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Who or how many are missing? – Toddlers experiencing numerical meaning in a game

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In this paper, we present results from an inquiry into how basic number meaning can be taught in preschool through a game. The game was designed in collaboration between preschool teachers from three preschools and researchers in accordance with both theoretical and empirically founded principles. Based on video observations of teacher and child interaction (27 toddlers, 179 video recordings) when playing the game, we elaborate on how the meaning of numbers is made possible to discern and what needs to be differentiated in order to make the meaning of numbers discernible. Results show that non-numerical features play a bridging role for using the game to teach the meaning of numbers.

Background and aim

This paper is part of a larger study[1] in which researchers and teachers collaborate to develop mathematics teaching activities and educational principles for the youngest children in early childhood education (1- to 3-year-olds). There are indeed studies of preschool activities that are supposed to support young children's mathematical learning, taking an educational perspective (Gejard, 2018; Sæbbe, 2019). However, there are few theoretically grounded studies focusing on what learning outcomes are made possible. The field of early mathematics education thus seems to lack discussions about *why* learning is made possible or not through particular activities.

In mathematics education, regardless of the age of the student, a critical question is how to teach the meaning of something that cannot be seen or perceptually experienced, such as numbers. To overcome this fundamental question, representations become necessary for mediating the meaning of the mathematical object. While we can never access mathematics without representations, the representations are not to be confused with the mathematical objects they represent (Duval, 2006). Duval's (2006) critical view on representations is essential: 'How can they [learning children] distinguish the represented object from the semiotic representation used if they cannot get access to the mathematical object apart from the semiotic representations?' (p. 107). In early childhood mathematics education, and particularly concerning the youngest

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toddlers, this is at the core of the educational endeavour since mathematical objects such as numbers are truly novel to the children.

This paper concerns the learning of numbers in early years and particularly the role of representations when drawing attention to and opening up for exploration of the meaning of numbers. The object of analysis is a specially designed game focusing on cardinality and numerical relations between small quantities. Two research questions are posed: RQ1) To what extent are numbers made the focus of attention in the designed game? and RQ2) What needs to be differentiated within the game in order to make discernment of the meaning of numbers possible? These questions are answered through analysis of empirical data consisting of 179 video recordings of the specially designed game played by 27 toddlers and their preschool teachers during one year.

Learning the meaning of numbers

Children encounter numbers early in their lives when communicating with others about quantifiable phenomena. From cognitive science we know that children are able to perceive differences in small quantities in a process called subitizing (see Clements et al., 2019; Wynn, 1998) and to estimate larger amounts such as the approximate number system (Dehaene, 1992; see also Ulrich & Norton, 2019). These are intuitive abilities that support human survival, but they may also be a foundation for learning to reason about changes in quantities with cultural tools such as counting words and symbols. Such reasoning skills are today highly valued, visible even in the mathematics curriculum for 1-year-olds entering their first step in the education system (see *Curriculum for the Preschool lpfö 18*, National Agency for Education, 2019).

Fuson (1992) describes contexts in which children encounter different number meanings: cardinal, ordinal, measure, sequential, symbolic and non-numerical. The meaning of numbers differs between these contexts, and Fuson concludes that it takes many years before children are expected to identify these different meanings. Thus, the learning of numbers is complex; for example, how to interpret numbers as ordinal when seeing a sign on a house but cardinal when reading a price tag and even non-numerical when looking for the number on a bus. However, emphasizing this complexity does not reveal *how* children learn these different meanings.

How children learn the meaning of numbers is the focus of a study by Björklund et al. (2021) showing that it is necessary for children to discern and attend to the aspects of cardinality, ordinality, part-whole relations and representations in order to understand numbers in a flexible and prosperous way. Depending on the situation in which numbers appear, one or more of these aspects may be foregrounded while others are kept in the background. The first three aspects (cardinality, ordinality, part-whole relations) need to be discerned in order to determine the number of a set of objects and also in order to reason about changes in quantities and to solve basic arithmetic problems. The fourth aspect, representation, is critical since it is both an aspect of numbers itself and a means to mediate numerical meaning (see also Duval, 2006 above). For example, counting words are verbal symbols that do not in themselves have numerical meaning. Verbal

symbols are often the primary representation for pointing out numerical meaning and thus important for learning the specific difference in meaning between, for example, 'two' and 'three'. Research in mathematics education has primarily focused on representations as means to mediate mathematical meaning, but a reasonable conclusion is that in early childhood mathematics education it becomes critical to not only use representations for visualizing mathematical meaning but also to show how a representation in itself is an aspect of mathematical concepts. Recent findings also reveal that young children who make use of several representations (for example, counting words and finger patterns), whether they use them in correct correspondence with a number of objects or not, are more attentive to learning the cardinal meaning of numbers (Gibson et al., 2019). Thus, representations play a significant role in the early learning of numbers and should not be restricted to verbal or graphical signs.

Theoretical frame for teaching the meaning of numbers

The study was conducted in accordance with variation theory of learning (VT) (Marton, 2015). According to VT, when a child discerns what s/he has not previously been aware of, this changes the child's way of experiencing the phenomenon; the child has learnt a new or more nuanced meaning of the phenomenon. When teaching, the teacher therefore needs to understand what the child has not yet 'seen' and thus what teaching should afford the child to discern. However, even when taking part in the same teaching activity, children tend to learn different things because they enter the activity with different experiences of the learning object. These different experiences impact what they 'see' and how they interpret what the teacher is trying to mediate. Thus, the children also leave the teaching situation with different experiences of the learning object. In this teaching-learning process, then, it is critical for the teacher to identify what the child has not yet discerned and to understand how to enable the child to discern what is necessary in order to change his/her way of experiencing the object of learning.

A variation theory approach to knowledge, teaching and learning helps us understand why children experience a phenomenon (numbers) differently. Children's encounters with numbers are indeed novel in the early years, but the children do have *some* experiences of numbers. These might be intuitive (e.g., perceived differences between sets of objects) or semiotic (use of counting words in certain situations). When the child encounters contrasting meanings of numbers, meanings that differ from the child's understanding, new ways of 'seeing' numbers may occur. The core principle of teaching is thereby, according to VT (Marton, 2015), to deliberately choose what aspect to make possible for the child to discern and design situations making that aspect discernible. This is done by varying what is intended to be discerned, for example, representations, and keeping other aspects, such as the number to be represented, invariant. If instead the cardinality of a set is to be discerned, it needs to be contrasted against another set with a different cardinality while other aspects such as representations are kept invariant (e.g., comparing two red balls with three red balls). Designing teaching activities in preschool that facilitate this learning is a delicate task, however. Research on toddlers (Palmér &

Björklund, 2019) has shown that before focusing on the learning object, it becomes important to first make sure that the toddlers are able to discern critical structural elements of the activity, such as rules of a game, turn-taking or accepted level of initiatives. If not, there is a risk that shared attention to an intended learning object may not be achieved, with implications for expected learning outcomes.

Methods

The above-described theoretical foundation provides a framework for how to design teaching. However, it is empirically challenging to accomplish teaching in the dynamic context of early childhood education and thus also to study how learning is facilitated among young children. How children experience a phenomenon, in our case numbers, has to be interpreted through observations of their interaction with numbers. In this study, two researchers and three preschool teachers developed teaching activities during three semesters. The activities were tried out and evaluated in authentic preschool environments in an iterative process, with a total of 27 toddlers (age between 12 and 27 months at the start of the study). The activities were documented with video, used for assessment and further development but also for thorough analyses of the children's learning opportunities. One of the activities is used as data for analysis in this paper.

Design of the game

With the aim of facilitating toddlers' learning of the meaning of numbers, we took as our departure point 'Kim's game'. This choice of game was based on observations in the three preschools where the children showed enjoyment of games where they themselves and/or objects were hidden. Using activities from observations in authentic settings provided external and ecological validity, making the results transferable to the educational setting (Cohen et al., 2018). In the traditional game, a set of (different) items is shown to the participants, for instance, on a tray. The items are then covered with a cloth and some of them are removed, unseen by the participants. When the cloth is taken away, the participants are to tell which items are missing. In the study, we redesigned the game in accordance with the following principles (see Palmér & Björklund, 2019):

Picture 1. Three conditions of the game: First using different items to introduce the rules of the game (all kinds of features vary), then using the same kind of items only differing in colour (a particular feature varies) and, in the last step, using identical items (only the numbers vary).

P1) the children need to understand the rules of the game to be able to participate, P2) the game needs to initiate interaction and engagement in order to enable shared attention to a specific learning object, and P3) numbers (within the subitizing range) are made possible to differentiate as the items are presented in ways that make necessary aspects of numbers discernible (see VT described above, Marton, 2015). Kim's game was redesigned in accordance with these three principles, ending up in a three-step model. The framing of the game was kept invariant but the props were altered to enable critical conditions for the principles to come through (see Picture 1).

Theoretical principles embedded in the game

The three conditions of the game were designed to allow the children to become acquainted with the game so that they knew what the rules were and what was expected from the participants (P1, P2). Through a theoretical lens of VT, the three conditions, and particularly their ordering, should give the children opportunities to discern certain aspects of numbers (P3). *The first condition* (different items) appeals to the familiarity of the items that the children recognize, while many aspects may vary between and within the items (different in all kinds of ways). In an explorative sense, the children can associate the items to certain characters' names, songs related to the figures, or any other familiar context. *The second condition* also appeals to the child's familiarity with the items but is now restricted to one kind of item. The features of the items are limited in that the general appearance and size are kept invariant and only the colours differ between the items. The difference between conditions 1 and 2 is that in the former all kinds of features vary while in the latter there is only one feature that distinguishes the items and, consequently, the child's attention is drawn to this particular feature. Since colour is one of the basic concepts preschool children are likely to encounter in daily communication with caregivers and teachers, this is also a familiar feature that most children know the word for (or as observed in our study, often supported with sign language). In both of these conditions of the game it is possible to enumerate the items as the numerical aspect is one possible feature of the game.

Enumerating the items becomes necessary only in *the third condition* where all features of the items are the same, which directs attention to them as a composite set. It then becomes relevant to represent them as parts (one or two) constituting a composite set (a whole of three) in words or gestures when completing the task of figuring out how many are hidden.

In conclusion, by gradually narrowing the features of the items that are likely to be foregrounded in the children's awareness while playing the game, the three conditions should, theoretically, support children in attending to numbers since the three-step model allows numbers to be discerned as a significant feature.

Data and analysis

The data for analysis consists of 179 video recordings of the three teachers playing the game with the 27 toddlers. The game was played both individually and with peers, sometimes playing through only one condition but mostly through two or all three

conditions. This was an expected irregularity because the data was collected in naturalistic teaching settings. The recordings were analysed in a two-step process.

In the first step, each recording was classified according to: 1) the three conditions (Picture 1) of the game, 2) how the child completed the game (not participating, partly participating, completing the task), and 3) how the child responded to the task in the game (no response, tries to reach the hidden items, marks the spot on the table where the hidden item was taken from, verbalizes the hidden item with feature or name, uses counting words or finger patterns). In each video recording there were most often several responses identified. This first analysis revealed to what extent numbers or some aspect of numbers was foregrounded in the child's awareness when playing the game.

In the second step, the analysis concerned only the recordings where numbers were expressed through counting words or finger patterns found in condition 2 and/or condition 3,[2] a sample of 14 recordings. This analysis focused on *how* counting words were used and interpreted, thus at the centre of this analysis was not merely the use of words or gestures per se but *what variation in meaning* the words or gestures expressed.

Results

The first analysis revealed a pattern in responses that gave support to the theoretical grounding of the activity, thus answering to what extent numbers were made the focus of attention in the designed game (RQ1). This, in relation to the child's tendency to complete the task, tells us how the designed game contributes to the potential for learning about numbers. Table 1 shows that in conditions 1 and 2, where the hidden items can be described in terms of features or names, the children tend to use the 'shortest way' and interpret the game as if its goal is to find *which item* is hidden. Thus, the numerical aspect of the game is rarely coming through. The more different the features of the items are, the more likely the children are to name them (see Table 1, D). In line with the VT principles, when visual features are kept invariant (condition 3), numbers come through as a possible feature to attend to in order to complete the task. This is shown in the toddlers using counting words or fingers (E) to a larger extent than in the other conditions. Marking the spot on the table from where the hidden object was

Classification of responses	**Condition 1**	**Condition 2**	**Condition 3**
A. No response	19 (29%)	16 (22%)	24 (20%)
B. Reaching for the item	1 (1.5%)	12 (16%)	27 (23%)
C. Marking the spot on the table	2 (3%)	10 (13%)	16 (13%)
D. Naming the hidden item	42 (65%)	20 (27%)	13 (11%)
E. Using counting words or fingers	1 (1.5%)	16 (22%)	40 (33%)
Total	65 (100%)	74 (100%)	120 (100%)

Table 1. Response frequency in the three conditions embedded in the game.

[2] A single response in condition 1 was not included in the further analysis.

taken (C) may be an indication of the toddler experiencing that the game centres around a group of objects but does not induce responses indicating that the toddler is able to single out any specific relation between what is hidden and what is visible. When toddlers are reaching for the item (B), the intention seems rather to be to find the missing object, as in a hide-and-seek game, which is also familiar to most preschoolers. Response A is quite evenly found across conditions, while responses B to E show distinctions between conditions.

As mentioned, the second analysis focused only on the recordings where numbers were expressed through counting words or finger use (57 documented responses in category E distributed among 14 recordings). Table 2 shows that, among five of these recordings (b, c, e, f and k), counting words or fingers were used in both conditions 2 and 3, but most common were different ways of responding to the task in the different conditions. This indicates that there are variations in ways of experiencing numbers also when the use of counting words or fingers is observed. Such variations may be related to the toddlers' opportunities to discern the intended meaning of numbers and are therefore of interest in this educational study.

	a	b	c	d	e	f	g	h	i	j	K	l	m	n
Cond. 2	B	E	E	D/E	E	E	E	B/E	B/E	E	E	E	D	D
Cond. 3	B/E	E	E	E	E	E	A	B/E	B	D/E	E	A	E	E

Table 2. Overview of 14 recordings of toddlers' use of counting words or fingers in two conditions.

Below follows a qualitative analysis of what meaning of numbers these observations contain and what then needs to be differentiated for the game to successfully facilitate toddlers' learning of numbers' meaning (RQ2). The analysis revealed four categories of description.

Numbers are words in a random sequence

Some toddlers are observed to use counting words but without apparent connection to the items on the table or the hidden ones. A typical response to the task of finding out how many are hidden is 'one two three four'. The teacher might then ask the toddler to count the visible items again and the toddler responds 'one two three four six' (obs. g).

Numbers as words ordered in a sequence

When one visible item is added, extending a set, some toddlers are observed saying the next word in the counting sequence. Enumerating is indeed one way of determining quantity, requiring discernment of both cardinality and ordinality. However, our observations show that in many cases there is a close connection to the physical items, and particularly actions on the items (taking out one item at a time from under the cloth), which indicates that the cardinality of the counting words is not necessarily discerned. For example, the teacher asks how many are hidden, and when the toddler does not respond, she asks how many the toddler wants her to take out. The toddler says 'one', she takes one item out, asks if the toddler wants more, and when the toddler answers

'two', she takes another item out saying, 'now you have two, do you want more?', and the toddler says 'three'. Thus, the toddler is not expressing cardinal meaning in his use of counting words (obs. d).

Numbers as labels of a visible set

Some toddlers are observed to instantly say 'two' or show two fingers simultaneously as a finger pattern when the cloth is removed, indicating their experiencing the set of items in a cardinal sense and how to represent the number. However, when asked to determine how many are hidden, the hidden items are labelled with identifiable features such as colour. For example, when the cloth is removed to reveal one blue bear, the toddler says 'a bear', takes the bear in her hand, kisses it and puts it back again. The teacher asks how many are hidden in the cloth and the toddler says 'one bear, one green bear and one red bear' (obs. n).

Numbers are related

Some toddlers are observed to use numbers in a way that indicates their experiencing numbers as generalized representations of sets of items that are related: If there are two visible, one is hidden; if there is one visible, more than one is hidden. For instance, one toddler sees one blue bear when the cloth is removed, saying 'only one' and shows his index finger. The teacher confirms 'now there's one there, how many do I have?' The toddler hesitates for a moment, then says 'eh, two' and unfolds his index and long finger (obs. b). This way of experiencing numbers, as describing a relation between sets, is observed also when the toddler uses finger patterns instead of counting words, indicating that the numerical relation is discerned.

Conclusion

Based on the observations of toddlers' ways of experiencing numbers (see categories above) we conclude that different ways of experiencing numbers influence the game's potential for their further learning of the meaning of numbers in terms of discerned aspects (see Marton, 2015). *First,* if toddlers are not discerning the cardinality of numbers, they turn to labelling items such as 'mommy, daddy' or in terms of colours (often expressed in condition 2). When they encounter the task in condition 3, the toddlers find labels to help them differentiate not how many items but which items are hidden, for instance, labelling items as 'a bear'. This further indicates their experiencing the items as single items (non-cardinal, one bear and another bear), since mainly non-numerical features are foregrounded, also making it harder to discern them as composed sets.

Second, even though toddlers are able to correctly label a set of visible items with a counting word (indicating that they are experiencing some meaning of cardinality in the counting words), they are labelling hidden items with names such as 'daddy'. This helps them keep their attention on the hidden item, but the numerical aspect is kept in the background. However, in order to find out the hidden *number* of items, the relation between the visible and hidden set has to be discerned simultaneously. Thus, to solve the task, toddlers are observed to attend to different features (both non-numerical and

numerical) that help them discern necessary clues of this relation. They may, for instance, name the items with a generalized label ‘bears’, which seems to help them discern bears as constituting a composed set, but also mixing features ‘three bears and one green’ (obs. m). The quote reflects an awareness of the total number of bears as a cardinal number, but the reference to ‘one green’ reflects that the set of three is not seen as a composition of related parts; the hidden bear is described by other features. These two conclusions point to the game: even though it is theoretically well designed to bring numerical aspects of numbers to the fore, it is challenging to the toddlers, and they often foreground non-numerical features while seeming to ignore the numerical feature.

Third, a critical issue becomes which set is asked for. When numerical aspects are discerned by the toddler, it becomes necessary to differentiate other aspects as well. In the game, the focus is directed towards the original set of three items; when the cloth is removed, it reveals that part of this set remains on the table and another part is hidden. This becomes critical in condition 3, where no visible features are available to help the toddler discern the composition of the sets, which demands an awareness that there is a numerical relation to be explored. The teacher’s directed questions and actions to direct attention to what part of the set is asked for then becomes significant for the toddler’s possibility to discern the particular part-whole relation.

In sum, the ways toddlers experience numbers’ meaning does not necessarily include any numerical aspects that would allow reasoning about relations between sets (number of visible–hidden items), as in toddlers’ experiencing numbers as words in a random or ordered sequence, or as labels of a visible set. Condition 3 (identical items) will therefore not facilitate learning of number relations if the child has not discerned aspects that are necessary for making use of the game’s designed condition. This becomes critical if Kim’s game is to have the best potential to facilitate the learning of numerical relations (in terms of part-whole relations). However, the non-numerical features do seem to bridge the toddlers’ discernment of items as constituting a composite set with an emerging sense of number relations. This indicates that non-numerical features play a significant role for toddlers’ possibilities to discern the critical numerical aspects, which is why representations (particularly the features the representations bring forward) seem to be key in this kind of teaching.

Discussion

In this paper, we set out to investigate how the meaning of numbers was experienced in a designed game and what had to be differentiated in order to make discernment of numbers’ meaning possible. Duval’s (2006) view on representations for mathematics learning has inspired our investigation and brought to light the importance of taking children’s perspectives as the outset when analysing learning opportunities. Together with a VT approach (Marton, 2015) to understanding how learning of mathematical objects such as numbers may be facilitated, our analyses may shed some light on the educational dilemma that Duval raised. Representations mediate meanings that are not

evidently discernible to all learners, but through an ordered sequence of reducing irrelevant features, numerical aspects may be discerned by young children.

The inquiry presented in this paper is both theoretical in that we aimed to contribute knowledge about how mathematical meaning emerges (expanding descriptions of context-related meaning of numbers as presented, for example, by Fuson, 1992) and empirical in our attempt to show *how* this is facilitated, on theoretical grounds, in an authentic preschool activity. In conclusion, our study shows that preschool activities may facilitate directed attention to numbers, but different conditions in terms of selection of representations afford this attention to various degrees, emphasizing the need to account for the learners' ways of experiencing numbers and the task at hand.

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