

Connections between natural language and mathematical symbols in mathematics textbooks

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This presentation will give an overview of a project focusing on mathematical symbols and natural language, which both can be found in mathematics textbooks. There often exists different types of connections between the two sign systems. In this study we examine these connections and changes over school years. Six types of connections were defined, and over 3000 connections were identified and coded in textbooks for school years 2, 5 and 8. The results show that connections are mostly based on spatial proximity in the early years (about 63%), and by symbols interwoven in sentences in later school years (about 80%). There is a development from separating natural language and symbols in year 2 to mixing the two sign systems in school year 8.

Introduction

Mathematics has a multisemiotic nature which means that different sign systems are active at the same time. O'Halloran (2011) mentions the natural language, the mathematical symbols, and the visual displays, and how these can be used in mathematical texts. The most noticeable semiotic resource in mathematics is the symbol system. Often, we find mathematical symbols and written natural language together, for example, in "the angle is 37° ". The symbols 3 and 7 and the symbol for degrees $^\circ$ are used interwoven in natural language. Students need to learn how to make meaning out of the natural language and the symbols at the same time, and also to make meaning of the relation between the two sign systems. Research has shown that combining text and symbols can cause difficulties for students (Österholm, 2006).

In this study we examine how natural language and the mathematical symbol system are connected in mathematical texts. We focus on different types of connections and to what extent these types can be found in textbooks for mathematics in school years 2, 5, and 8.

Method

A framework with six types of connections were constructed, drawing on Cohn (2013). The connections were: *Interwoven-NL* (a symbol or symbol expression is inserted into natural language), *Interwoven-MSym* (a word or a phrase is inserted into a symbol expression), *Chunked* (a line with natural language is preceded or followed by a line with symbol expressions), *Adjoined* (natural language and symbol expressions are

connected by spatial proximity only), *Marked* (a visual indication of a connection, for example, an arrow), and *Referenced* (a semantic reference to a specific symbol or symbol expression elsewhere in the text).

180 pages from nine textbooks, three from each of school year two, five, and eight, were randomly selected and all symbols and symbolic expressions were analysed. When a connection of any of the types described above was present, it was coded and counted. In total, 3623 connections were identified and coded (see Table 1).

Tentative results

Two of the connection types vary more over school years than the others. The *Adjoined* type was the most common in school year 2 (63.3%) and then going down to 16.0% in school year 8. The *Interwoven-NL* followed the opposite pattern, changing from 16.7% in school year 2 up to 65.0% in school year 8. Both these changes were significant between all school years.

Type	Year 2	Year 5	Year 8	Total
Adjoined	63.3	33.0	16.0	33.0
Chunked	0.8	1.5	2.9	1.9
Interwoven-NL	16.7	47.4	65.0	47.5
Interwoven-Msym	13.6	12.0	14.8	13.7
Marked	1.7	0.5	1.0	1.0
Referenced	3.9	5.6	0.4	2.8
Total	100	100	100	100

Table 1. Distribution of connection types over school years (percentages)

If we take into consideration that students should generally develop over time, it seems reasonable that in school year 2, many connections are indicated by spatial proximity. This is then followed by an increased number of situations where the two semiotic systems are interwoven with each other. Finally, in school year 8, many connections consist of mathematical symbols inserted in natural language. We see a development from separating to mixing the two sign systems.

During the presentation we will give examples of the different types of connections and how these were coded.

References

- Cohn, N. (2013). Beyond speech balloons and thought bubbles: The integration of text and image. *Semiotica*, 2013(197), 35-63.
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