemistry and chemical engineering/Computer Science and ICT (including 10 credits of subject didactics)	60
Interdisciplinary courses	20
Optional courses	10
Thesis	20

The programme was designed to comply with the following course guidelines:

- All Swedish teacher-training programmes should contain a common core of 30 credits corresponding to the first line in Table 1.
- Teacher-training programmes should contain 30 credits of interdisciplinary courses in order to prepare the teachers for interdisciplinary cooperation. In the CL-programme there are 20 credits of courses in this category, for example *Engineering Science* that looks at the engineering profession from different perspectives, and *Communication and Media*. The thesis, as described above, is also of an interdisciplinary nature. This makes up for 10 more credits in this category.
- A total of 20 credits of subject didactics are included in the subject matter.
- At least 30 credits should be directly related to studies and practical work in school. In the CL-programme this means a number of shorter periods of 1-2 weeks of field studies and initial practical teacher-training during the first four years of the programme, and a longer period of approximately 8 weeks of practical training during the fifth year, which should contain the thesis work with further field studies and/or practical training.

Teacher-training specific courses are integrated in the programme from day one. The first year is common to all students, with mathematics, pedagogy, interdisciplinary courses, computer programming and practical training in upper secondary school and on a science centre. Years 2 – 4 follow the same pattern, with the addition of the second main subject (physics, chemistry or computer science and ICT). The last year, as described above, is mainly devoted to practical training and thesis work.

There is a definite ambition to work across the department and institution lines. For example, during the first year there is collaboration on a task in the didactics of mathematics between the mathematics department at KTH and an introductory course in pedagogy given at LHS. In another project, teachers from KTH and LHS work together with the supervising teachers, from the upper secondary schools where students do their practical training, developing assignments for practical training. Tasks in subject didactics may also be integrated into courses in general pedagogy and didactics. Furthermore, we support and encourage pedagogical enhancement of the standard engineering course compulsory to the CL-students; this could for example mean activities to enhance capabilities in oral and written communication.

Mathematics Courses

Mathematics is a common subject for all students on the program, and the courses in mathematics are developed especially for this programme. Compared to standard engineering mathematics courses on KTH, there is an additional emphasis on written and oral communication skills and on the use of technology, and there are also didactic tasks integrated in the curriculum. Compulsory courses in mathematics include:

- Mathematics 1. Pre-calculus, linear algebra and single variable calculus. (8 credits)
- Mathematics 2. Linear algebra and calculus in several variables. (8 credits)
- Differential Equations and Transforms. (4 credits)
- Programming. (4 credits)
- Numerical methods. (4 credits)
- Discrete mathematics. (5 credits)
- Probability and mathematical statistics. (4 credits)
- Mathematics for teachers. On different number systems and their properties. Concepts from real and complex analysis. Geometry. (5 credits)
- Mathematics for physics/chemistry/computer science. (4 credits)
- The history of mathematics. (5 credits)
- Didactics of mathematics. (10 credits)

In addition to this there is of course a significant amount of applied mathematics, problem solving and modelling within the standard engineering courses in the programme.

The Students and Their Background, Expectations and Motivations

Nominally there are 60 places in the programme each year. The first year attracted approximately 35 students, but the second year 67 new students were accepted. These enrolments occurred in a period of relatively low recruitment to Swedish teacher-training programmes in science. During the last three years, the CL-programme has attracted more students to upper secondary teacher training in chemistry than all other Swedish teacher training programmes taken together, when comparing the subjects chosen by new students. Please note that Swedish upper secondary teacher trainees normally take two main subjects, and that the choice of the second subject often is made once trainees have entered the programme, so these figures must be interpreted with caution. There are similar but less extreme figures for physics, while statistics for computer science is still under investigation. Mathematics teacher programmes usually do better in recruiting in most teacher training facilities.

There is approximately the same number of male and female students. Most students are young; 86% of those who started their studies in 2003 were then under 25 years of age. We have surveyed the student's background, expectations and motivations. The result can be summarized as follows.

- Most of the students would have chosen a traditional engineering education, in the absence of the CL option. In a web survey 23 of the students who entered the programme 2003 answered questions on which other education programmes they had applied for. Eighteen students had a traditional engineering education as second alternative, while only 1 of them had a teacher-training programme as a second option. Results for students who entered 2002 are similar.

- Most of the students expressed a commitment to teaching. In response to a question about the motivation for applying to the programme 19 of the 23 respondents declared that they were motivated by the combination of an engineering and teacher-education, 3 of them were motivated by the engineering aspect, and 1 by the teacher-training aspect. Once again we found similar interests in the group who entered the programme in 2002.

Although the response rate to this survey was low, it does seem to indicate that the CL-programme has succeeded in recruiting new groups of students to engage in a math/science teacher-training and that most students expect a professional life where teaching is a large component. An ongoing extensive study of all entering students' preferences confirms that the programme has to a large extent attracted students who would have chosen an engineering programme rather than a teacher training programme in the absence of the CL-option.

Recent Development, Dropouts and Future Plans

In September 2004, 57 new students entered the programme. Of the approximately 35 students who entered the programme in 2002, 24 remain in the programme two and half years later. Most of those who left did so during the first year. Among those 67 students who entered in 2003, 41 were still following the programme in 2004. In other words, we have had a dropout rate of approximately 1/3 during the first year. This is substantially more than for other engineering educations at KTH, where typically 20-25% of students change programmes or abort their studies during the first year. We have performed an analysis of those who entered 2003.

- 26 students among the 67 admitted 2003 had left the programme one year later.
- Among those 26, 8 left almost immediately, and 4 more retained their place for a possible re-entrance later on. This leaves us with 14 students who started studying on the programme, but decided to leave during the first year. Interviews with this group reveal different reasons, but the following were recurrent answers: (i) personal reasons or change of interests, (ii) programme too focused on teacher training (iii) uncertainty about the professional status of the programmes engineering identity.

Being a new programme, educating towards a new type of engineer who is an expert in communication, teaching and learning within his/her field as reflected in reasons (ii) and (iii) are perhaps not unexpected. Students tend to compare with established engineering programmes in search for their identity, and since they know that for example KTH's programme in Engineering Physics is well-established with high professional status, they may feel insecure when they realize that they will not achieve the same level of expertise in engineering physics and feel reluctant to trust that their special competence in technology and learning will have the same value on the market. This indicates that the content and the profile of the programme need to be better communicated when recruiting new students.

A major revision of the programme has been carried out during 2004, affecting the structure rather than the content. Previously the first semester contained a course in mathematics which was not well suited to some of students' background in mathematics, and a course in pedagogy, which was experienced as being too abstract and having too much focus on the development of younger children. One aim of the revision is to make the first year more accessible, and to strengthen the students' identity as prospective teachers *and* engineers. To achieve this, the first semester now includes two introductory courses for the engineering profession and teaching profession respectively. We have also strengthened the first mathematics course with 2 credits to enable a smoother transition from upper secondary math. In parallel they study didactics of mathematics with integrated practical training; many students expressed the need for early practical experiences of teaching.

We also have endeavoured to improve the integration of the school practicum within the university programme. The logistics of synchronizing schedules for two university institutions, involving a number of different programmes with the practical training has been extremely challenging. In the new structure we have block times where practical training is abundant, and during these periods students do not take courses that have to be synchronised with other programmes.

Similar Initiatives

A similar combined education programme exists at NTNU, the technical university in Trondheim, Norway. Partly inspired by the work at KTH and LHS their programme opened in the academic year 2003-2004. There is already a cooperation developing between KTH-LHS in Stockholm and NTNU in Trondheim. At Mälardalen University in Eskilstuna, Sweden, a combined engineering and teacher-training programme started in 2004, leading to an upper secondary teacher's degree in technology. At present we are not aware of any other similar programmes, neither in Sweden, nor abroad.

Conclusion

A combined engineering and teacher-training programme in Stockholm coordinated jointly by The Royal Institute of Technology (KTH) and The Stockholm Institute of Education (LHS) has been briefly described. Experiences from the first two years suggest that this programme does well in recruiting new groups of students to teacher training. It has become apparent that it is very important to communicate the goals of this new programme as clearly as possible to prospective students, in order to give appropriate expectations, and to structure the programme in such a way that students' double identity as engineers and teachers is strengthened. A positive side effect of this initiative has been the stimulating and challenging meeting between two academic worlds, the engineering culture at KTH and teacher-training culture at LHS.

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Thematic Afternoon-A: TEACHERS OF MATHEMATICS: Recruitment and Retention, Professional Development and Identity[34]

Glenda Anthony and Barbro Grevholm

Organisers of the Thematic Afternoon: Glenda Anthony (Massey University), Mellony Graven (University of the Witwatersrand), Barbro Grevholm (Agder University College) & Toshiakira Fujii (Tokyo Gakugei University).

The focus of Thematic Afternoon—mathematics teachers—reflected the renewed research interest in mathematics teachers and teaching noted by several of the plenary speakers: The Survey team, chaired by Jill Adler, reported increased research on teacher development, learning and associated curriculum reform. Sfard also noted the change of research focus from learners to teachers. The acknowledgement of the central role of teachers in students' learning of mathematics has encouraged research to consider more closely the nature of the teaching demands, the ways in which teachers manage these demands in the realities of their classrooms (Stein, 2001, Strässer et al, 2004)). With respect to mathematics reform in particular, studies include: investigating teachers as learners, a more critical examination of the pre- and in-service development provisions and the associated formation of teacher identity. Research presented in the Thematic Afternoon reflected this closer examination of the professional formation of teachers. The papers[35], offering a variety of theoretical frameworks and models, highlighted the collaborative nature of emerging research methodologies.

Recruitment, Supply and Retention of Mathematics Teachers

The issue of recruitment, supply and retention of mathematics teachers was addressed by contributions from England and Sweden. The small number of contributions offered for this strand is possibly indicative of the relative scarcity of related research. Collective concerns were the decrease in the number of students studying mathematics courses, the quality of mathematics teachers' qualifications, and teacher attrition related to work conditions and aging teacher populations. Contributors argued that all of these issues impacted on the quality of teaching within schools. An additional concern raised by Johnston-Wilder (TA) related to difficulties of engaging teachers in 'out of school' curriculum development projects. Schools, faced with difficulties finding relief teachers and fears of teachers not wanting to return to school after project involvement, were becoming increasingly reluctant to release quality teachers for curriculum development projects.

Solutions offered within the English context to address recruitment included changes in schools to address workload issues, the adoption of an entitlement of continuing professional development (Zhang, TA, 2004), and diversification of routes to qualified

[34] This report is included in the Proceedings of ICME10 and was written in 2004 by Glenda Anthony and Barbro Grevholm.

[35] Papers referenced by (TA, 2004) refer to the Thematic Afternoon presentations and are available on the ICME 10 website (http://www.icme10.dk/) programme page.

teacher status, including flexible training options. Angier (TA, 2004), reporting on the experiences of students completing a flexible Post Graduate Certificate in Education, claimed that such courses may make little impact on recruitment numbers. However, on a positive note she argued that the impact of flexible pedagogies may better equip teachers to "enjoy the complexities and challenges of teaching" and thus improve teacher retention. Using a similar strategy involving changes in initial teacher education programmes, Thunberg (TA, 2004) reported a Swedish initiative to combine engineering and teacher training culminating in a double diploma qualification involving practicum experiences in both schools and science centres.

Pre – and In – service Education of Mathematics Teachers

The need to understand and support with appropriate professional development all stages of the Teacher Professional Learning Continuum—pre-service, induction, early career, and experienced—was advanced in several contributions within this strand. Van Zoest (TA, 2004) posited that the third stage, approximately years 4-7 in a teacher's career, may well be a time of experimentation and consolidation that shapes the future teacher.

In light of current reforms the need for effective teacher education and increased knowledge about what and how teachers might learn was a central issue addressed by several papers. While some of the papers provided examples of ways in which progress is being made, others also highlighted challenges still to be addressed. Sztajn, White, Hackenberg, and Allexsaht-Snider (TA, 2004) emphasised the need to develop trust within professional development programmes: trust between the facilitators and the participants. Van Zoest warned that the quest for the ideal model of professional development needed to be clearly linked with outcomes, arguing that we need to more clearly understand and articulate the nature of transformation in teachers' knowledge, understandings, skills and commitments. In this respect, Morony (TA, 2004) considered the potential of recently developed professional teaching standards (AAMT, 2002) as a tool for professional development and Baber (TA, 2004) noted the role of professional teacher associations in developing "networks of learning".

Contributions also highlighted the various models of teacher education across the international spectrum. Peterson's (TA, 2004) study compared expectations of pre-service practicum in both Japan and US. Cultural differences at a discipline level were also highlighted within Groves' (TA, 2004) discussion of integrated curriculum studies. Initially introduced as a response to a crowded curriculum, the integrated curriculum studies course compounded growing concerns about the adequacy of time available to support mathematics education within initial teacher education. Continued reports such as Groves are needed to monitor this trend and are clearly linked to the wider issues of teacher knowledge expressed in the parallel strand.

Professional development using distance learning and associated technologies was explored by da Ponte (TA, 2004). Within the virtual community, the strong presence of collaboration and reflective writing led da Ponte to question the impact on teachers' professional identity: the fundamental roles, norms and values of the mathematics teacher.

Missing from this strand were studies that focused on the early years of teaching. Given the concerns expressed about retention of mathematics teachers there appears to be much scope for studies that examine the nature and effectiveness of support for beginning teachers.

Mathematics Teachers' Identity

Contributions in this strand interpreted the issues related to mathematics teachers' identity in many ways, and from a range of theoretical perspectives. Some papers raised issues that mirrored discussions on teacher recruitment, particularly in relation to the potential disjunction of identities and related images of the mathematics teacher:

Alignment with the mathematics community—in the sense of doing well in your degree and taking on the characteristics of a mathematical person—may well be at odds with alignment to school teaching. (Rodd et al., 2003, cited in Winbourne, TA, 2004).

Thornton (TA, 2004) provided an official version derived from teacher input of teachers' identities (AAMT, 2002). Integrating the standards document into assessment and portfolio tasks, Thornton argued that the signposts and guidelines enable student teachers to effectively map their developing teacher identity against a vision of what it means to be an excellent teacher. Likewise, Wilson (TA, 2004) provided observations on teacher excellence in relation to a sense of self in terms of motivation, commitment and feelings about teaching. However, both Proulx (TA, 2004) and Parker (TA, 2004) challenged the use of pre-designed official identities. Proulx suggested that student teachers appropriate teacher education programs in unique ways—their identities continuously unfold as new opportunities and possibilities are realised. Based on a series of interviews, Proulx provided a range of characterisations of pre-service teacher as 'Technician', 'Mimic', 'Self-assured', 'Reflective practitioner' and 'Natural teacher'. Applying Bernstein's theory Parker argued that local teacher identities emerge within specific pedagogic contexts as a 'form of consciousness' embedded in the social practices of a community. Within the context of South Africa Parker discussed the duality of identity formation experienced by novice teachers: that of a mathematics teacher and a mathematics learner. Also mindful of the multiplicity of identities, Winbourne applied Wenger's (1998) theory of participation with the notion of 'figured worlds' (Holland et al., 2001) to develop a theory of identity formation within a community of practice.

The question of how the emerging work on teacher identities might be usefully used within teacher education was a recurring focus. Reflection on the characterisations offered in the papers was seen as a positive way of increasing student teachers awareness of the development of identity, not only enabling teachers to become the teacher they want to be, but also being able to articulate and justify this.

The Mathematical Competency of Teachers

Today, in a climate of reform, many teachers are being asked to teach in ways that are very different from how they learned, and the expectations of teacher knowledge often outstrips that which teachers, especially those in generalist roles, can confidently realise. While acknowledging the many factors involved in effective teaching, the papers in this strand addressed the central role of teacher knowledge, both in terms of classroom practices and issues of competency related to expectations of professional standards.

Case studies (e.g., Christiansen (TA, 2004), (Kaldrimidou, Sakonidis, & Tzekaki, TA, 2004)) focusing on the complexity of the teaching process highlighted the importance of effective teacher scaffolding, interactions and the creation of space and time for student learning. Explorations centred on teachers' ability to 'notice'—to have a sense of when something happens that can carry the learning forward—and the nature of interventions in relation to student difficulties and errors. Kaldrimidou et al. noted the need to focus on the subject-matter structure within lessons, claiming an interplay between the epistemological organisation of the mathematical content and the organisation of the mathematics classroom.

While the majority of papers focused on mathematical knowledge and pedagogy, Forgasz (TA, 2004) presented research from the Australian context indicating the need to address teachers' beliefs. Reviewing studies from a range of school sectors Forgasz noted that despite changes in contemporary students' beliefs about the gendering of mathematics (Leder & Forgasz, 2002), gender-stereotype expectations remain prevalent among teachers, especially in relation to the interaction of technology and mathematics.

Assessment of teachers' competency is increasingly becoming a focus of government agency within a range of countries. Fraser and Morony (TA, 2004) discussed the AAMT Teaching Standards Assessment Evaluation Project aimed at the development of a process for acknowledging outstanding teachers. Assessed through a portfolio and interview, knowledge of students, knowledge of mathematics and knowledge of students' learning of mathematics all contributed to the Professional Knowledge domain. Concerns about pre-service teachers' mathematical knowledge base were also addressed in several papers (e.g., Oh (TA, 2004); Arvidson (TA, 2004)). Amato (TA, 2004) reported an action research project involving pre-service teachers' exploration of a series of children's activities. Increases in mathematical understanding were attributed to the unlearning and re-learning process that facilitated student teachers ability to work backwards from their symbolic ways of representing mathematics to more informal representations.

Conclusions

The papers in the thematic afternoon provided a snapshot of the issues and directions that we as a community are concerned with. This focus on mathematics teachers, their knowledge, their identity and their learning will play a critical role in ensuring quality teaching and effective learning of mathematics. However, the papers also indicate gaps and questions still to be addressed. Despite advances in our research capability and increased focus on reform teaching practices, there remains the interminable challenge to provide equitable mathematical access to all children irrespective of culture, ethnicity, gender, economic and social positions.

The panel debate triggered important questions from participants, such as "We talk about 'beneficial, efficient, excellent, improve, change, develop' without making clear what we mean by these words. Teachers are not good but need to become good. Do we know what we are aiming at?" Future research needs to listen to such questions and try to include them and address them in the work.

This challenge makes issues of recruitment, teacher education and retention of quality teachers all the more pressing. Using a metaphor of teachers "clearing the path on which they walk" van Zoest (TA, 2004) reminded us that the journey to reform is difficult and exhausting. For example, within current reforms in South Africa, Parker (TA, 2004) argued that the focus on mathematical practices (e.g., investigating, making conjectures, justifying, generalising etc.) and on making meaning, rather than simply skills and product, has created new demands on mathematical competencies to teachers. Within this context, teachers need to develop new images of 'good practice' for mathematics teaching and new pedagogic identities. Although our research efforts must clearly be directed to making the pathway less hazardous, it is evident that we must be patient in our efforts to reach the destination. The interest expressed and generated in this thematic strand bodes well for the forthcoming ICMI Study: The Professional Education and Development of Teachers of Mathematics.

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SMDF ISBN 978-91-973934-7-8
ISSN 1651-3274