Problem-solving in Swedish curricula in a time of change

Johan Sidenvall[1,2], Anna Ida Säfström[1,3], Erika Boström[1,3]

[1]Umeå Mathematics Education Research Centre, [2]Municipality of Hudiksvall, [3]Umeå University

It is long known that students' learning in mathematics is facilitated by problem-solving activities, and school authorities all over the world have incorporated problem-solving in their curricula. However, problem-solving does not have a clear definition, and its meaning risks being watered down in the process of implementation. In this study, we examine how problem-solving is described and used in Swedish syllabi, commentary materials and national tests for school year 6–10, before and after the 2021 and 2022 revisions. Our results show that 'problem-solving' is increasingly conceptualised as a goal rather than a means for learning, and that as a goal problem-solving competency is reduced. As a guidance for teachers the policy documents are often vague and even contradictory. Implications for teaching practice and Swedish students are discussed.

Mathematics education research agrees that teaching through problem-solving has benefits (Munter & Correnti, 2017). Teaching mathematics is often regulated by governing text (for a longer discussion see e.g. Boesen et al., 2014). Sweden has foregrounded problem-solving in mathematics in curricula since 1994 (Utbildningsdepartementet, 1994). However, the term 'problem' is given different meanings by different people—not all associated with learning benefits—and the meaning and role of problem-solving can be diluted during curricular reforms (Gravemeijer et al., 2016). Lately, Swedish mathematics syllabi have been revised every year, with the most recent changes being implemented in 2021 for upper secondary school (years 10-12) and in 2022 for compulsory school (years 1-9). In this study, we examine the meanings of problem-solving expressed in Swedish school curricula, by analysing available syllabi, commentary materials and national tests before and after the latest revisions.

Problem-solving in mathematics education research

In this study, we define 'problems' as tasks where the solution method is not known in advance, in contrast to routine tasks that only requires the use of a known procedure (NCTM, 2000; Schoenfeld, 1985). In this sense of the word, solving problems is a far more complex process than solving routine tasks. This complexity is reflected in the various models for problem-solving processes developed by researchers, describing different phases, such as interpretation, exploration and suggestion of potentially fruitful approaches (Rott et al., 2021). The importance of evaluation, including describing and verifying the solution, and identifying alternative approaches, is also often highlighted as key for learning (Koichu et al., 2021).

Some researchers focus on prerequisites of problem-solving or foreground problem-solving as a goal of mathematics education (e.g., Schoenfeld, 1985). While it is acknowledged that problem-solving could be seen as a competency and assessed as a learning outcome, it is still an open question how this should be done (Niss & Højgaard, 2019). In fact, mathematics tests mostly consist of short, structured items that do not assess problem-solving competency (Jones & Inglis, 2015). Mathematics education research more commonly frames problem-solving as a learning activity and it has been shown that problem-solving give richer opportunities to develop mathematical competence than applying given methods on routine tasks (Boaler, 2014; Downton & Sullivan, 2017; Jonsson et al., 2014; Lester & Cai, 2016. In sum, the research field supports the use of problem-solving as a means for teaching and learning, while effects of assessing problem-solving competency is seldom studied or advocated.

Problem-solving in Swedish curricular reforms

Problem-solving was introduced in Swedish curricula at the same time as the school management model changed from a rule-based to a goal-based system (SOU 1992:94; SOU 2007:28; Utbildningsdepartementet, 1994). The intent of this reform was to refrain from directly regulating teachers' practice and instead govern schools through testing students' performance in relation to given goals. This was initially done by two means: the curriculum, including learning goals within the syllabi for each subject, and the national tests. The first syllabi specified "goals to strive for" that were meant to form a basis for the planning and design of teaching, and "goals to attain" that were meant to be assessed (Skolverket, 2004), with problem-solving competency included in the former. However, evaluations showed that the goals were hard to interpret for teachers and that the teaching was based on the goals to attain rather than the goals to strive for (Boesen et al., 2014; Skolverket, 2004; SOU 2007:28). This led to calls for commentary materials further explaining the goals and teachers turning to the national tests for help in interpreting the syllabi (SOU 2007:28). In 2011, the two sets of goals were reformulated as mandatory content and knowledge requirements in terms of competencies, of which problem-solving is one. The goal-oriented system still remains, and the guidance for teachers now consists of the syllabus, the commentary materials and national tests for grade 3, 6, 9 and each course in upper secondary school. In contrast to research, the Swedish system thus foregrounds problem-solving as a competency to be assessed, while the choice and development of means for teaching and learning is left to teachers.

Aim and research question

The Swedish curriculum and the national tests play a key role in shaping teachers' view of teaching and learning in mathematics in general, and problems and problem-solving in particular. This view may in turn impact the teaching offered to students. Therefore, we aim to investigate the concept ‘problem-solving’ in Swedish national policy documents, by focusing the following research question: How is ‘problem-solving’ described and used in syllabi, commentary materials and national tests for school year 6–10 before and after the 2021 and 2022 revisions?

Methods

To answer our questions, we conducted a content analysis according to Nuopponen’s (2010) four steps: compilation of the material, elaboration of a preliminary concept system, systematic analysis of each sources separately, and synthesis and comparison of the results for each source. The analysis was both descriptive: describing the state and use of concepts before and after the revisions respectively, and contrastive: exploring and clarifying similarities and differences between the previous and revised policy documents (Nuopponen, 2010). The four steps were conducted as follows below.

Compilation of the material

As we aim to study the concept problem-solving in national policy documents, our analysis was source-restricted (Nuopponen, 2010) to the three policy documents governing Swedish mathematics instruction: the mathematics syllabi, the commentary materials for those syllabi, and the national tests. We further restricted our analysis to school year 6–10. We excluded year 11–12 since a large part of students do not study mathematics in those years, and year 3 as the national test for year 3 does not specify what tasks assess problem-solving. For year 10 we chose the syllabus and test for the intermediate track course Mathematics 1b. There are no national tests available based on the new syllabi, and such tests could therefore not be included in the data collection.

Our data thus consisted of the mathematics syllabi for compulsory and upper secondary school from before and after the revision (Skolverket, 2019; 2021a; n.d.a), the commentary materials for these syllabi (Skolverket, 2017; 2021b; 2021c; n.d.b), and the most recent publicly available national mathematics tests and scoring instructions for school years 6, 9 and the course Mathematics 1b. Each syllabus consists of a short introduction and an aim, followed by a list of mandatory content and knowledge requirements specified for grade levels E, C and A. From the tests only the tasks assessing problem-solving competency according to the scoring instructions were included, a total of 55 tasks.

Elaboration of a preliminary concept system

An early step in concept analysis is to form preliminary concept definitions and relations to use as a framework for the analysis. Since the commentary material and the national tests are intended to interpret the syllabi, we used the syllabi to form our preliminary concept system:

As stated, problems are tasks where the solution method is not known in advance. Problem-solving is the activity students engage in when trying to solve problems and a means for learning. Problem-solving competency is the set of abilities needed to engage in problem-solving and a goal for learning. Three aspects of problem-solving competency are: solving problems, describing and evaluating solutions, and suggesting alternative approaches. These three aspects and their intended progression over years as well as different levels of them are further explicated in the left side of Figure 1.

Systematic analysis of each source separately

As the national tests is a different kind of source than the syllabi and commentary materials, we devised separate methods of analysis for these two types of sources.

Analysis of syllabi and commentary materials

First, all instances of the term 'problem', including compounds, were marked. In the commentary material, mere quotes from the syllabi were excluded. Next, each sentence was coded as regarding either problems, problem-solving or problem-solving competency. The syllabi sentences coded as problem-solving competency were the knowledge requirements and were not further analysed as they had already been used to create the preliminary concept system (left side of Figure 1). The sentences in the commentary material concerning problem-solving competency were coded as related to either progression or grade level, or both. Each progression and/or level sentence was then analysed in relation to the three aspects of problem-solving competency, identifying both further explanation of and contradictions to the knowledge requirements.

Analysis of National Tests

Of the 55 tasks 25 were rewarded points on grade level E, 30 on grade level C, and 18 on grade level A. In order to understand the tasks' requirements, we first formulated the solution steps students most likely would take to get the point specified in the teachers' scoring instruction (Jäder et al., 2020). Then, we used the preliminary concept system to analyse the tasks, scoring instructions and plausible solutions, considering: 1) whether the task was a problem by identifying what (if anything) the student needed to do besides methods included in the mandatory content; 2) what aspects of problem-solving competency the task assessed by identifying: a) the context of the task, b) whether the student had to solve the whole task to receive the point, c) whether the student had to describe or evaluate any part of the solution was considered, and d) whether the student had to suggest alternative approaches. The analysis first considered each task, then summarised the result for the set of tasks giving points at each grade level and school year, and finally identified similarities and differences between grade levels and school years.

Synthesis and comparison of results regarding each source

In the final step of the analysis, the results from the separate analyses of the syllabi, the commentary materials and the tests were synthesised and compared. This was done by identifying further explanations, additional examples and contradictions between different sources concerning the same syllabus and what had changed or stayed the same in the revisions.

Results

For all grades and both before and after the revisions, the syllabi describe problem-solving as both a means and goal for learning. However, the commentary materials have shifted in focus. Before the revisions, more sentences focused on problem-solving as a means (48 for year 6–9, 39 for year 10) than on problem-solving as a goal (17, 0). After the revisions the focus has changed (means: 31, 16; goal: 34, 22). While formulations regarding problem-solving in the aim and mandatory content of the syllabi are largely the same before and after the revisions, the knowledge requirements have changed remarkedly. Below, we present in more detail how the knowledge requirements are stated in the syllabi (Figure 1) and interpreted by the commentary materials and national tests first before and then after the revisions, by attending to each of the main aspects of problem-solving competency – ‘solving problems’, ‘describing and evaluating’, and ‘alternative approaches’.

	BEFORE THE REVISIONS			AFTER THE REVISIONS		
	School year 6	School year 9	School year 10	School year 6	School year 9	School year 10
SOLVING PROBLEMS	The students can solve simple problems in situations close to the student	The student can solve different problems in familiar situations	The student can formulate, analyse and solve mathematical problems	The student solves		
	E in a mainly functioning way		E of simple character	E simple problems		
	C in a relatively well functioning way		C	C relatively complex problems		
	A in a well-functioning way		A of complex character	A complex problems		
	by selecting and using strategies and methods with		These problems include			within the different areas of the course.
	E some adaptation to the character of the problem.		E a few concepts and requires simple interpretations.			
	C relatively good adaptation to the character of the problem.		C several concepts and requires			
	A good adaptation to the character of the problem.		A advanced interpretations			
DESCRIBING AND EVALUATING	The student describes approaches			The student		
	E in a mainly			evaluates the plausibility of the results and	evaluates strategies and the plausibility of the results	assesses the plausibility of the results.
	C in a relatively well					
	A in a well					
	functioning way and reasons	The student reasons	The student can, with		E in a simple way	
	E in a simple and somewhat substantiated way		E simple judgements		C in a developed way	
	C in a developed and relatively well-substantiated way		C simple judgements		A in a well-developed way	
	A in a well-developed and well substantiated way		A nuanced judgements			
		about approaches and	evaluate the plausibility of the results as well as selected models, strategies and methods			
	about the plausibility of the results in relation to the problem situation, and is able to				and	
ALT. APPR	E contribute to some suggestion for an alternative approach.		E	E contributes to some suggestion		
	C give some suggestion for an alternative approach.		C and alternatives to them.	C give some suggestion		
	A give suggestions for alternative approaches.		A	A give suggestions		
				for alternative approaches.		

Figure 1. An overview of the syllabi knowledge requirements in school year 6, 9 and 10 before and after the revisions (author’s translation).

Problem-solving competency before the revisions

The knowledge requirements specifying the progression and different levels of problem-solving competency grade 6–10 are shown on the left side of Figure 1. For year 6–9, the commentary material gives some further explanations of the progression, as well as the differences between grades. However, for year 10, no sentences concerning progression or grades were found, and hence no references to the commentary material for year 10 are made below.

Solving problems

From year 6 to 10, the main progression lies within the character of the problems students solve. In year 6, students solve simple problems in situations “close to the student”. The commentary material describes such situations as familiar to the student and as everyday situations; it thus seems like the problems in year 6, should be connected to situations that students have personal experience of. The analysis shows that half of the tasks in the national test has an everyday context, but it is unlikely that the students have had a personal experience of many of the task contexts. The syllabus states that students in year 9 should solve different problems in familiar situations. The commentary material describes this as a broadening of problem-solving, in the sense that students can encounter more complex everyday situations that the students have not necessarily experienced personally, for example making a budget. Also in the test for year 9, however, half of the tasks are purely mathematical. In year 10, students solve mathematical problems, which implies that problems do not have to have a real-world context. Still, a majority of the problems in the year 10 test have an everyday context. In fact, the proportion of tasks that are purely mathematical, i.e. lacked an everyday context, slightly decrease over school years, contradicting the commentary materials’ interpretation of progression.

The grade levels are characterised in different ways in compulsory and upper secondary school. In year 10, the difference between grades lies in students being able to solve problems of different complexity. This implies that students at different grade levels solve different problems, which is also reflected in the test. In year 6 and 9, the difference lies in the quality of students’ solutions, implying that students at all grade levels are to work on the same problems but solve them with various success and sophistication. The commentary material and the year 6 and 9 tests give a different view, more in line with the year 10 requirements. The commentary material states that:

> A student that have gotten far in their knowledge development can view a task as a routine task if he or she knows a solution method. Another student encountering the same task can, however, need to explore and try different approaches to reach a solution. (p. 7, Skolverket, 2017, authors’ translation)

The quote reflect the idea that tasks can be problems for students at lower grade levels but not for students at higher grade levels, also in year 6 and 9. This view is supported by the year 6 and 9 tests, in that they mostly reward points for different grades for different tasks. While about a third of the tasks in year 6 and half of the tasks in year 9 are rewarded problem-solving points for a partial solution, the complete solution will in those cases most often not give higher level problem-solving points. Only in 16 tasks do students have the opportunity to show problem-solving competency at two or three levels in the same task. In fact, an increase in complexity of tasks from E to A level can be identified in the tests for all school years. At C- and A-level, students need to, to a greater degree, interpret, and structure the information in the task and make calculations so that the right conclusions can be drawn to find a solution.

The analysis also showed that students in year 6 and 10 can, not seldom, receive E-level problem-solving points by presenting a full solution that merely applies methods included in the mandatory content. This signals that students at E-level are not expected to learn standard methods to a level where they can recognise when they are applicable and use them confidently.

Describing and evaluating

Only a few sentences in the year 6 and 9 commentary material concerns the progression or level of describing and evaluating. One sentence states that the "depth of students' reasoning regarding approaches and plausibility of results" should increase over school years and grades, and another that students should develop an awareness of the efficiency of their approaches with respect to the problem, through being "given knowledge of evaluation of choices", but this knowedge is not further described. Of the 55 tasks analysed, 36 prompted students to write down her solution, but no examples where found that explicitly asked for evaluation of solutions. The apparent idea of a progression starting in description and successively adding more evaluation and reasoning regarding one's approaches is not further elaborated in neither commentary material nor national tests.

Alternative approaches

Alternative approaches are only mentioned once in the commentary material for year 6 and 9, and then as a mere statement that higher grades require higher degrees of contribution from the student. No tasks on the national tests explicitly required students to formulate alternative approaches.

Problem-solving competency after the revisions

In the revisions, the knowledge requirements have been substantially shortened. The progression over school years and the differences between grades is now easier to overview, while the three aspects are still represented (right side of Figure 1). A larger proportion of the sentences about problems and problem-solving in the commentary materials now concern progression and differences between grades for both compulsory and upper secondary school.

Solving problems

The requirements for different grades in year 10 has now replaced the requirements for all school years. This means that the difference between grades in year 6 and 9 is no longer the quality of students' solutions, but the complexity of the problems students are able to solve. This also means that there is no progression over years evident in the knowledge requirements. However, the commentary material for year 6 and 9 relates increased complexity to both progression and grades. For year 10, the increased complexity required for higher grades is exemplified in seven aspects. For year 6 and 9, three of those aspects are used to specify difference in grade level (familiarity, level of abstraction, combining content), while three of the aspects specify progression (more advanced interpretations, more advanced calculations, more advanced concepts) and one exemplifies both (less guidance).

Describing and evaluating
There is only one sentence specifying the knowledge requirements related to 'evaluation' in each of the commentary materials. For year 6 and 9, evaluation still concerns both results and strategies, but the commentary material only specifies evaluation of results: "the student needs to relate conclusions and results to the original mathematical questions". For year 10, evaluation is now restricted to results and is described as "the student as a rule, identifies absurdities, even if there can be exceptions".

Alternative approaches
Being able to assess 'alternative approaches' has been removed as a criteria from year 10, but contributions to alternative approaches is still required in year 6 and 9. Only one sentence in the commentary material comments on this, but does not further explicate or exemplify different levels of contribution: "the student can see the problem and the methods and strategies used from different perspectives".

Conclusion and discussion

Our results show that 'problem-solving' is increasingly conceptualised as a goal rather than a means for learning in Swedish policy documents, and that as a goal, problem-solving competency is reduced in complexity. The commentary material only provide vague guidance for teachers, due to the documents' absence of rich descriptions of how to handle the different aspect of problem-solving. In addition, descriptions and examples given in the syllabi, commentary material and the national tests are often contradictory.

Before the revisions, the descriptions of problem-solving competency were rather complex, but reflected important aspects of problem-solving as a process and as a learning activity. They gave value to less efficient approaches, contributing to collective solution processes and reasoning about strategies and results. In this respect, they reflected what we know about problem-solving phases (Rott et al., 2021; Schoenfeld, 1985) and the practices that enhance mathematics learning (Munter & Correnti, 2017). However, neither the commentary materials nor the national tests fully reflected these aspects. Specifically, they gave very limited guidance regarding two of the three aspects: 'describing and evaluating', and 'alternative approaches', even though they are known to be both important and hard to engage students in (Koichu et al., 2021). Rather than providing the called for support for teachers' understanding of the complexity of problem-solving (SOU 2007:28), the revised curricula has simplified and narrowed the description of what problem-solving entails.

A significant change in the syllabus for compulsory school is that grades are now separated by the level of complexity of problems students solve. The previous syllabus implied that students at all grade levels should work with the same problems, but might contribute at different levels of proficiency. Such an approach facilitates other activities that are beneficial for learning, such as communicating, comparing, contrasting and critiquing one's own and others' ideas (Munter & Correnti, 2017). If students instead solve different problems, such activities can become much more difficult to orchestrate.

Furthermore, our results show that before the revisions, E-level tasks are seldom require more than application of methods included in the mandatory content, which – if problems are to be tasks for which the solution method is unknown – implies either that students at E-level are not expected to know the mandatory methods taught or that problem-solving is not required at E-level at all. The new requirements risk increasing the difference between learning opportunities given to different students, and as a result hamper students' development of mathematical competence, counteracting the aim of the subject as stated in the curricula (Skolverket, 2021a; n.d.a) and contradicting research showing that students at all ages and levels benefit from problem-solving activities (Boaler, 2014; Downton & Sullivan, 2017; Jonsson et al., 2014; Ridlon, 2009).

Another significant change is that the knowledge requirements no longer reflect any progression with respect to the types of problems students should be able to solve in different school years, aside from covering different mandatory content. Since it is long known that problem-solving competency goes beyond application of given methods (Schoenfeld, 1985), it is unclear whether this can be seen as a progression in problem-solving—especially for students who stay at E-level over years. In this sence, the policy documents provide questionable guidance for teachers' development of students' problem-solving competency.

In sum, this study raises doubts regarding the revised curriculas' potential for supporting teachers in creating rich opportunities for learning mathematics, especially for struggling students. Based on the results of the study we suggest an enrichment of the syllabi concerning problem-solving, hence teachers can get a deeper understanding what is to be taught and how, that in turn can support richer teaching of mathematics.

References

Boaler, J. (2014). The road to reasoning. In K. Brodie (Ed.), *Teaching mathematical reasoning in secondary school classrooms* (pp. v–vii). Springer.

Boesen, J., Helenius, O., Bergqvist, E., Bergqvist, T., Lithner, J., Palm, T., & Palmberg, B. (2014) Developing mathematical competence: From the intended to the enacted curriculum. *Journal of Mathematical Behavior, 33*, 72–87.

Downton, A., & Sullivan, P. (2017). Posing complex problems requiring multiplicative thinking prompts students to use sophisticated strategies and build mathematical connections. *Educational Studies in Mathematics, 95*(3), 303–328.

Gravemeijer, K., Bruin-Muurling, G., Kraemer, J.–M. & van Stiphout, I. (2016). Shortcomings of mathematics education reform in The Netherlands: A paradigm case? *Mathematical Thinking and Learning, 18*(1), 25–44.

Jones, I., & Inglis, M. (2015). The problem of assessing problem solving: can comparative judgement help? *Educational Studies in Mathematics, 89*, 337–355.

Jonsson, B., Norqvist, M., Lithner, J., & Liljekvist, Y. (2014). Learning mathematics through algorithmic and creative reasoning. *Journal of Mathematical Behavior, 36*, 20–32.

Jäder, J., Lithner, J., & Sidenvall, J. (2020). Mathematical problem solving in textbooks from twelve countries. *International Journal of Mathematical Education in Science and technology*, *51*(7).

Koichu, B., Parasha, R., & Tabach, M. (2021). Who-is-right tasks as a means for supporting collective looking-back practices. *ZDM – Mathematics Education, 53*(4), 831–846.

Lester, F. K., & Cai, J. (2016). Can mathematical problem solving be taught? Preliminary answers from 30 years of research. In P. Felmer et al. (Ed.), *Posing and Solving Mathematical Problems* (pp. 117–135). Springer.

Munter, C., & Correnti, R. (2017). Examining relations between mathematics teachers' instructional vision and knowledge and change in practice. *American Journal of Education*, *123*, 171–202.

Niss, M., & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics, 102*(1), 9–28.

NCTM (2000). *Principles and standards for school mathematics*. NCTM.

Nuopponen, A. (2010). Methods of concept analysis – Towards systematic concept analysis. *LSP Journal, 1*(2), 5–15.

Ridlon, C. L. (2009). Learning mathematics via a problem-centered approach: A two-year study. *Mathematical Thinking and Learning*, *11*(4), 188–225.

Rott, B., Specht, B., & Knipping, C. (2021). A descriptive phase model of problem-solving processes. *ZDM – Mathematics Education, 53*(4), 737–752.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Academic Press.

Skolverket (2004). *Nationella utvärderingen av grundskolan 2003. Huvudrapport – svenska/svenska som andra språk, engelska, matematik och undersökningen i årskurs 5* [The national evaluation of compulsory school 2003]. Fritzes.

Skolverket (2017). *Kommentarmaterial till kursplanen i matematik. Reviderad 2017.* [Commentary material for the mathematics syllabus]. shorturl.at/yAILP

Skolverket (2019). *Läroplan för grundskolan, förskoleklassen och fritidshemmet 2011. Reviderad 2019.* [Curriculum for compulsory school, preschool class and after-school center 2011. Revised 2019]. Nordstedts.

Skolverket (2021a). *Matematik* [Mathematics]. http://shorturl.at/txzJ2

Skolverket (2021b). *Kommentarmaterial till kursplanen i matematik. Grundskolan.* [Commentary material to the compulsory school mathematics syllabus]. https://www.skolverket.se/getFile?file=7840

Skolverket (2021c). *Kommentarsmaterial till ämnesplanen i matematik. Gymnasieskolan och kommunal vuxenutbildning på gymnasial nivå.* [Commentary material for the upper secondary mathematics syllabus. https://www.skolverket.se/getFile?file=7841

Skolverket (n.d.a). *Jämför ämnesplanerna. Ämnesplan för matematik på gymnasial nivå.* [Compare the syllabi. Syllabus for mathematics on upper secondary level]. http://shorturl.at/qwCMU

Skolverket (n.d.b). *Om ämnet matematik* [About the subject mathematics].

SOU 1992:94. *Skola för bildning* [School for education]. Nordstedts.

SOU 2007:28. *Tydliga mål och kunskapskrav i grundskolan. Förslag till nytt mål- och uppföljningssystem* [Clear goals and knowledge requirements in compulsory school. Suggestions for a new goal-based system]. Fritzes.

Utbildningsdepartementet (1994). *Kursplaner för grundskolan* [Syllabi for compulsory school]. https://gupea.ub.gu.se/handle/2077/30959