Analysing argumentative processes during mathematical problem solving in small groups

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This presentation reports from a study of young students' use of justifications when solving a mathematical problem in small groups. In the study, video recordings from two grade 6 classrooms were analysed, using Harel and Sowder's taxonomy of proof schemes as a base for an analytical tool. The analysis showed similarities between the young students' justifications and results of previous research on adult students' proofs, for instance regarding students' use of examples. The analysis also showed how students sometimes used calculations that contradicted empirical examinations, which could not be explained by the taxonomy of proof schemes. An ethnomethodological approach is suggested, to further the analysis of argumentative processes taking place during mathematical problem solving in groups.

This presentation reports from a study of grade 6 students' use of justifications during mathematical problem solving in small groups (Fredriksdotter et al., forthcoming). The aim of the presentation is to suggest an approach to further the analysis of argumentative processes that took place when students engaged in solving the mathematical problem.

The empirical material of the study consisted of video recordings from two classrooms, where students solved a combinatorial problem about the number of queues that two, three and four persons can form. The students were also asked to formulate a general rule for calculating the number of queues. In the analysis of the students' argumentation, Harel and Sowder's taxonomy of proof schemes (Sowder & Harel, 1998) was used as an analytical tool. The taxonomy contains three classes: *Externally based* proof schemes, where students refer to factors outside of themselves, *Empirical* proof schemes, where students rely on examples and on their perception, and *Analytic* proof schemes, characterized by general and logical reasoning. The results showed that students often referred to examples, which is common among all ages. Students also used general justifications. However, several students' suggested the incorrect solution of squaring the number of persons in order to find the number of queues. This sometimes occurred even when an example showed the correct number of permutations. The first example shows how Anna agreed with Bella's suggestion that three persons can form nine queues, although Anna's own example showed the correct number six:

Anna: I believe it's like that (points at her table representing six "queues").

Bella: Either six or nine […] because nine, then it is three times three, that makes all of them be different.

Anna: Mm (looks at Bella, adds "9" to her notes).

The second example shows how Carla and Dennis ignored Elton's objection:

Carla: Why do we do three times three? Because they can stand in three different places. Nice!

Dennis: Eh, yeah, yeah.

Carla: Everyone can stand in three different places, therefore it's three.

Elton: But we weren't allowed to do it like that, were we?

Dennis: Well, I wrote that you do the number times itself.

Carla: I wrote that you do the number of persons times the number of persons.

Dennis: Yeah, exactly, exactly, but that's what I meant.

The third example shows how Fred's peers challenged his correct explanation of the calculation $3 \cdot 2$, but instead of pursuing the discussion the group changed topic of talk:

Fred: There are three different, but those behind can change places, and then it's three times two if everyone changes as you can make two of each.

(Omitted: peers' comments on whether they understand Fred's solution or not.)

Greg: I think it is three times three.

Hanna: Me too.

Greg: But in the first one it's two, everyone knows that, right?

The examples contain the same mathematical content, in the sense that $3 \cdot 3$ was suggested. However, the groups handled the suggestion of performing the operation of squaring quite differently (as in agreeing, ignoring objections or changing the topic of talk) which also affected their subsequent process of solving the mathematical problem.

The justifications of $3 \cdot 3$ may be categorized in accordance with the taxonomy of proof schemes, but the notion of proof scheme cannot explain why students not only suggested but also maintained the operation of squaring. Moreover, the taxonomy is not a sufficient tool for analysing students' various ways of handling the suggestion of squaring. The argumentative processes may instead be analysed through the use of an ethnomethodological approach, where situations are considered to be co-constructed by participants' actions. When students act in a certain way, such as justifying a solution to a mathematical problem, they are contributing to the context for the next participant's action, as well as showing their understanding of the situation (Ingram, 2018). Analysing students' argumentation on a turn-by-turn basis may therefore not only show students' individual understanding of the situation, but also reveal how argumentative processes may affect students' methods of solving the mathematical problem.

References

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