

Exemplifying different methodological approaches of analysing textbooks in mathematics

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In this symposium, we will discuss different ways of analysing mathematics textbooks from a methodological point of view. The discussion will be based on examples from five separate ongoing analyses of Swedish textbooks divided into two methodological approaches; one where analysis is conducted within an established theoretical framework, and one where analytical tools are constructed through combining aspects of different theories. The symposium will be held at the conference MADIF13.

Textbooks play an important role as mediators between syllabi and teachers in the classrooms, or similarly, in translating policy to pedagogy (Valverde et al., 2002). By now, it is well established that mathematics textbooks have a great impact on teaching since they serve as an important resource for mathematics teachers (Lepik et al., 2015), form learning opportunities for students (Stein et al., 2007), and may even define mathematics as a school subject (Rezat & Strässer, 2014). Over the past two decades, textbooks have received increasing attention in the research field of mathematics education (Jablonka & Johansson, 2010; Rezat et al., 2019), especially considering the rapid development of digital teaching materials (Pepin et al., 2017). However, the field of mathematics textbook research is still developing and strives to establish its philosophical foundations, theoretical frameworks, and research methods (Fan, 2013).

In an attempt to classify literature on mathematics textbook analysis, Fan et al. (2013) came up with the following five categories: (1) content and topic; (2) cognition and pedagogy; (3) gender, ethnicity, equity, etc.; (4) comparison of textbooks; and (5) conceptualization and methodology. In the symposium, we will focus on methodology in relation to content analyses and comparisons of textbooks. Our aim is to increase the knowledge of how theory and method can be applied when analysing textbooks by discussing five ongoing Swedish textbook analyses. The studies exemplify two methodological approaches, where the first covers analyses where theoretical concepts and analytical tools are embedded in a broader theory, while the second covers analyses that combine and/or modify different theories in order to form an analytical tool.

Approach I: Analysing within one theoretical framework

We present two textbook analyses that are positioned within different theoretical frames; Variation theory and the Anthropological Theory of the Didactic (ATD).

Study 1 builds on variation theory and investigates mathematics textbooks for upper secondary vocational education, with the aim of exploring learning opportunities afforded to students with respect to both general mathematical knowledge, and its' use in vocational practice. In particular, characteristic properties of geometry tasks will be contrasted with the corresponding topics in textbooks for other programmes, as well as older vocational mathematics textbooks. According to variation theory, the object of learning and its corresponding critical aspects are in the focus of attention. To discern an aspect, the theory states that students must have the opportunity to experience a potential variation in that aspect (Marton, 2015). Therefore, patterns of variation are analysed in order to evaluate learning opportunities afforded to students by sets of textbook tasks (Kullberg & Skodras, 2018; Sun, 2011).

Study 2 is embedded in the ATD. By using praxeology as an analytical tool (Chevallard, 2006), algebra content in Swedish upper secondary textbooks for academic preparatory and vocational programmes is analysed and compared. Internationally, school algebra has already been investigated from an ATD perspective. For instance, it has been shown that secondary school algebra is often reduced to equation solving and manipulation of algebraic expressions (Bosch, 2015). It is therefore of interest to examine this from a Swedish perspective. The study forms part of a larger project exploring socioeconomic aspects of the didactic transposition (Chevallard, 2006) of algebra in Sweden. In the present study, the focus is on the didactic transposition of algebra from 'knowledge to be taught' to 'taught knowledge'. The aim is to unpack differences in how algebraic content is offered in textbooks for academic preparatory and vocational programmes, and to discuss how these differences may affect students' opportunities for future education and participation in society.

Approach II: Analysing by modifying and/or combining theoretical frameworks

Within this approach, three studies are described. The first concerns digital teaching platforms and the following two programming content at different school levels.

In Study 3, the utilization of dynamic elements in digital teaching platforms is analysed. Building on a pre-existing model of 15 different elements present in digital textbooks (Pohl & Schacht, 2017), a typology for describing and analysing dynamic functions in digital textbooks in mathematics was developed. A development into 23 elements was needed since the existing model did not capture all features deemed important. The typology was developed in relation to an analysis of the dynamic functions present in seven digital textbooks. The sections were divided into text elements, for example a paragraph or an image, and categorised according to their characteristics in terms of semiotic resource, modality, degree of dynamic function, interactivity, and degree of feedback. The 23 categories were then structured into a typology of five dynamic types (Bergvall & Dyrvold, 2021). These types define five steps of increasing dynamic function and increasing invitation to interact; from the first type defining static presence to the last type defining continuous dynamic feedback.

In Study 4, programming content in mathematics textbooks for grades 1-9 is analysed, with the aim of characterizing the programming content and discussing how

it may affect students' opportunities to learn mathematics. In an initial study, programming tasks in textbooks for grades 1-6 were analysed (Bråting & Kilhamn, 2021) using an analytical tool that combines a theoretical framework of computational thinking (Brennan & Resnick, 2012) and one of actions (Benton et al., 2017). Initially, the programming tasks in the textbooks were analysed using the two frameworks separately, but neither of them could describe the content in a comprehensive way. The first framework did not highlight mathematical ideas, and the second did not sufficiently elaborate on computational concepts. An analytical tool was therefore constructed as a combination of concepts and actions, which, in addition, could be used to discuss the links between programming and mathematics. In an ongoing study, textbooks for grades 7-9 are analysed. Taking the previously constructed analytical tool as a point of departure, further developments are made to accommodate for text-based programming.

In Study 5, programming content in mathematics' textbooks for upper secondary school is analysed. As the theoretical frame an instrumental approach is used (Trouche, 2004). The key idea is the difference between an artifact and an instrument: Within the activity of a subject, an artifact becomes an instrument through an individual genesis, the so-called instrumental genesis (Verillon & Rabardel, 1995). Thus, the instrument does not exist in itself – it is made up of an artifact and a psychological component. In order to investigate in what ways the textbooks can offer support for students' instrumental genesis, programming tasks are investigated at three levels inspired by the already mentioned framework of Brennan and Resnick (2012). At the first level the artifact itself is in focus, when the computational concepts treated in the tasks are investigated. The second level is a psychological one, where the actions that students are requested to perform are investigated. At the third level, the categories emerge from the data, based on the different ways in which programming is used as a tool for simulating situations, constructing algorithms, and using different representations.

Concluding remarks

In the symposium, we will compare the different approaches in order to discuss affordances and constraints that they bring on board. From a methodological perspective, we raise questions concerning the clear but at the same time restrictive frame created by the first approach and issues of validation and coherence in the second approach.

Moreover, although all five studies basically analyse what learning opportunities the textbooks offer, there is a variation in aims and foci. For instance, Studies 2 and 4 are primarily concerned with the mathematical content, while Studies 1, 3 and 5 look more at how the textbooks are, or potentially could be, used. Another difference can be noticed between the two analyses of programming content; while Study 4 aims to find out the role of programming in mathematics education, in Study 5 it is already assumed that the students are supposed to use programming as a tool in mathematics.

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