Spatial relations and other text features in the connections between mathematical symbols and written language

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In this presentation, ways to communicate connections between mathematical symbols and natural language in text are described. In mathematical textbooks, the two semiotic resources are often used together to introduce and explain mathematical ideas, concepts, and methods. To make sense of the texts, readers need to combine language descriptions and symbolic expressions, and when reading order is not adamant, spatial relations become important. We present different categories of connections between written language and mathematical symbols that are based on spatial relationships together with other features, for example, visual links, primary syntax, and reading order.

Introduction

In mathematics, we expect students to read mathematical texts and we want them to make sense of mathematical symbols and written language, not only one by one but also in combination. That is part of the mathematical reading competency.

Written language is (in the Western world) normally read from left to right and from top to bottom. When combining written language and mathematical symbols in modern mathematics textbooks there are many possibilities to abandon normal reading order, at least partly. Connections can be created based on spatial relations, often in combination with other text features. Five different categories of such connections are presented in this paper. The categories can be used as a tool, for example, when classifying the occasions in mathematical texts where the reader is invited to switch between reading of written language to reading of symbols, and vice versa.

Development of categories

Inspiration for the present classification was found in Cohn's descriptions of written language-image relations in comics (Cohn, 2013). The descriptions include spatiality, syntax, and visual links, and was found to be a suitable starting point. Initially, analogues to the connections described by Cohn were identified in textbook texts. Examples were collected from Swedish mathematics textbooks for year 1-12 (a convenience selection) by the first author and discussed with the other authors. Based on the analogues

identified, a first description of four connection categories was formulated including *interwoven, adjoined, marked,* and *referenced*. During the first test analyses, one type of connections used in mathematics texts was found to be excluded. To include that type, a new category was added: *chunked*. After the addition, several rounds of analysis and discussion were performed to set the details of the five category descriptions.

Category descriptions based on spatial relations and other text features

A coherent text will include a variety of connections on different levels. Below, we describe five different connection categories that will invite a reader to switch between the reading of written language and the reading of symbols.

Interwoven connections are of two types. In one type, symbols or symbolic expressions are used as substitutes for words in a sentence, e.g., "Variabeln x ersätts med talet 3". In the other, words or phrases are used as entities in a formula, e.g, "circumference = $\pi \cdot$ diameter". Both types have one primary syntax (and a secondary), and mathematical symbols and written language share the location in the text.

Chunked connections are connections between successive lines in a text where one line use the language syntax as primary syntax, while the other line follows the syntax of the mathematical symbol system. Written language and mathematical symbols are adjacently located and additionally connected by reading order.

Marked connections are formed when written language and symbols are adjacent and the connection is further emphasised by a visual link, e.g., an arrow, brackets, or colour coding.

Adjoined connections, similar to the marked connections, are built on proximity between language and mathematical symbols but without visible links or a fixed reading order. This category includes headers connected to mathematical formulas, comments to worked examples, instructions to arithmetic exercises, etc.

The *referenced* connections presume written language and symbols to be quite distant and without any visual links between them. Instead, the language description is painting a picture of what to find and in which location or direction, for example, "The formula above can be used to calculate [...]".

Although somewhat different in appearance, all categories described above could be found in textbooks from different educational stages. The presentation will include examples from the different categories.

References

Cohn, N. (2013). Beyond speech balloons and thought bubbles: The integration of text and image. *Semiotica*, *2013*(197), 35-63.