

How natural language gives meaning to mathematical symbols in textbooks at different school years

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This presentation focuses on an on-going study where we examine if and how there is any type of progression over school years in the use of natural language when addressing symbols in textbooks. The analyses produce a set of categories that describe different ways of using natural language for giving meaning to symbols, for example, how symbols are defined or explained in a very explicit language or how natural language can give information about a symbol in a bit more implicit or indirect manner.

Introduction

Research shows that natural language (for example Swedish) is central to students' meaningful understanding of mathematical symbols. For example, empirical studies on the use of different natural languages show that the interpretation and use of numerical symbols depend on the characteristics of the natural language (Pixner et al., 2011). Empirical studies have also demonstrated the importance of the natural language as a starting point for the development of knowledge of the symbolic language, especially in relation to issues of objectification (Caspi & Sfard, 2012). However, there are very few theories or frameworks that describe the learning of the symbolic language, except when it is limited to a single symbol, such as the equal sign. Arcavi (1994) and Hiebert (1988) seem to be the only ones describing a more comprehensive perspective on the learning of the symbolic language. Also, empirical studies so far only address understanding in single student groups for certain types of symbols, with no longitudinal perspective.

This ongoing study is part of a larger project concerning the role of natural language when learning the mathematical symbolic language. In this project, we aim to create a model describing how progression in the use of natural language can relate to advancements in the use and understanding of the symbolic language. Such a model would give valuable information to understand students' difficulties in learning the symbolic language and help in the design of measures to overcome these difficulties. We focus here on textbooks, if and how they show any type of progression over school years concerning the use of natural language when addressing symbols. More specifically, we aim to answer the following research questions (RQ):

1. To what extent does natural language in mathematics textbooks give meaning to mathematical symbols in different school years?
2. What type of meaning of mathematical symbols is given by natural language

in mathematics textbooks in different school years?

A central notion in our research questions is *giving meaning to symbols*, which can be done by specifying a referent to mathematical symbols (cf. Arcavi, 1994; Hiebert, 1988). We delimit our study to situations when some (property of a) referent is specified through natural language. For example, in the statement “ x is a number”, a referent to x is specified through the association to the word “number”.

Method

For the analyses in this study, we used three different textbook series commonly used in Sweden, from school years 2, 5, and 9, giving a total of nine textbooks. We randomly selected 20 pages from each textbook, resulting in a total of 180 pages in our data set. Any use of natural language in relation to symbols can be said to contribute to giving meaning to symbols, but we delimit our study to situations that are more explicit. Our analyses started by locating explicit statements about symbols. Then, through iterative analyses of subsets of our data, we located other types of situations, somewhat more implicit or indirect, which also specify some (property of a) referent to a symbol. Through this process, a set of categories were created, which describe different ways of using natural language to give meaning to symbols. The categories themselves address the *types of meaning* mentioned in RQ2, while RQ1 can be answered by counting how often the use of natural language in relation to symbols falls in any category.

Results

Analyses are ongoing, but preliminary results have produced different categories of how natural language gives meaning to symbols. For example:

- A symbol can be defined or explained in a very explicit language. For example, statements as “ x stands for the number of cars”, where information is directly given about the referent for x (number of cars). Another example is “13 is odd”, where information is given about a property (being odd) of the referent.
- Natural language can also give information about a symbol in a bit more implicit or indirect manner, such as “the number x is odd”, where a referent to x is specified by labelling it as “number”, or “the movie was 2 hours long”, where “long” specifies that “2” stands for the length of the movie.

References

- Arcavi, A. (1994). Symbol sense: Informal sense-making in formal mathematics. *For the Learning of Mathematics*, 14(3), 24-35.
- Caspi, S., & Sfard, A. (2012). Spontaneous meta-arithmetic as a first step toward school algebra. *International Journal of Educational Research*, 51–52, 45-65.
- Hiebert, J. (1988). A theory of developing competence with written mathematical symbols. *Educational Studies in Mathematics*, 19, 333-355.
- Pixner, S., et al. (2011). Whorf reloaded: Language effects on nonverbal number processing in first grade - A trilingual study. *Journal of Experimental Child Psychology*, 108, 371-382.