Mathematics Teachers and the Role of Physical Environment

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The physical environment (PE) affects the teaching and learning in school. Research is conclusive that different characteristics of PE can be enabling or hindering learning activities. Still, we need to know more about the role of PE in mathematics education to utilize what a good PE can offer and to avoid the hindering situations. The aim of this study is to characterize the different roles PE play in relation to the teacher, the student, the learning content, and their interactions. For this, mathematics teachers' stories about their experiences of PE in teaching are analysed. The results show that teachers often try to prevent disturbances or distractions from insufficiencies in PE. The results also suggest that aspects of classroom PE, such as classroom layout, sustain classroom norms whereas other elements in PE can be an aid in breaking norms.

Introduction

The physical environment (PE) affects the teaching and learning in school (Earthman & Lemasters, 2009; Tanner, 2009; Woolner et al., 2007). When examining mathematics education research, PE could either have an enabling or hindering role in the mathematics classroom (Fahlström, 2017; Fahlström & Sumpter, 2018). The enabling role can be found when the conditions in PE are sufficient and satisfactory for the educational activity. In less successful cases, the teaching can be hindered by poor building status or insufficient classroom size, which eventually affect wellbeing and learning outcomes (Earthman & Lemasters, 2009; Tanner, 2009; Woolner et al., 2007). In these cases, the insufficiencies in PE are predominantly known preconditions, however not fully controllable for the teacher (Fahlström & Sumpter, 2018). Hence, we need to know more about the role of PE in mathematics education in order to avoid PE from hindering the learning activity and preferably utilize the benefits that come from good PE conditions.

Studies regarding the physical learning environment in schools have different objectives in focus. One such objective is to look for evidence linking technical aspects and other specific criteria of school PE to wellbeing, behaviours, and large-scale learning outcomes, for instance measured by standardized tests (Tanner, 2009; Uline & Tschannen-Moran, 2008; Woolner et al., 2007). Another type is research that starts out in ideas about specific teaching practices and examines whether the physical classroom settings promote or hinder such teaching practices (Cleveland & Fisher, 2014). In

relation to this category there is research examining whether alterations made to school buildings and classrooms have had the desired effects on teaching practices. Most of these studies conclude that involving teachers and students in the process of building new, or altering existing, physical school environments is crucial for long-lasting changes to the classroom practices (Cleveland & Fisher, 2014; Uline et al., 2010; Woolner et al., 2012; Woolner et al., 2018). There are also studies that focus on teachers' views on school PE and these studies often picture school designers, school authority, and related research on the receiving end of their research (Barrett & Zhang, 2012; Earthman & Lemasters, 2009). The present study falls into this category, where mathematics teachers' stories about their experiences of PE in different teaching situations will be studied. PE is defined as the collection of physical objects that we can touch and physical phenomena that we can sense to this end.

The aim of this study is to characterize the different roles PE play in the stories of different teaching situations told by the participating mathematics teachers. A role is defined as the purpose or influence of someone or something in a particular situation. The identified roles that PE play will be characterized in relation to the actors, the teacher, the student, the learning content, and the pairwise interactions between them. The characterizations will also include the direction of influence or purpose in the role, if the role is enabling or hindering, and whether the role is active before, during, or after the described teaching situation.

Background

In a previous study, the didactical triangle (see figure 1) was extended to include PE as a fourth actor together with the three original actors the teacher, the student, and the content. With the content being mathematics, the extension could be used as an analytical tool (see figure 2) which allowed for the investigation of the role of PE in mathematics education (Fahlström & Sumpter, 2018).

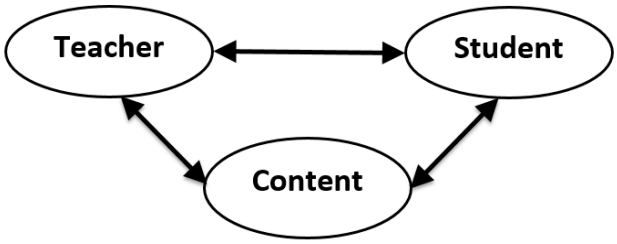

Figure 1. The didactical triangle illustrating its actors and their interactions

The idea to extend the didactical triangle was inspired by an extension with a socio-cultural perspective added to the didactical triangle (Rezat & Sträßer, 2012). Another inspiration was the problematizing of the didactical triangle by Schoenfeld (2012). In this paper a broadened view on the classroom activities related in the didactical triangle is suggested and one such view is to reflect on the classroom as a place for doing mathematics (Schoenfeld, 2012). Place is a keyword here, and aspects of the classroom as a place for doing mathematics is parallel to the roles PE can play in the teaching situations described in the mathematics teachers' stories investigated in this study.

In Fahlström and Sumpter (2018) it was suggested that the extension (see figure 2) of the didactical triangle could be used as a theoretical model to talk about, as well as to analyse, the role of PE in mathematics education, other subjects, and education in general. PE is placed in the middle in this extension where the teacher, the student, and the content remain at the implied vertices of the triangle. The pairwise interactions between the three original actors are indicated with dashed double-pointed arrows. The possible roles of PE in relation to these three actors and their pairwise interactions are indicated with solid line double-pointed arrows (see figure 2).

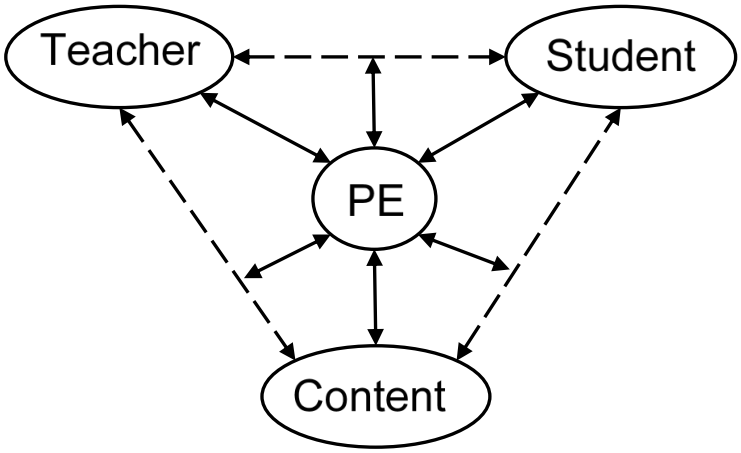

Figure 2. Extension of the didactical triangle with PE as a fourth actor

Research concerning PE and mathematics education will be presented here and related to the actors and relations in the theoretical model (see figure 2). The connection to mathematics as the content can be immediate, such as to a classroom activity like mathematical problem solving. The connection can also be indirect, as in cases where research has shown that students learning outcomes in mathematics, English, and other learning subjects are affected by the status and design of the school building (Tanner, 2009; Uline & Tschannen-Moran, 2008). Good school building status can enhance the quest for academic achievement and the forming of a student identity. The building can also inspire students' freedom to move around and explore as well as provide a secure feeling and sense of belonging (Uline & Tschannen-Moran, 2008; Uline et al., 2009). In the case of teachers, a good school building status can promote development of professional identity and contentment together with increased involvement, commitment, sense of security and raised expectations. Student-teacher interactions are also shaped in a positive way by a good school building status (Uline et al., 2009).

In the opposite case with, for instance insufficient space or crowded areas, the PE status can obstruct or limit the interactions between students and teachers. A classroom can for instance be perceived as having insufficient space due to technical equipment occupying considerable space in the classroom (Uline et al., 2010). In addition, a crowded or narrow school building can make students feel claustrophobic, distracted, and cause a sense of inequity (Uline et al., 2010). Poor school building status can also have a hindering influence on teachers. This influence can be to trigger compensation, extra effort, and innovation to improve (Johnston, 2012; Uline et al., 2010). The school building design and classroom layout shapes learning activities such as group work and

reading as well as time on task in mathematics and other learning subjects (Uline et al., 2009). Individual task solving in mathematics, mainly in silence, is more sensitive to noise disturbances than other, non-silent activities (Fahlström, 2017). Teachers consider and adjust content during the planning and preparation before teaching depending on classroom layout, configuration, and status (Johnston, 2012; Soygenis & Erktin, 2010). In mathematical problem solving, where students stand up and use vertical black or whiteboards to write on, more engaged and inclusive student behaviour can be observed. Students can see solutions or parts of solutions written on the different vertical writing surfaces. Students can also hear mathematical communication from other students and new student-content interactions evolve (Johnston, 2012; Liljedahl, 2016). The geometrical aspects of the classroom, school building, and the surroundings can serve as mathematical content in the classroom and increase the students' awareness of the built physical environment and surroundings (Soygenis & Erktin, 2010).

Recognition of the significance of PE during the educational activity can also be found in mathematics education research where PE is not primarily targeted. One such example is where student workspace and room organization are two out of nine focus elements of mathematics teaching practices (Liljedahl, 2016). Another such example is where the teacher's and the students' spatial positions and the classroom seating arrangement are included in a proposed model for social classroom climate (Kuzle & Glasnović Gracin, 2021).

Method

The aim of this study is to characterize the different roles PE play in the stories of different teaching situations told by the participating mathematics teachers. It was decided to analyse the data used in Fahlström (2017) with a different method. The 2017 study focused on what factors and effects could be identified in mathematics teachers' conception of PE in teaching. The model developed in Fahlström and Sumpter (2018) will be used as an analytical tool in this study (see figure 2). This choice of method of analysis enables the characterization of the identified roles of PE in relation to the teacher, the student, the learning content, and the pairwise interactions between them. The data set was generated through semi-structured interviews where the participating mathematics teachers were asked to describe different teaching situations related to elements in PE. The seven participating mathematics teachers taught middle school, lower secondary, or upper secondary school. They worked as mathematics teachers in different parts of Sweden, and their teaching experience ranged from one year to 18 years. During the interviews, the teachers were asked to describe teaching situations, both with and without, disturbance or distractions from elements in PE according to their interpretation. Follow-up questions were posed for clarification at times, for instance, regarding who was affected in a described situation. The interviews were recorded and lasted between 30 minutes and 60 minutes. The interview recordings were transcribed verbatim (Fahlström, 2017).

The first step of the analysis applied in the present study consisted of identifying and coding keywords in the transcripts relating to physical objects that we can touch or

physical phenomena that we can sense according to the definition of PE used here. In the following step, the text surrounding the coded keywords was examined for words relating to the actors the teacher, the student(s), or the content. The content can be explicit such as mathematics or other subject content but also implicit such as task or textbook. The indications of the teacher or students can also be explicit or implicit in the interview transcripts. When the actors had been identified, the situation could be classified as belonging to one of the six relations indicated with double-pointed solid line arrows in the analytical tool. The next step was to determine the direction of influence or purpose in the identified role, i.e., identifying who or what influences or has a purpose on who or what. This direction is represented by one of the two ends of the solid line double-pointed arrows in the analytical tool (see figure 2). The roles in the coded situations were then categorized as enabling or hindering and whether they are active before, during, or after the described teaching situations. Several of the situations described by the different mathematics teachers are explicitly not related to mathematics teaching and therefore excluded from the results. Situations related, although not exclusive to mathematics teaching, are included though.

Results

One coded role in a situation from each relational category is shown in this first part of the result section. In the role coding, (see table 1) PE – T means an influence from an element in PE towards the teacher T. An interaction between students S and content C with an influence towards an element in PE is coded as SC – PE.

Situation	Role of PE
Technical equipment working flawlessly reduces stress and the number of interrupts for the teacher	PE – T enabling before and during the educational activity
Students put their mobile phones away to avoid distractions or disruptions	S – PE enabling during the educational activity
Students' mobile phone good aid in math class	PE – C enabling during the educational activity
It is easier for the teacher to keep the order if the classroom has proper sound insulation	PE – ST enabling during the educational activity
Students makes noise when getting learning materials – teacher might avoid using them	SC – PE hindering during the educational activity
Teacher carry/transport learning materials to and from the classroom	TC – PE enabling during and hindering (tiresome) before and after the educational activity

Table 1. Examples of role coding with influence categorization

In the following part, the remaining situations relating to mathematics teaching are summarized and presented together in one of the six relational categories with an indication of direction of influence when feasible. Transcript excerpts are shown to illustrate one situation from each relational category. It should be noted that some of the transcripts contain more than one role of PE. These excerpts were translated from Swedish to English. During the translation some in-complete sentences were completed and adjusted to avoid ambiguity and to increase comprehension.

Teachers and PE

In the direction towards teachers, we find that teachers appreciate flexibility in the classroom organization, effortless access to teaching materials, and group rooms in the vicinity of the main classroom (see transcript below). Proper board lighting, air ventilation, and a classroom adapted to the teaching subject with satisfying acoustics are also something appreciated by the teachers.

> Teacher: Of course, it helps to have small group rooms that are soundproofed adjacent to the classroom. Someone [a student who missed a previous mathematics test] may need to sit and take a re-test.

In the direction towards PE, we find that teachers often take actions to improve the PE properties. For instance, reorganize seating, tidy if messy, arrange phone storage for students' mobiles, bring extra wall hangings for acoustical reasons, and open classroom windows to let fresh air into the classroom.

Students and PE

In the direction towards students, we find that tidy, uniform, and ergonomic classroom furniture increase student comfort and have a calming effect on students. The arranged phone storage, where students can put away their mobiles, helps avoiding disruptions. Privacy window tints can prevent visual distractions from outside the classroom windows. For some students listening to music on earphones can help them concentrate on the task (see transcript below). Rocker chairs can also be a good aid for some students.

> Teacher: Sometimes we also use them [students' mobiles] for students who need to have … for concentration. When we have ... it's time to work [solve routine tasks individually in students' mathematics textbooks], then they usually get to sit down and listen to music [on earphones from a mobile] … those who need it. They concentrate better then.

Content and PE

In this category PE is represented by technical equipment dedicated to subject content. Students' mobiles can be used purposely in mathematics class when specific mathematics applications are installed. Video projectors are often used to show subject content in mathematics, science, and other teaching subjects (see transcript below).

> Teacher: And this machine [pointing upwards] is something that, not the least in secondary school science classes, I have used a lot. I'm referring to the video projector in the ceiling.

PE and student-teacher interaction

In the direction from PE, we find that classroom discussions and the general classroom social climate benefits from proper classroom acoustics, good air quality, and lighting. Extra sound proofing for students with hearing impairments is beneficial for everybody in the same classroom (see transcript below). Good furniture hight and ample space between student seats promote student-teacher interaction at each student's seat in mathematics class.

> Teacher: In classes where I have students with hearing impairments in different ways, they are very disturbed by the chair scraping when someone else changes their position or moves. Now we are testing tennis balls on the chairs in two classrooms.
>
> Interviewer: On the feet?
>
> Teacher: On the feet of the chairs. It has become a completely different sound environment in fact.
>
> Interviewer: Does it also benefit those who are not hearing impaired?
>
> Teacher: It benefits everyone.

PE and student-content interaction

In the direction from PE, we find that mini whiteboards can be used by students to write on and also to show written content to other students and the teacher during mathematics class (see transcript below). There is often mathematical content displayed on the classroom main whiteboard that students can interact with as well. In the latter case it is important with proper directed board light. Students often take photos with their mobiles of the mathematical content written on the whiteboard for later recap.

> Teacher: Because I work a little with … now I do not have one here to show you … these small tablets. You know … whiteboards in mini format. So that they [the students] can write themselves. And then you need to look at what your friend has written.

PE and teacher-content interaction

The situations from the interviews in this category are indirect interaction with the content when teachers decide which teaching materials and textbooks to carry or transport to and from the mathematics classroom (see transcript below).

> Teacher: Yes, but oh, now we need this [learning material or book] … now I'm going to go and look in ... it does not work like that … I am not so flexible that … oh … that I needed that now huh, but yes, but wait I'm going to go and get it.

Discussion

Several of the situations presented in the result section are, regarded individually, not unique to mathematics classrooms. The situations in the results, regarded as a whole, are relevant to mathematics education though since they stem from stories of mathematics teaching told by mathematics teachers.

The aim of this study was to characterize the different roles PE play in the stories of different teaching situations told by the participating mathematics teachers. These characterizations were enabled by using the model developed in Fahlström and Sumpter (2018) as an analytical tool. The results are presented in the relations indicated in the model (see figure 2). In the first relation between the teacher and PE, we find specific examples of elements in PE that are enabling according to the teachers. Such examples are technical equipment working flawlessly, a classroom adapted to the teaching subject, and a roomy classroom with good air quality. In this relation, we also find teachers who make an extra effort to improve elements of PE. For example, tidy if messy or bring wall hangings for acoustical reasons. In the relation between students and PE the teacher stories predominantly contain arrangements in PE with the intent to prevent students from being distracted. For instance, some students listen to music on earphones to help them focus and to prevent them from distracting other students. These results show that teachers often devote a considerable part of their energy and attention to compensating for or preventing disturbances or distractions from insufficiencies in PE. Comparable findings can be found in previous research (Johnston, 2012; Uline et al., 2010). These extra or overhead efforts made by teachers are something worth addressing as a problem in itself, but also important to consider when planning and carrying out professional development programmes involving mathematics teachers' teaching practices. An implication of the present study is that these overhead efforts can be recognised and included in what is regarded as teaching practices and increase the potential of successful outcomes from implemented development programmes. The implication also involves interventions in the category that mainly rely on changing classroom PE, as in Cleveland and Fisher (2014), with the aim of changing teaching practices. In essence, this can be formulated as when looking to change teaching practices, always consider PE but avoid relying solely on PE for changes to take place. One example for such undertakings are the nine focus elements of mathematics teaching practices in Liljedahl's (2016) framework.

Results in the relation between PE and content contain examples of technical equipment dedicated to subject content, for instance, mobile phone applications suitable for practising specific mathematical content or video projectors to display content. Students' mobile phones are involved in several PE roles. One is mentioned above with an enabling role of mathematics applications. Another enabling role is when students use their mobile to take photos of mathematical content on the whiteboard for later recap. A third enabling role is when some students listen to music on earphones from their mobile to help them concentrate during individual mathematical task solving. A fourth role is a potential hindering role where students get distracted from their mobiles. This is often avoided by the teacher who collects the students' mobiles before class. There are no examples related to mobile phones in the previous research presented in the background. No examples of where aspects of the school building were used as mathematical content were found in the stories told by the mathematics teachers in this study. This is a suggestion for further studies since Soygenis and Erktin (2010) reported an increase in students' awareness of the built physical environment when used as

mathematical content in the classroom. Efforts in this direction could also address issues reported in this study related to teaching materials. Issues such as when teachers have to transport or carry materials to and from the classroom and that the situation can be noisy when the students bring the materials to their seats. If aspects of the school building serve as content it may reduce the need for some teaching materials.

Many of the situations described in the interviews illustrate a classroom practice with a one-to-many communication pattern between the teacher and the students. For instance, when the teacher shows content on a screen with a video projector or writes content on the main whiteboard. Such situations are often followed by individual student work. For example, the situation when some students can listen to music when it is time to work or the appreciation of good furniture hight and ample space between student seats for one-to-one teacher student interaction at each student's seat. This pattern of practice is often the norm and sustained by how classroom PE is configured. A common classroom configuration is with the main whiteboard, projector screen, and teacher position at the front of the classroom and the students seated in rows facing forward. Forward or the front, being at one short end of a rectangular classroom. There are also examples in the results indicating another type of classroom practice, or deviation from the norm practice. These examples include an enabling role of PE. For instance, teachers that appreciate flexibility in the classroom and reorganize the seating. The appreciated flexibility in the classroom organization indicates that something needs to be changed before certain activities can be initiated. This also indicates that it takes an effort to do the change from norm organization and therefor the teacher might avoid activities that needs a change of classroom organization. Another example with an enabling role of PE is when mini whiteboards are used. In this situation, students can write content on their mini whiteboard so that other students and the teacher can see what is written from a distance. In this case, PE in the form of mini whiteboards enables student-content interaction, student-student interaction, and student-teacher interaction. Results from this study indicates that certain aspects of classroom PE can be sustaining classroom norms and other elements in PE can be a help in breaking norms. If we want more of the type of classroom activities that are rich of verbal communication between students and students interacting with tangible learning materials, we need a PE that allows and enables it.

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