SMDF

Swedish Society for Research in Mathematics Education
Svensk Förening för **M**atematik**D**idaktisk **F**orskning

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Editor – *Christer Bergsten*

Newsletter no. 9

Under the heading *Some words from...* , the chair and the vice chair of the *Swedish Association for Research in Mathematics Education*, *SMDF,* give a special welcome to international readers of this special ICME10 issue of the Newsletter of the society.

The members of SMDF, as well as other readers of the newsletter, are invited to submit contributions to the newsletter (in Swedish or in English)[1], such as shorter or long research papers, personal reports from conferences, a general discussion on some critical topic, or other writings of interest and relevance to the members of the society.

In this issue the intention has been to give the international reader a quick look at what is going on in the field in Sweden. One important issue concerns assessment of students' mathematical knowledge, an area that has a long tradition in Sweden on the national level. In the paper *Assessing students' knowledge – national tests in Sweden,* a research group from the Stockholm Institute of Education gives an overview of recent trends in this field. From another domain of research, the role of technology in mathematics education, Tomas Bergqvist, Mikael Holmquist and Thomas Lingefjärd discuss some critical issues in their report *The role of technology when teaching mathematics*. To present an overview of Swedish research in general in our field, the plenary presentation by Christer Bergsten at the MADIF2 conference in Göteborg in 2000, *Faces of Swedish research in mathematics education,* is reprinted here. During the first years since the establishment of SMDF, the international MADIF conference every second year has been the major arrangement by the society, along with seminars in a smaller format. To update on what is going on at present, some licentiate theses from the national graduate school in mathematics education are presented by Christer Bergsten and Barbro Grevholm in *New faces of Swedish research in mathematics education*.

/ Christer Bergsten

[1] Your contribution should be submitted to the editor Christer Bergsten at *chber@mai.liu.se*

Some words from …

This issue of the *SMDF Newsletter* (which is the 9th issue since the start in 1999) is the first one ever in English (though some earlier single contributions have been in English, of course), and the obvious reason for this is that ICME-10 is taking place in Copenhagen in July 2004. The fact that the Nordic countries in cooperation are hosting the tenth ICME is and will be important for the development of mathematics education in the Nordic countries.

For about five years many persons in the Nordic countries have been involved in all kinds of preparations for ICME-10. Teachers have been running development projects in order to present them at ICME-10, teacher educators have prepared programmes of different kinds together with student teachers, doctoral students have been writing papers for topic study groups, discussion groups or thematic afternoon programmes and researchers have prepared work for presentation. The Nordic Contact Committee has cared especially for the Nordic flavour of the congress and has prepared a rich programme for the National Presentations time slot at the congress. Papers have been written and reviewed. A symposium for research on teacher education has been held and documented[2]. A pre ICME-10 conference has been held and documented[3]. Books have been edited and published and journals have made special issues for ICME-10. Posters have been designed and oral presentations prepared and rehearsed. All these preparations will culminate during one single week in July. The Nordic journal Nomad, *Nordic Studies in Mathematics Education* has been revived and is open especially for Nordic doctoral students and researchers. The collected efforts of all the preparations will of course have an impact on mathematics education in the Nordic countries. A Nordic school mathematics competition, *KappAbel*, has been established during the preparations, and the final this year will take place during ICME-10[4]. A Nordic Graduate School of Mathematics Education has come to existence during the preparations among other things due to the important personal links between researchers that have been created as a consequence of the cooperation. In all these preparations members of SMDF have been very active.

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[2] See programmer at www.mai.liu.se/SMDF/Malmo3.pdf . A book of proceedings has been produced and will be available at ICME-10.
[3] See *Proceedings of the Nordic pre-conference to ICME10 at Växjö University, Sweden, May 9-11 2003*. Växjö University & National Center of Mathematics Education, 2003.
[4] See web page at www.kappabel.com

What will happen after ICME-10? It is very important to maintain the links that have been created during the preparations and to continue to cooperate. The Nordic Graduate School will certainly have a life after ICME-10 and offer opportunities for doctoral students and supervisors to meet and cooperate in a Nordic environment. The national associations for research in mathematics education will still have relations and collaboration.

Unfortunately we live in a time when schools and universities have budget problems and many more teachers and researchers would have liked to participate in ICME-10 but have not been able to get funding for their costs. Although both the Research Councils and other funding bodies have been generous many have not succeeded in getting cover for the costs. This is regrettable and very short sighted. A congress like ICME-10 with a qualified programme is a very efficient competence development for the participants. And it will, most likely, not be another offer with a key congress like ICME so close to the Nordic countries for many years.

Still, we are convinced that the relations between individuals in mathematics education that have been created during the preparations will continue and result in further collaboration during years to come. Many teachers will have relevant knowledge about activities in all the Nordic countries and continue to make use of them. Maybe the Nordic countries will also become more visible on the international scene of mathematics education.

In the last five years graduate schools of mathematics education have been created in several of the Nordic countries and a large group of doctoral students are on their way to a Ph D in mathematics education. This means that there is a new generation growing into the field and we will in a few years have access to many new doctoral theses and research papers, with PhDs active in many positions. The international community of researchers in mathematics education has shown a great interest in this development and willingness to contribute to the work. We hope that all these efforts in the Nordic countries will live up to high demands for quality of research in mathematics education and contribute to the building of the international knowledge in the field.

We wish you welcome to ICME-10 and to share with us this issue of the SMDF Newletter,

… Barbro Grevholm, chair of SMDF,
Christer Bergsten, vice chair of SMDF

Assessing students’ knowledge – National tests in Sweden

In Sweden there is no external examination when students leave secondary or upper secondary school. The teachers do all assessing and grading. The national tests are not intended to steer the teachers in their grading but to help them to assess whether and to what extent the individual student has attained the stated goals for the subject. The purpose of having national tests is also to create a basis for assessment that is as uniform as possible across the country. In this paper we describe the Swedish curriculum and the assessment systems. We also give examples of different aspects of assessment.

Theoretical background

The role of assessment in the learning process

People learn in many different ways. Learning is dependent on various factors but also on how and on what aspects one is assessed. Students focus their learning according to the content and methods in the assessment. How the subject is presented to the students in the assessment will therefore influence students’ experiences, beliefs and views in regard to the subject (see also Black & Wiliam, 1998, 2001; Gipps, 1994; Leder, 1992; Shepard, 2000). We all know from our own experience that not all teaching and assessment is stimulating and supportive for learning. Assessment is not only a “receipt” for knowledge displayed, but it also influences an individual’s learning, his/her self-esteem and confidence in his/her knowledge. Assessment, if relevantly used, can provide a great potential for learning. But what does assessment mean for the individual? The consequences of assessment can be illustrated by the following figure:

(Pettersson, 2004, p 99)

An assessment that supports and stimulates learning means that a student's knowledge is analysed and evaluated in such a way that the student progresses in his/her learning and feels self-confidence in his/her own ability (I can, I want to, dare to), instead of an assessment that leads to a judgement and perhaps a condemnation (I cannot, do not want to, dare not)

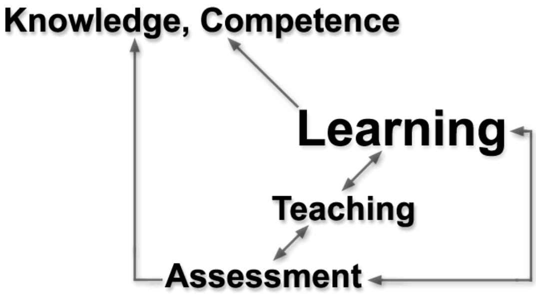
If the students are placed in a variety of situations they will get more possibilities to show their competence in different ways. A consequence of this is that the instruments for assessment should be characterised by great flexibility. In the past, solutions have been either right or wrong and scoring has been based merely on the number of correct answers. If the result of a test only gives the number of correct answers you lose detailed information about student´s performances across and within domains. (Black & Wiliam, 1998, 2001; Gipps, 1994)

Educational assessment

Gipps (1994, 2001) uses the term educational assessment, which means a wide variety of methods for testing, measuring and evaluating students' achievements. An integral part of this is also students' self-evaluation, evaluation of peers and of the learning environment. She states that this change in the practice of assessment reflects a changed view of how knowledge is constructed. Knowledge is actively constructed by the learner, and not passively received. The individual's construction of knowledge takes place through social interaction with others. Through communication with others the individual modifies his/her understanding and interpretations and this interaction is seen as a catalyst for independent development of knowledge.

Gipps (1994) argues that students ought to be active in the assessment process. The basic view is that the student is responsible for his/her own learning and thereby a participant in the assessment processes. It is important to develop an assessment that focuses both on qualitative descriptions of the learning process for the student (process assessment), and on quantitative information (product assessment). Gipps (2001) shows that assessment has played and still plays a key role in the cultural reproduction and social stratification in society.

Assessment can be seen from two perspectives. One is the summative perspective, to determine what knowledge and what level of knowledge an individual or group has on a given occasion. The level of knowledge is related either to stipulated criteria or to other students. The other perspective, the formative perspective, regards assessment as an integral part of the learning process for the individual (Erickson & Börjesson, 2001). It has been shown that formative assessment can have a considerable effect on a person's learning (Black, 2001; Black & William, 1998, 2001).



(Pettersson, 2003, p 62)

The figure above illustrates these two perspectives. On the left side, the arrow from assessment to knowledge and competence illustrates summative assessment. An example of summative assessment is the Swedish drivers' test. On the right side, the arrow from assessment via teaching and learning to knowledge and competence illustrates formative assessment, in which consideration is given to assessing the learning process for the student. An example of this type of assessment is the teacher's continual observation and documentation of the student's knowledge development, using various types of diagnostic material.

The government writes in its plan for development (1996/97:112, page 106):

> *The view of knowledge described in the curricula and expressed in various ways in the course syllabi and grading criteria provides entirely new premises for evaluating knowledge. Test items may no longer consist of "simple" measures of traditional style. They must also analyse what abilities those who have a particular education possess, and not merely list achieved rote memory knowledge. They ought to encompass problem solving, applications and combinations of different areas of knowledge. Results of learning are shown better as comprehensive competence and attitudes rather than reporting of facts. This is especially valid for the broad goals and moral value-system of the curriculum but also for the subject goals in the course syllabi. This requires new ways of constructing bases for evaluation and of analysing results of learning.*

The following considerations are presented in the Swedish curricula as being essential for students to achieve:

> *Knowledge which is durable over time; to be able to orient oneself in a complex reality with a huge flow of information; the ability to independently deal with sources of information and knowledge; to be able to critically examine facts and conditions and envision the consequences of different*

actions and decisions: to reflect upon problems and gradually develop an increasingly scientific attitude. (Utbildningsdepartementet, 1994, p 7)

The governments plan for development and considerations in the curriculum emphasise the process of generating knowledge. Another consequence is that the students must be given opportunities to demonstrate their knowledge in various ways. When we, the test constructors, develop the national test in Sweden we strive towards making the essentials assessable and not the most easily -assessed things the most essential.

Another important base in the construction of national tests is of course the ongoing development that is taking place in mathematics teaching and in international and national research in this area. To an increasing degree, countries like Australia and the Netherlands use more extensive tasks, so-called authentic problems. There are different reasons for this (Wiliam, 1994). The authentic problems represent mathematical competence better than other types of problems. It is believed that better prediction of future success in higher studies is facilitated using authentic problems. And finally, an investigative studying approach is an important part of the subject of mathematics and authentic problems in tests encourage teachers to work in a corresponding fashion (de Lange, 1992).

Steering documents in Sweden

Curriculum and syllabus

A new national curriculum for compulsory schools and upper secondary schools came into effect in the autumn of 1994 (Utbildningsdepartementet, 1994). This curriculum is criterion-referenced and it defines the underlying values and basic objectives and guidelines of the school system. In addition, there is a nationally defined syllabus for each individual subject. The compulsory school syllabi indicate the purpose, content and objectives for teaching in each individual subject. These are of two kinds: those that the schools should strive towards and those which the schools have the responsibility of ensuring that all students attain.

Examples of goals to strive towards in the curriculum for the compulsory school:

The school should strive to ensure that all pupils

- *learn to listen, discuss, reason and use their knowledge as a tool to formulate and test assumptions as well as solve problems* (Utbildningsdepartementet, 1994, p 9)

- *develop the ability to assess their results themselves and to place their own assessments as well as those of others in relation to their own achievements and circumstances.* (ibid. p 16)

Example of a goal to be attained in the compulsory school:

The school is responsible for ensuring that all pupils completing compulsory school have mastered basic mathematical principles and can use these in everyday life. (ibid, p 10)

In the mathematics syllabi for compulsory schools there are two kinds of goals to strive towards. One kind is more comprehensive and the other kind describes the mathematical content. Examples of goals that are more comprehensive:

The school in its teaching of mathematics should aim to ensure that pupils

- *develop an interest in mathematics, as well as confidence in their own thinking and their own ability to learn and use mathematics in different situations,*
- *develop their ability to understand, carry out and use logical reasoning, draw conclusions and generalise, as well as orally and in writing explain and provide the arguments for their thinking,*
- *develop their ability to formulate, represent and solve problems with the help of mathematics, as well as interpret, compare and evaluate solutions in relation to the original problem situation.* (Skolverket, 2000, p 26)

In addition to these goals there are goals that pupils should have attained by the end of Grade 5, by the end of Grade 9 and by the end of the different courses in upper secondary school. These goals are related to different areas of competence in mathematics, such as number sense and statistics.

Assessment in the subject of mathematics

From the middle of 1940 until the middle of 1990 the system of grades was norm-referenced in Sweden. At the beginning there were norm-referenced tests in Grades 2, 4 and 6 in the compulsory school. Later there were compulsory achievement tests in mathematics only in Grade 9 and in Year 3 for two of the programmes in the upper secondary school. The aim of the tests was to achieve a grading as uniform as possible among all teachers throughout the country. Teachers were expected to use the results of the tests to gain information about the average grade level and the distribution of grades in their classes in relation to classes throughout the country.

To coincide with the introduction of the new curricula and syllabi, a new system of grading come into effect in 1994. Under this system, grades are awarded on a three-level scale from the eighth year of schooling onwards. The grades are

Pass, Pass with distinction and Pass with special distinction. In the upper secondary school the grade Fail is added and the students in upper secondary school are graded after every course. The grading is criterion-related; i.e. the grades relate the knowledge and achievements of students to the criteria stipulated in the syllabus. According to the curriculum, when assigning grades the teacher should make use of all available information about the student's knowledge in relation to the requirements stated in the syllabus and make a comprehensive assessment of the knowledge acquired by the student. We have no external examinations for grading – grades are assigned exclusively by the teacher.

The assessment of the pupils' knowledge in the subject of mathematics refers to the following qualities:

- *The ability to use, develop and express mathematical knowledge.*
- *The ability to follow, understand and examine mathematical reasoning.*
- *The ability to reflect on the significance of mathematics in cultural and social life.* (Skolverket, 2000, p 29)

Examples of grading criteria in mathematics for the compulsory school:

Criteria for Pass with distinction

– *Pupils demonstrate accuracy in their problem-solving work and use different methods and procedures.*
– *Pupils follow and understand mathematical reasoning.*

Criteria for Pass with special distinction

– *Pupils develop problems and use general strategies for the planning and execution of the tasks and analyse and present their work in a structured way with correct mathematical language.*
– *Pupils participate in the arguments of others and based on these arguments propose mathematical ideas of their own.* (ibid, p 30)

The present testing system

The starting point when constructing a test is the view of knowledge stated in the curriculum, the view of the subject in the syllabus and in the criteria for the different grades. The test does not enable the teacher to determine the level for his/her class as compared with other classes in the country but it helps the teacher to find out to what extent each student has met the demands of the curriculum and syllabi. The test paper consists of several different parts in order to give the students an opportunity to show as many aspects as possible of their competence in mathematics. We, as test constructors, try to make our tests as balanced as possible with various kinds of tasks in a variety of contexts and a range of response formats. According to the curriculum, mathematical competence is much more than merely knowing certain mathematical content and skills. It is important to use mathematical strategies, models and methods within one's present knowledge and skills and also to be able to develop new skills and methods utilising a range of facts, concepts and processes. Making inferences and drawing conclusions is also necessary. It is also essential to be able to communicate one's mathematical knowledge both in written and in oral forms. The tasks in the testing material should be designed in such a way that the student is given opportunities to demonstrate different areas of competence and different qualitative levels in mathematics. It is only the regular classroom teacher who marks and assesses the students' national tests.

The following schematic schedule shows the different test materials for the compulsory school and the upper secondary school in Sweden.

Diagnostic materials Not compulsory	Pre-school, and up to Grade 6	Assessment scheme for analysis of mathematics A bank of tasks
Diagnostic materials Not compulsory	From Grade 6 up to Grade 9	Assessment scheme for analysis of mathematics A bank of tasks
Subject test Not compulsory	Grade 5	4–5 different parts + group-tasks and self-assessment
Subject test Compulsory	Grade 9	2–3 different parts + group-works or/and oral test
Course test Compulsory, see below.	A–D	2–3 different parts

The diagnostic materials in mathematics have a formative purpose and they consist of two parts, one part for use in pre-school and up to Grade 6 and one part from Grade 6 to Grade 9. Each part consists of an assessment scheme for analysis and a bank of diagnostic tasks. The purpose of the materials for analysis is to help teachers to analyse and document the student's knowledge in mathematics. The same scheme is to be used for students of different ages. By using this scheme of analysis the development of a student can be traced over several years. Students are allowed to express their knowledge in different ways:

actions, pictures, words and symbols. Three different areas are focussed upon in the first scheme of analysis: “measuring and spatial sense”, “sorting, tables and diagrams” and “number sense”. The second scheme for Grades 6–9 focuses on four areas: ‘Measuring, spatial sense and geometrical relations’, ‘Statistics and probability’, ‘Number sense’ and ‘Patterns and relations’. This structure is intended to make it easy for the teacher to navigate in the scheme and is not to be considered as a lesson plan. The order of the various parts of the scheme is not an indication of the degree of difficulty of the content of different areas of knowledge.

At the end of Grade 5, national tests in mathematics are held. These tests are not compulsory. Their main purpose is to assist the teachers in assessing whether the pupils have met the demands of the curriculum and syllabi. This purpose is summative but the tests also have a formative purpose. The teachers are supposed to assess the test holistically and not use scoring points. It is important to analyse how the pupils solve the problems and to examine the quality of their work in order to identify their strengths and weaknesses in mathematics. The teachers are then supposed to consider both their assessment of the pupils’ work in the subject test as well as their overall assessment of the pupils’ mathematical knowledge. To describe each pupil’s mathematical knowledge the teachers can use a ‘Competency Profile’. With the help of the profile, the teacher can gain a more balanced picture of the pupils’ knowledge in mathematics. In the test material there is also a scheme for self-assessment. How the students assess their own knowledge is not a part of the summative assessment, but can be an essential part of the formative assessment.

The national tests in the upper secondary school and the subject test for grade 9 have summative purposes. The national subject test in mathematics for Grade 9 is compulsory for all students. The main purpose of the test is to help the teacher assess to what extent the students have reached the goals set up in the syllabus and to provide support for teachers in setting grades. The test paper consists of several different parts in order to give the student an opportunity to show as many aspects as possible of her/his competence in mathematics. The student can show this by different ways of working (individually or in pairs/groups) and by different ways of answering (oral, written answers with or without explanations and calculations). The teacher should use different ways of marking/assessing (right/wrong, scoring rubric and analytic scoring). Two parts are more traditional in nature and in the first part no calculator is allowed. The third part consists of one more extensive task and the fourth part is often an oral test.

In the upper secondary school we have national tests for every course, A–D. The Course A examination is obligatory for all while the other course examinations (B–D) are obligatory only for the final course in a given program. In all these tests there is a variety of kinds of tasks. All tests have one rather extensive task and in one part no calculator is allowed.

Aspects of assessment

An important element in qualitatively assessing pupils' knowledge is analysing, through various tasks and situations, how the pupils work with and master an area of mathematics. We examine whether the pupils have tried to find a solution to the task, how well they have understood the task, which concepts they have displayed and in what ways the pupils have dealt with the task.

Concepts and misconceptions

The pupils can work on tasks in many different ways. We have seen that those who arrive at correct results may have used different strategies, either ones that are dependent on the context, or ones that are more general. The pupils who arrive at incorrect results may display errors, which are *accidental* in nature, that is, they are not found systematically but are of a more random character. There are also errors, which are *systematic*, that is, they occur consistently. This often indicates deficiencies in grasping concepts. There are many possible reasons for reaching incorrect results. The pupil might have misunderstood the task, or might have understood the task but used an incorrect method or might have understood the task, used an appropriate method but made a miscalculation (Pettersson, 1990).

There are different components to be assessed in mathematics. We examine knowledge of facts, knowledge of skills, knowledge of concepts, "higher order skills" and the pupils' conceptions about mathematics and of their own learning.
We have found that some pupils have difficulties with numbers in fractional and decimal form and cannot reach the goals which the pupils are to have attained by the end of Grade 5 (Alm, 2004): "Pupils should have a basic understanding of numbers, including natural numbers and simple numbers in fractional and decimal form." (Skolverket, 2000, p 28)

In the test of year 2000 for Grade 5 there are two tasks involving fractions. The first is: "Amir is in school for a quarter of a day and Linda for a third of a day. Who has the longest school day? Write down how you worked out your answer." Seventy-five percent of the children in Grade 5 solve this task correctly. The most common way they use to show their understanding is by pictures (38 %), then by calculations (29 %) and also by explanations with words (6 %). The most common misconception, for 8 % of the pupils, is that 1/4 is greater than 1/3 because "4 is greater than 3" or "4 is more than 3". The pupils look at the denominator as a whole number. (Alm & Björklund, 2000)

Self-assessment

The curriculum states that each student should develop the ability to critically evaluate his/her own results and place his/her own results in relation to his/her own achievements (Utbildningsdepartementet 1994). In the test for Grade 5 there is a section entitled "You and mathematics", in which students are to judge how confident they feel in various situations where they use mathematics. The answer alternatives are: "certain", "quite certain", "uncertain", "very uncertain". The students start with this part and are to evaluate themselves without referring to their answers to the testing materials. A comparison can then be made with their results on similar tasks on the test and thus provide a basis for knowledge about the student's self-confidence in using mathematics and how realistic this degree of confidence is – which, as mentioned earlier, is a part of the formative and not the summative assessment. Here are the situations where the greatest proportion of the students indicated the alternative "certain" in the test:

"feel certain"	***1996***	***1997***	***1998***	***1999***	***2000***	***2002***
In calculating 8 – __ = 3	*88 %*	*86 %*	*88 %*	*81 %*	*87 %*	*86 %*
In calculating with your calculator	*76 %*	*75 %*	*75 %*	*89 %*	*86 %*	*87 %*
In looking at the newspaper and finding out how long a TV program lasts			*80 %*	*75 %*	*79 %*	*76 %*
In working together with someone.	*68 %*	*69 %*	*71 %*	*70 %*	*74 %*	*72 %*

(Alm, 2002, p 14)

Many teachers use the section "You and mathematics" in conversation with students and even with parents in a so-called "Progress discussion". At least once per term the teacher, the pupil and the pupils guardians must have a meeting to discuss how the pupils progress in learning and social matters can best be supported. Self-assessment as an important part of the learning process for each student is emphasised in Sweden (Skolverket 2003), and can be seen in the following quotation: *"Students who think about and discuss their progress toward the achievement of class goals and the learning of mathematical concepts, on the basis of the evidence they see in their own work, will build better understanding and control of their own success"* (Stenmark, 1991, p 56).

Assessing an extensive task

A distinguishing feature of more extensive tasks, often placed in authentic contexts, is that they are evaluated and assessed using methods different from those used for ordinary tasks. The teachers assess/evaluate these tasks with the support of assessment matrices.

More than other tasks these more extensive tasks assess the students ability to do independent work, to be creative, to systematise, to carry out mathematical reasoning, to create mathematical models, to formulate and test assumptions, as well as to draw conclusions. A requirement for these tasks is that they give all students the opportunity of at least starting a solution, while at the same time the task should be sufficiently challenging that a good solution can show qualities at the highest-grade level. It should be possible to assess the students' entire solution on different qualitative levels.

Example of an extensive task

On the next page we present a task from the national test for course A in the upper secondary school. Course A, which is compulsory for all students, is the first mathematics course in upper secondary school. This task was given in May 2000, just a few weeks before the opening of the Öresund Bridge and the information about the fares was given in many Swedish newspapers. The intention was that the students should use about one hour to work with this task.

By car across the Öresund Bridge

There will be tolls for travelling by car across the Öresund Bridge. The traveller may choose between several different alternatives to pay the toll fee. These have been set up to suit all types of travellers.

Information about car fares across the Öresund Bridge

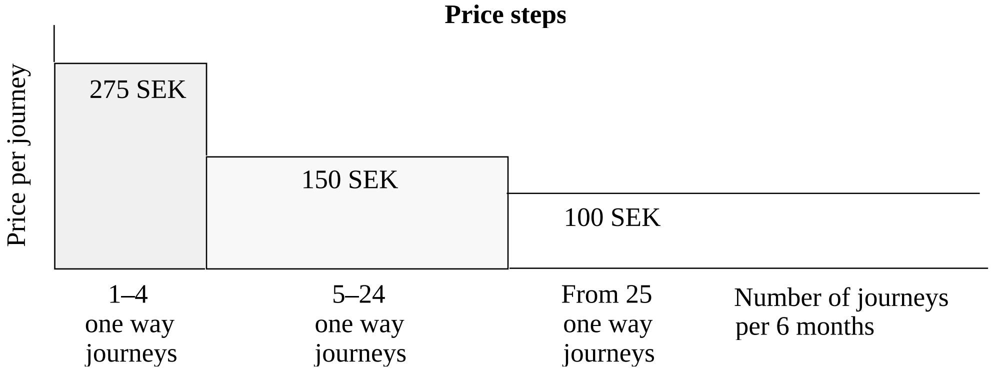
Normal price

The normal price for a one-way journey by private car is 275 SEK (Swedish crowns) but there are two different possibilities of getting a reduced price.

Öresund bonus

With an Öresund bonus agreement it gets cheaper and cheaper after the first four one-way journeys.

Price steps



The agreement is valid for an unlimited number of journeys during *a period of six months*. At the end of the period one starts again and pays the normal price for the first four journeys.

Öresund commuter

The Öresund commuter may sign *a monthly agreement*. It costs 4 080 SEK for a private car per month and includes up to 50 one-way journeys.
Nothing extra is charged for signing an agreement. The deposit one pays for the "Brobizz" (the electronic identity card) is refunded when the card is returned.

Exercises

Imagine that you have got a summer job in an information booth at the Öresund Bridge. There you are to help travellers to interpret the information (see page 2) and you must also be able to suggest the cheapest alternative for payment.

❖ Prepare for your job by *doing calculations and/or making diagrams*, which can help you to quickly suggest the cheapest alternative for different types of car travellers, for example:

- the traveller who goes across only occasionally, for pleasure
- the shopping traveller, who regularly goes across for shopping
- the commuter who lives in Malmö but works on the other side of Öresund.

❖ Explore also how many journeys you have to make per month or per six months to make the Öresund commuter alternative advantageous for you.

The development of the assessment matrix

This kind of task was first used in the national tests in 1995 and in the assessment guidelines the teachers were advised to make holistic assessments of the students' work. To aid the teacher in assessing and grading there were descriptions of students' work at different achievement levels as well as authentic students' work, graded by researchers and a group of teachers (Kjellström & Olofsson, 2001).

We have a long tradition of teacher-assessed national tests in Sweden and teachers are accustomed to having detailed assessment/scoring guidelines. The holistic approach in assessing students' performance turned out to be difficult for the teachers and some of them found it to be subjective and unfair to their students. Research carried out concerning examinations has shown that essay-type questions are less reliably marked than structured, analytically marked questions (Murphy, 1982). The greater the amount of structure in the marking scheme the more likely it is that two people will agree on the marking. This is also supported by research which shows that even for performance-based tasks, when assessors are trained and scoring rubrics are provided, inter-inter reliability can be high (Brown, 1992; Shavelson *et al*,1992).

We started looking for other assessment models than holistic ones and found among others analytic scoring scales. Analytic scoring is particularly useful in assessing students' problem-solving efforts. When you use analytic scoring you look at the different phases in or aspects of problem solving. An analytic scoring scale includes specific criteria for awarding partial credit for each aspect or phase.

Communication, which is very important in our grading criteria, is not included in analytic scoring. In Romberg (1992) we found another way of assessing problem solving in Vermont's work with assessing portfolios using a matrix. This matrix was the starting point for the development of our first general assessment matrix. We organised the different aspects of the problem-solving process according to our syllabus in mathematics and the grading criteria. The quality levels were described in three levels according to the criteria for different grades. One of the purposes of the assessment matrix was also that it would be possible to use it independently of mathematical content and in different courses.

After the first test, for which the assessment matrix was used, we evaluated teachers' opinions in a teacher questionnaire. About 50 percent of the teachers answered that the matrix facilitates assessment and that it was more faire to the students than holistic assessment. Many teachers thought, however, that the assessment was time-consuming but 50 percent of the teachers answered that it was worth the effort but they wanted more detailed matrices (Kjellström & Olofsson, 2001). We always pre-test all tasks for our national tests. For the tests in 2000/2001 and onwards we have developed task-specific matrices for each more extensive task (see page 12). The first step in the process of developing a matrix for a task involves an in-depth analysis of the students' work. We started with the general assessment matrix and with the help of this analysis we made task-specific descriptions in the matrix. Together with the matrix we also publish authentic student work assessed and graded by researchers and a group of teachers. From the teacher questionnaire we know that almost all teachers think that assessing/marking with a task-specific matrix is worth the effort and that it facilitated the marking.

The purpose of the matrix is also to show the students the different aspects of knowledge that can be assessed as well as to describe the different qualitative levels within each knowledge aspect. The students as well as their teachers can find examples of more extensive tasks as well as assessment matrices at our web page (www.lhs.se/prim).

Assessing oral communication

The oral test for Grade 9 is carried out in groups of 3-4 students who work well together. Our intention with this is that there should be a conversation between the students with as little interference as possible from the teacher. The teacher selects the groups. It is quite a challenge to construct problems and make assessment directives for the oral part. The aim is to construct problems that are more suitable for oral communication than for written solutions. The exercise consists of two sub-sections. First every student gives an account of "her/his problem" to the other students in the group. Then the whole group discusses all the problems.

One of the models we have used

Model 1 – Similarities and differences (two examples)

Every student gets a sheet of paper with *one pair* of figures and is then given approximately five minutes for preparation. Figures can be chosen which are compatible with the capacity of the student. The students report in turn to the other students in the group. To help the student to advance the teacher might insert short questions. After each report the other students in the group are invited to ask questions, make comments and additions. Finally all figures are put on the table and the students are invited to discuss together similarities and differences between as many figures as possible.

The different pairs of figures are of varying difficulty to compare. In assessing the student's performance one should therefore take into account what pair of figures the student has compared. This exercise determines whether the student knows how to describe and compare geometrical figures and to find similarities and differences between the figures for example concerning perimeter and area. The exercise also tests to what extent the student can use mathematical language, see relations, argue and accept arguments from others. The teacher should make a holistic assessment of a student's performance with the support of a task-specific assessment matrix.

Task-specific assessment matrix for "Similarities and differences"

The assessment concerns	**Qualitative levels** **Lower** ⟶		**Higher**
Understanding *To what extent the student shows an understanding for the task and motivates his/her conclusions* *To what extent the student uses relations and generalisations*	Shows understanding for either of the concepts perimeter and area, i.e. by describing how you can calculate perimeter or area for some of the figures.	Shows a good understanding for both the concepts perimeter and area, i.e. by explaining how you can see that perimeter and area are the same/ not the same for the two figures.	Shows a good understanding for both the concepts perimeter and area and relations between them. i.e. by carrying out general reasoning and extending the problem.
Language *How clear and distinct the student's account is.* *To what extent the student uses relevant mathematical language and terminology .*	Comprehensible and possible to follow.	Easy to follow. Uses acceptable mathematical language and terminology.	Well-structured and clear with relevant mathematical language and terminology.
Participation *To what extent the student participates in discussions with mathematically based ideas and explanations.*	Accounts only for one's own figures.	Contributes with his/her own ideas and explanations after explanations of others or in the final discussion.	Is involved in the arguments of others. Carries the discussion forward.

Teachers and assessment

We have studied teachers' assessments in the national tests in Grades 5 and 9 and also in the national test for Course A in the upper secondary school.

For Grade 5 we have analysed competency profiles and the teachers' assessments of some student solutions to the tasks. This data has been examined both qualitatively and quantitatively. We have analysed the teachers' answers to a questionnaire as well. We find that that most teachers in the survey do not verbalise their assessments of pupils' knowledge in the various areas of the subject. Many teachers that in fact do make verbal comments on their assessment reports tend to focus on weaknesses of the pupils. Another conclusion is that most teachers in the survey, with the help of a subject test in Grade 5, assess their pupils' performance on individual tasks satisfactorily. (Björklund, 2004)

From the tests for Grade 9 and Course A (2001) we have chosen some tasks, which require complete solutions. We have randomly chosen one hundred student solutions of these tasks and reported the results of the teachers' assessments. Four teachers have then re-marked these solutions according to the marking guidelines for the test. Teachers in Sweden often believe that there are greater discrepancies between teachers marking of a more extensive task than of more "ordinary" problems. In comparing the scores from different kinds of tasks we found, of course, that there is some variation between teachers marking. But this variation or divergence is not greater for more extensive tasks than for "ordinary" tasks with the same number of available scoring points. The variation depends more on the fact that some teachers consistently mark all solutions more severely than others do. What we found is confirmed by Gipps (2001), who writes:

> *We are social beings who construe the world according to our values and perceptions, thus our biographies are central to what we see and how we interpret it. Similarly in assessment, performance is not 'objective'; it is construed according to the perspectives and values of the assessor, whether the assessor is the one who designs the assessment and its 'objective' marking scheme, or the one who grades open-ended performances.* (p 36)

In analysis of teachers' answers to the teacher questionnaires that are included in the test materials, we find that most of the comments on the various parts of the tests are positive. Some teachers express their opinions like this:

> *"They require a lot of work but are very good."* (Alm & Björklund 1999, p 12)
>
> *"The test deals with all areas and the students are able to demonstrate what they know in many different ways."* (Kjellström 2002, p 19)

”The preparation and analyses afterwards are very time consuming.” (Alm & Björklund, 1999, p 13)

”They reveal individual strengths and weaknesses for each pupil but they also show which topics and areas I need to give more attention for the whole group.” (Alm, 2003, p 12)

”I understand better now, than I did some years ago, what the purpose and intention of the test is.” (Olofsson, 2002, p 37)

”I think that the test provides good assistance in setting grades. The result does not deviate much from my own assessments of the pupils.” (Olofsson, 2003, p 36)

”The test reflects well the view of the subject as expressed in syllabi.” (Kjellström, 2003, p 15)

Teachers are generally pleased with the structure and content of the national tests. The teachers feel that the tests give them support in assessing the performances of the students and that the assessment guidelines provide sufficient help. (Alm & Björklund, 1999; Alm, 2003; Kjellström, 2002, 2003; Olofsson, 2002, 2003)

The Swedish system

As described earlier, the teacher has the exclusive responsibility of assessing the students’ knowledge throughout all school years in Sweden. Since we have no external examinations, the situation concerning assessment is rather unique in Sweden, in comparison with other countries,

Of course there are both disadvantages and advantages with the Swedish system. For the teachers, assessment of their students’ knowledge takes a lot of time to perform and when it comes to assessment and grading, the Swedish students are in the hands of their teachers. As a matter of fact we can’t be sure that there is adequate agreement between different teachers’ assessment and grading. Thus it is important that the teachers have sufficient competence to carry out the assessment satisfactorily. One means towards this end is to provide more time for teachers to do assessment and above all, time for discussions about assessment. It has been shown that the quality of assessment is enhanced if teachers discuss assessment in concrete terms with each other (Gipps, 1994). Professional development of teachers in the area of assessment would also be a desirable measure. In the Swedish system the teachers are expected to do proper assessment, one might say that the teachers are relied upon for assessment. Our contacts with teachers at several lectures and courses, and also in thousands of questionnaires, have given us the impression that most teachers really want to do a good job concerning assessment of their students' knowledge. With the teacher as assessor the student has many opportunities both in the testing situation and in teaching situations to demonstrate her/his abilities. One underlying purpose of the national tests is to reflect the values and ideas described in the national curriculum and syllabus. In this way the national tests in mathematics can stimulate discussion about assessment in general and about the teaching of mathematics in particular. If the national tests have any influence on the teaching of mathematics, hopefully it is in a direction that gives more students the opportunity to experience mathematics as meaningful and interesting.

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The role of technology when teaching mathematics

Introduction

The availability of technology has changed enormously during the last 20-30 years. From the large and clumsy desk calculators of the 1970's over to the so called functional calculators in the 1980's and during the last 10-15 years we have got graphing calculators as well as symbolic calculators. There are also calculators which include software for geometrical constructions on the market today. If we shift to computers, the availability of software for all kinds of undergraduate mathematics is enormous, both commercial and freeware is easy to get. This significant change in availability naturally raise questions concerning mathematics education at all different levels, questions concerning content, teaching & learning, and assessment.

Most curricula around the world probably include technology at different levels in some way. Here are some excerpts from the Swedish compulsory school (our translation):

> *The teaching of mathematics should strive for that students with familiarity and sound judgment can see and use the potentials of calculators and computers* (Utbildningsdepartementet, 2000, p. 15, our translation).
>
> *Students should develop their knowledge about how mathematics is used within information technology, as well as how technology can be used in problem solving, in visualizing, and when investigating mathematical models.* (Skolverket, 2000, p. 2, our translation).

The use of technology in mathematics teaching at the college and university level today is harder to describe. Most university students at Gothenburg University seem to have calculators from their upper secondary studies, as well as easy access to computers and Internet both at campus and in their private environment. Many study halls at the university include the possibility to make wireless network connections and since many students carry laptops with them in school, it is almost as if these students have instant access to tools for calculations and visualization. This is probably true for most university students in Sweden and in the Nordic countries.

A survey of the use of computers in undergraduate mathematics education was done at the end of the 1990s (Bergsten, 1999), showing that, at that time, computers were used in about half of the math courses. A computer lab was the dominant format, assigned tasks and projects less common, and demonstrations in class by the teacher rare. Reasons for using computers in math courses were pedagogical, information, pragmatic, vocational, and to foster computer knowledge. Lack of resources, ineffectiveness, lack of appropriate software, not needed, or tradition were given as reasons not to use computers in the course. Handheld calculators were allowed on exams in a third of the courses, in basic calculus more or less forbidden, by pedagogical or pragmatic reasons. During the three years before the study was conducted the computer use was doubled, indicating that the situation today has already changed significantly in relation to the survey reported.

It is obvious that in the Nordic countries the access to technology is not the problem, as it very well might be in other parts of the world. It leads to the following educational issues to be discussed:

- Why should I use technology when teaching my part of the mathematical curriculum?
- What kind of technology should I use when teaching my part of the mathematical curriculum?
- How should I use technology when teaching my part of the mathematical curriculum?

These issues will raise new issues, like the following:

- Is it necessary to know exactly the same content today, compared with the days before modern information technology?
- What can be learnt instead?
- What can be added into the curriculum as a result of technology?

We do not intend to pretend that these issues have any final solutions, but we do intend to discuss some central implications in this article. In this paper some of these issues will be discussed from a Swedish perspective, with a focus on possibilities and limitations of modern information technology within school mathematics.

Technology in the mathematics classroom

The use of technology when studying mathematics is not a new issue, since humankind always has been looking for solutions to avoid time consuming routine work. Maybe the definition of routine work has changed, since the implementation of modern computers. Today we can not only get help with long and complicated calculations, we can also use computers and modern software to simulate and model complex situations described by mathematical structures.

The views on how information and communication technology can be used to support learning of mathematics have changed over time. When the new curriculum for the Swedish compulsory school was implemented in 1980, there was a new subject introduced called computer science. Mainly teachers of mathematics, Swedish and social science were involved in this work. Since the 1980's, the importance of computer support in the teaching and learning of mathematics has been emphasized more and more. It can be seen in the steering document for the curriculum we have for the present Swedish compulsory school. The view on how technology should be used in the mathematical class-room is also affected by other forces, as for instance strong expectations that our present schools should mirror what goes on in the surrounding society in this respect.

> *…there are expectations from the research society as well as from school politicians that the technology may affect the teaching and learning in mathematics in a positive way. The computer supported teaching of mathematics is still infantile. The possibilities and conditions for teaching that the technology bring does not always meet appreciation from teachers, who probably are far too busy with all issues involved in their everyday teaching.* (Samuelsson, 2003, p. 223, our translation).

In this context it is highly relevant to ask the question about how the teaching and learning of mathematics may be affected when modern technology is available. When it comes to computer support, Samuelsson (2003) addressed the role of the computer as an agent of change and focused on the following issues:

1. What new methods will be used by the teachers because of the technology?

a. When teachers use technology to introduce the concept of the equation of a straight line ($y = kx + m$), the teaching will be different compared to an ordinary introduction. The students will be able to easily investigate how different values for the parameters will affect the graph of the function.

b. The students learn more mathematics through visualization and models in the computer.

c. The students will learn effective procedures that help them to solve problems more efficiently, mainly due to the element of game and competition offered by the computer.

2. What new goals/results will be in focus for the teacher thanks to the possibilities offered by technology?

d. The students will be able to learn to handle technology in different mathematical situations.

e. Different aspects of knowledge can be acknowledged, for instance in statistics and algebra, if technology is used when students solve mathematical problems.

3. What in the everyday technology-enriched teaching of mathematics supports the overall visions that exist for school mathematics?

f. The student's views of mathematics may be affected in a positive way if the teaching is pursued with technology

g. Learning oriented teaching with utility programs seems to support the vision that the teacher should focus more on conceptual learning and less on the learning of skills and procedures.

h. See answers a, d, and e.

4. What in the everyday technology-enriched teaching of mathematics interfere with the overall visions that exist for school mathematics?

i. To work with drill and practice software supports an old form of teaching mathematics, which the modern school is leaving behind. The technology could thus be assimilated into old traditions.

j. The students' possibilities to reflect over the content in the problems are reduced by the element of game and competition offered by the computer.

k. Technology offers a variety of different distractions, which results in students doing other things than mathematics in the classroom, surfing on the web, checking e-mail, and so forth. Technology becomes amusement.
(Samuelsson, 2003, p. 221-222, our translation).

What we have here is evidence that today's questions are not about the favor or disfavor of technology. Instead, it takes a modulated attitude toward what possibilities and obstacles that follows from the use of modern technology in the teaching of mathematics.

> *…a further discussion leads to the question about how well a specific content in mathematics defends itself when using modern technology to illustrate, calculate and demonstrate solutions and results* (Lingefjärd & Holmquist, 2001, p. 298, our translation).

This issue concerns the fact that facilities of the kind that modern technology constitutes raise different demands on tasks and problems that the students work with. A positive interpretation of this is that it can lead to possibilities to stimulate and enthuse students with new challenges beyond what we consider natural. In other words, today it is not the access of technology that limits the use of technology in the teaching and learning of mathematics, instead it is the views and opinions with the user.

> *The usefulness and thereby the users importance and influence when addressing the results of technology supported teaching has not been acknowledged enough, thereby most likely one of the reasons why there exists*

so many different perspectives about the meaningfulness of technology-enriched learning. (Farkell-Bååthe, 2000, p. 159, our translation)

An important group in the process of using technology in the teaching and learning process is inevitably the teachers of mathematics in our schools. At the same time as the responsibility for computers at schools has shifted from the mathematics teacher who mastered the schools computers in the 1980's over to quite other groups of teachers today, the use of technology among students and people outside school has increased enormously. So the use of technology to do for instance calculations are today much more present outside school than inside. Inside modern schools, the use of technology for search on the Internet, word processing and a variety of communication techniques has increased (KK-stiftelsen, 2003). This change over the last 20 years emphasizes the importance of running a continuous discussion of possibilities and obstacles with the use of technology when teaching, learning, and assessing mathematics.

Benefits from handheld graphing technology

In a large survey from Michigan State University (Burril, 2002), one of the central findings is the following:

Given supporting conditions, the evidence indicates that handheld graphing technology can be an important factor in helping students develop a better understanding of mathematical concepts, score higher on performance measures, and raise the level of their problem solving skills"(p. i).

The improvement in problem solving skills consists mainly of a larger set of tools for problem solving, and a higher tendency to explore the problems and to use graphical solution methods. However, it is also said in the survey that the gains in student learning when using graphing technology is a function not only of the existence of the technology, but also of how it is used in the classroom.

The use of the technology is central. What can be done with graphing technology that cannot be done without it? One issue is of course the possibilities for visualization of concepts. This is illustrated in a quote from Pommerantz (1997):

In the past, students studied advanced mathematics (calculus) to learn how to draw graphs. Now computer-generated graphs can be used to study important mathematical concepts.

Turning the teaching upside down that way is a serious clash with the traditional way of teaching calculus. The teacher must change her or his approach from an algebraic to a teaching, which is more based on visualizations and illustrations of concepts. If the teacher is interested in using the benefits from the technology, that is. There is a danger involved here. If the technology is available but not a part of the teaching, the students might be led into using it only to carry out routine arithmetic calculations, something that might have a negative impact on their learning. All the authors of this paper have witnessed students looking for their calculator when asked to d simple arithmetic.

It is also important to be aware of the differences between different kinds of technology. In Swedish school mathematics, handheld technology is predominant. These are mainly of two types, graphing calculators or scientific calculators. For educational purposes graphing calculators are much more useful and student-friendly than scientific calculators. The following two simple examples will illustrate this:

1. A scientific calculator has a button marked with an equality-sign. The meaning of the equal-sign then becomes "calculate this". A student which adapts that meaning might encounter difficulties when confronted with an equation which has x on both sides, since there is nothing to calculate.
2. On the graphing calculator, sine and cosine functions are written in the same way as by hand, cos(60) is entered exactly that way. On a scientific calculator you press 60 and then the cosine button. This might further confuse students who have a tendency to mix up the function and the argument.

Swedish research about the use of technology when teaching mathematics

There are several Swedish studies who have addressed the question about using technology when teaching mathematics quite differently. In 1986 Lars-Erik Björk and Hans Brolin (1995) initiated a long term research project, the ADM-project (Analys av Datorns konsekvenser för Matematikundervisningen), where tool-kit computer programmes were used to enhance the conceptual understanding of functions. The time the students practiced routine skills was reduced, and the students were instead given the opportunity to use a computer software, "Matematikverkstad" ('the mathematics tool kit'). The programme made it possible for the students to work with functions, numerical solutions of equations, calculation of integrals, and problem solving on different levels. At the end of the project, a test was used to evaluate the students' progress. Compared to a reference group of students, the test classes had significantly better results on tasks concerning conceptual understanding and on problem solving tasks, while no difference could be found concerning routine skills.

Dahland (1998) did research on the connection between the technology, the students, and the teacher. Research on the use of graphing calculators and possibile consequences for learning mathematics has been reported by Dahland and Lingefjärd (1996), with focus on students’ use of technology when solving problems. In a study by Bergqvist (1999), students were encouraged to use investigations on a graphing calculator in order to better understand the factor theorem by graphing a quadratic function together with the two linear functions of its factorisation. Some evidence was found that students could benefit from investigations on a graphing calculator when trying to understand factorisation and the factor theorem. Another result was that students could get a better understanding for the connection between graphical and algebraic representations of functions.

Lingefjärd (2000) presented his doctoral thesis based on three consecutive studies that were conducted to investigate prospective mathematics teachers' understanding of mathematical modeling when using technology to solve a variety of problems. The intent was to develop a framework for exploring the students’ difficulties with mathematical modeling by observing and interviewing them in the context of a regular, if unique, course on mathematical modeling. The framework illustrates how different sources of authority as well as conceptions and misconceptions of mathematics and mathematics modeling play different roles in the mathematical modeling process. Technology acted both as a tool and as a source of authority in this process. A transformation of authority took place in the first weeks of the course so that the students became uncritical of the results they got from the computer and graphing calculator. This happened despite the fact that they had been urged to be very cautious when using software to select models (*ibid*.).

The use of computers for visualizing and studying geometrical objects was a vital part of early research on students of age 11-12 years using Logo programming, made by Hedrén (1990). By an experimental design it was found that working with the software did foster an active, exploring and investigative style of learning resulting in improved knowledge in arithmetic and geometry. It was also found that the role of teacher was critical for this. Further on the research has been more focused on students’ use of tools for dynamic geometry, software like The Geometer’s Sketchpad and Cabri-géomètre (Lingefjärd & Holmquist, 1997; Holmquist & Lingefjärd, 1999). Research on how new tools for studying, learning and teaching geometry influence mathematics teachers’ education has also been reported by Lingefjärd and Holmquist (2003).

Taken together, these studies provide empirical support for possible benefits of using technology in the mathematics classroom, at different age levels, but also point to inherent risks and limitations.

Possibilities and limitations

Besides the possibilities and limitations that exist when using modern technology when teaching and learning mathematics, there might also exist possibilities and limitations in the external constraints that decide what - and shapes how - mathematics should be taught.

> *Several teachers in the Gymnasium have ideas about different content and different teaching methods, which they would like to develop and try out. Examples are for instance to develop lessons where graphing calculators or suitable software are used to analyze functions, to develop ways to extend students knowledge about the language of mathematics, or to start collaborative work with social science and history in order to create statistical investigation and Gallup surveys. The major obstacles for such a progress are that it would steal too much time from the courses in mathematics, according to the teachers. Especially more advanced courses in mathematics are considered to be inflexible in terms of content and under constant time pressure* (Skolverket, 2003, p. 45, our translation).

The complexity in using modern and advanced technology in mathematics education implies that this is an area in need of further research about different issues. There is also a substantial need of development concerning different ways to train and educate all those who are involved in decisions about and realization of education in mathematics.

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/ Tomas Bergqvist, Mikael Holmquist and Thomas Lingefjärd

Faces of Swedish research in mathematics education[1]

Introduction

The recent appearance of international research reviews (Grouws, 1992; Bishop et al, 1996) as well as national (e g Arzarello & Bartolini Bussi, 1998; Blum et al, 1992; Douady & Mercier, 1992; Kieran & Dawson, 1992) in the field of mathematics education is a significant mark of the state of the art, showing that the field has become so vast and complex that it has become difficult to overlook. This is also underlined by the fact that its state as a scientific discipline has been the focus of a recent ICMI study (Sierpinska & Kilpatrick, 1998). Swedish research studies were reviewed already in the general overview by Werdelin (1973). More recently, information about the work of present Swedish researchers, based on answers to questionnaires, was given in Johansson (1991, 1994). A survey of recent Swedish research on mathematical disabilities is made in Sahlin (1997). See also Engström (1999).

This presentation is based on looking at Swedish research publications from the past, on the answers to questionnaires that were sent to Swedish researchers in the end of 1997 (reply from 30 persons) and to departments of teacher training, education, and mathematics in 1999 (reply from 10 departments), and comments from colleagues. By showing different faces of Swedish research in mathematics education the product set of their characteristics will display the varieties and potentials of the research efforts. After a look at history, a short review of research problems and methods, examples of research results, research communities, and future perspectives will be presented.

Beginning with a general outlook on the object of the research interest, i e the teaching and learning of mathematics in Sweden, it has been said that before the time of the ”new math” there had been very little change in the way mathematics was taught at schools (Magne, 1986). The teacher showed the students, using explanations and visual tools (object lessons), how a mathematical technique worked (for example two digit multiplication), and the students worked, under the guidance and help from the teacher, through a number of tasks to practice

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[1] This paper is reprinted (with due permission) from C. Bergsten, B. Grevholm & G. Dahland (Eds) (2002). *Action and Research in the Mathematics Classroom. Proceedings of MADIF2, Göteborg, 26-27 January 2000 (pp. 21-36).* Linköping: SMDF.

this technique from a book of exercises, which was the only ”textbook”. This was called teaching by rules, and was considered the most efficient method for mathematics teaching in the upper grades of compulsory school. In the lower grades the heuristic method of learning[2] was considered to be more adapted to the level of thinking of young children (Bergsten, 1939). Both these methods were rooted in the strong Swedish tradition of elementary mathematics teaching to keep the learning activities close to the reality of the young child, using concrete materials and visual tools to make children develop their mathematical skills through their own activities. (See e g Wigforss, 1952) The ideas behind the new math were therefore strongly criticised (Lindström, 1968) already before its curriculum came into play in 1969, but nevertheless it put a stop to a promising development of mathematics teaching in the country.[3] Now, it did not take long before the poor results of the new programme, mainly on computation skills, were highlighted in the PUMP project (Kilborn, 1979), and the low Swedish results (in international comparison) from the second IEA study 1980 (Skolöverstyrelsen, 1983), started intense activities on a national level to continuing education of mathematics teachers in compulsory school. The emergence of the electronic calculator during this time ”complicated” the situation, and the time since then in mathematics education in Sweden has been, as I see it, ”a search for identity”. The need for research in mathematics education has therefore only been increasing since the new math changed the scene, even the curriculum from 1980 (”back to basics”), and from 1994 (again with more emphasis on mathematical thinking and understanding) have tried to put mathematics education in Sweden on a positive development.[4] The Swedish results in the international TIMSS study (Skolverket, 1996) indicated that the efforts made had got a positive pay off.

Face 1: *History*

The development of Swedish research and developmental studies in mathematics education could be termed as a move from content oriented and experience based ”guidance” to research based ”didactics” (*didaktik*). The title of the last

[2] The student should first get so simple tasks that he is able to solve them by himself, with his own methods, then by systematic training with more and more difficult tasks he will develop mathematical skills. The key word was learning by practice. (Bergsten, 1939)
[3] See Johansson (1985/86, p. 9), who also includes a valuable list of early Swedish research and developmental literature in mathematics education.
[4] See Wyndhamn (1997) for an analysis of the mathematics curriculum development in Sweden; see Håstad (1978) for a description of the development of mathematics education in Sweden 1950-1980.

book from one of the nationally most well known mathematics educators in Sweden during the 19th century, Karl Petter Nordlund, is ”Vägledning vid den första undervisningen i räkning” (”A guidance to early arithmetic teaching”, my translation; Nordlund, 1910).

On the international level this move or shift parallels the paradigm shift in mathematics education research around 1980, when positivistic models of hypothesis testing gave way to methods more apt to the practitioner's perspective. To quote Kilpatrick (1992, p. 31), ”research in mathematics education was moving out of the library and laboratory and into the classroom and school”. This also explains the shift: When scientific research put itself outside the (strong) guidance tradition it did not have any effect on it, when moving inside, it did. It was also at this period when research journals in mathematics education appeared[5] as well as research institutes[6].

The book by Nordlund (1910) is a very detailed description of what lessons in elementary mathematics topics should look like, with extensive use of concrete material and student activity. It is interesting to note that the work by for example Bergsten (1939) and much later the textbook by Anderberg (1992) still belong to the same tradition. At the turn of the century (i.e. 1900) there was a strong movement to make mathematics teaching more ”åskådlig” (it is hard to find an appropriate English word for this Swedish word, which means making mathematics more clear or lucid, for example by using visual means for displaying mathematical meaning), by mathematics educators like Ehlin, Kruse and Setterberg, influenced by the Perry-movement in England (Wistedt et al, 1992). Nordlund had already used that principle for 40 years by then (his first mathematics textbook dates from 1867), possibly carrying on the tradition from Comenius' work in Sweden 200 years earlier[7].

Wigforss extended this tradition of ”åskådlighet” (*lucidity*) by the development of diagnostic testing materials of high quality that became widely used (e.g. Wigforss, 1946). The development of standardized tests on a national scale, by Wigforss, to support the marking system, was another early significant contri-

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[5] Journal for Research in Mathematics Education, Educational Studies in Mathematics, Zentralblatt für Didaktik der Mathematik

[6] Shell Centre, Institut für Didaktik der Mathematik, Instituts de Recherche de l'Ensignement des Mathématiques.

[7] Nordlund's introduction of the heuristic method for teaching, as an alternative to the mechanistic teaching methods ”by rules” that were common at that time, was inspired by his teacher Kjelldahl in Uppsala (Bergsten, 1939).

bution (see Kilpatrick & Johansson, 1994). Other research studies during the period before the new math were few and individual products rather than long term results from mathematical education research groups or milieus, for example the ingenious early interview study by Jonsson (1919) and the powerful factor analytical studies by Werdelin (1958, 1961).

The shift in mathematics teaching with *the new math* initiated, by the problems it evoked, an increased interest in the nature of mathematical skills and knowledge, and also in teacher training. This is shown by the increased number of projects and studies in the field that appeared (e.g. Kilborn, 1979). Also in the US, there was an explosion in the number of articles that appeared in the field (Kilpatrick, 1992). Subject matter based *didaktik* was the focus of a conference in Marstrand (Marton, 1986), after which the term *matematikdidaktik*[8] was beginning to be widely used. The first course at university level in Sweden by the name *matematikdidaktik* was organized by Wyndhamn and Unenge in Linköping in 1985 (10 credit points). During the 15 years that have followed more than 15 PhD works have been presented, the largest number within the phenomenographic approach, along with many other studies. Textbooks for teacher training in this new research based *matematikdidaktik* like Unenge et al (1994) or Bergsten et al (1997) look very different from those mentioned above in the ”guidance” tradition.

In some of the national research reviews that have been presented a ”national tradition” or ”trend” has been able to identify. In Germany, for example, there is the *Stoffdidaktik* tradition since the 1950:s (vom Hoffe, 1998), in Italy the two trends of *concept-based didactics* and *innovation in the classroom* were identified (Arzarello & Bartolini Bussi), in the Netherlands there is the *realistic approach* (DeCorte & Vershafel, 1986), in France there is the well known conception of *didactic engineering* (see the paper by Artigue in this volume). And so forth. Is it possible to find a typical ”Swedish” tradition in mathematics education research, an identifiable trend that dominates, or has dominated, the national scene?

Face 2: *Research problems*

The range of problems that are studied in the field of mathematics education research can be listed along several dimensions – focus on different actors (students, teachers...), organisation of teaching (groupings, individualisation...), mental processes (reasoning, visualisation...), focus on topic areas (geometry,

[8] English translation *didactics of mathematics* or *mathematics education*.

algebra...), and many more. Within each dimension the focus can be on a general level (how does understanding in geometry develop?) or on a more specific level (how do students in grade 9 understand the concept of similarity?). A look at 23 Swedish PhD works during the period 1919-1999 (of which 15 are from the last ten years) that could be classified as belonging to the field of mathematics education, showed an almost non-overlapping distribution of research problem areas: problem solving (Wyndhamn, 1993), arithmetic with school beginners (Neuman, 1987; Ahlberg, 1992; Ekeblad, 1996), structure of mathematical knowledge (Werdelin, 1958; Bergsten, 1990), computers in mathematics education (Hedrén, 1990; Dahland, 1998), long term development of mathematical ability (Pettersson, 1990), students ways of solving arithmetic tasks (Jonsson, 1919), the organisation of learning (Ekman, 1968; Dunkels, 1996), fractions and reflective thinking (Engström, 1997), effects of curriculum change (Håstad, 1978; Kristiansson, 1979; Hellström, 1985), individualisation (Larsson, 1973), mathematical modelling (Wikström, 1997), understanding graphs (Åberg-Bengtsson, 1998), teachers' and students' conceptions of mathematics/ teaching and learning mathematics (Löthman, 1994, Sandahl, 1997), teachers' different ways of handling content (Runesson, 1999), influence of social factors on mathematics achievement (Chen, 1996). Other studies focus, additional to the above mentioned areas, on gender, communication between students and between students and tasks, informal (”everyday”) knowledge, quality of children's mathematical thinking, teacher students, mathematical disabilities, and undergraduate mathematics education.

From a quantitative point of view, a big proportion of the Swedish research has been made within a number of projects, sometimes on a large scale, often funded by the National Agency for Education. Examples of such projects are (in alphabetical order) ADM, ALM, ARK (including DIM and RIMM), BIM, DIS, DOS, GEM, GUMA, HÖJMA, MYT, PUMP, and ”Matematik i en skola för alla” (Mathematics in a school for all), ”Problemlösning som metafor och praktik” (Problem solving as metaphor and practice), and ”Vardagskunskaper och skolmatematik” (Every day knowledge and school mathematics).[9]

Problem areas in mathematics education that recently have attracted most interest and research attention in Sweden are (according to the questionnaire from 1997 as mentioned above) young children's conceptions of mathematics and early number learning, teachers' and teacher students' conceptions of teaching and learning, assessment and evaluation of knowledge, technology in mathematics education, problem solving and communication in the classroom,

[9] For explanations of acronyms and references please contact present author.

mathematical disabilities, gender issues, and research approaches such as phenomenography and constructivism. Learning issues in undergraduate mathematics have recently come into focus in some studies. Topic oriented studies are very few, as well as theoretical studies of epistemological character, and there is no *stoffdidaktik* tradition in the German sense (exceptions are for example the work by Kilborn, 1979, and by Bergsten, 1990). Qualitative methods are dominating, in particular in combination with the phenomenographic approach, and more or less well structured methods of triangulation are often used to increase the validity of the studies.

Face 3: *Research methods and results*

Methods of research in mathematics education in Sweden have followed the international trend, i e from an early dominance of quantitative studies towards an increasing number of qualitative studies. Experimental designs, with experimental and control groups, using pre- and post-test techniques, have been used by for example in Werdelin (1968), Hedrén (1990) and Ahlberg (1992). Examples of studies using psychometric methods are Werdelin (1958, 1961), Bergsten (1990), Pettersson (1990), and Chen (1996). A number of national survey studies have been produced by the National School Board, for example on grades 5 and 9 in 1992 (Skolverket 1993a, 1993b). Some interesting longitudinal studies have been conducted, for example on early number conceptions (Neuman, 1987) and alternatives to standard algorithms (Hedrén, 2000). Today interview techniques are dominating the scene, but Jonsson's early study (Jonsson, 1919) proves there is a long tradition in Sweden for qualitative methods. Modern techniques such as video recordings have been used in Löthman (1994) and Dunkels (1996).

Phenomenography (Marton, 1981) has a strong position in mathematics education in Sweden, as in the studies by Neuman (1987), Ahlberg (1992, 1997), Ekeblad (1996), and others. Piagetian constructivism has fewer exponents (e g Engström, 1997), and so has the Vygotskyan perspective (Wyndhamn, 1993; the paper by Säljö in this volume). Studies on a more subject oriented theoretical level are also less frequent (e g Kilborn, 1979; Bergsten, 1990).

It is often said, by practitioners, that research is very interesting but not of so much use in the daily work in classroom reality. Now, research results are always of a theoretical nature, and even if they sometimes are on a very general level, they can nevertheless form a basis for designing curricula, as well as teaching or learning situations. For example, knowledge at different levels of the education system of the five examples of major findings of research in mathematics education, given by Niss (1998), would be a safe basis to avoid many mistakes in the design of teaching and learning activities.

It is not possible, in this limited format, to list the ”Swedish results”. Instead some sharp results that seem to have obvious teaching implications will be shown, ordered chronologically from 1919 to 2000.

Mental calculation methods

In a study by Jonsson (1919) a series of interviews on mental calculation methods was conducted. His results are still relevant for the discussion today when the teaching of formal calculation methods (‘standard’ algorithms for addition, multiplication and so on) in elementary school is questioned, in favour of building on children’s own methods. Jonsson found that despite the extensive training on only one formal method for doing additions fourteen out of fifteen students interviewed in grade 2 used other methods when they were free to choose. He also found that students chose the methods, in each calculation, that needed as little thinking effort and time as possible.[10]

Rules first or discovery learning?

The effects on learning of different organisations of learning situations have been much studied in pedagogical research. Since mathematics is, among other things, a rule-using discipline, it has been common to use two different methods of instruction, i.e. rule first (given by teacher or book) – then practice, or let the students themselves discover the rule from the material (discovery or heuristic learning). Studies on this problem were of interest for example for the design of programmed teaching in the sixties. This was studied for example in the BIM project. In an experimental study by Werdelin (1968) with 211 grade 6 students, one group (A) got the principle (law of distribution) before the examples, group B first some examples, then the principle, and more examples, and finally group C only examples. To measure the effects of the different treatments one test was given immediately after the experiment, one test two weeks later, and one test to measure transfer effects. There was a significant difference at the immediate test to the advantage of group A, a difference that however disappeared after two weeks. There was no difference on the transfer test. The main conclusion (from this and other studies) was that the advantages of the heuristic method become visible in a long term perspective, when combined with other important more

[10] See Johansson (1985/86) for a short presentation of Jonsson’s study.

general aspects of learning, such as drawing your own conclusions and make a synthesis of what you experience. One could comment that this research provides scientific support to the ideas of Nordlund in the 19th century (cf above).

Doing mathematics without using understanding

In a study on the different steps involved in solving a complex mathematical task, Ekenstam and Nilsson (1977) showed how students strongly depend on their familiarity with certain standard patterns when solving mathematical tasks, rather than trying to think what the problem was about, or consider the meaning of the mathematical expressions as starting point for their reasoning. To construct their tests, Ekenstam and Nilsson chose a 'top task', e g an equation like $\frac{3x-2}{2}=\frac{x}{3}$ and broke it down to the small steps that were used in solving the task. This produced a series of tasks like $3(3x-2)=2x$, $9x-6=2x$, $7x-6=0$ and $7x=6$. To find out at which step the students had problems, each of these five tasks were included in the test items, along with some similar tasks that changed e.g. the *x* to a *t*, the – to a +, or the particular numbers involved. From 10 top tasks 130 items were thus constructed, distributed over 10 tests of mixed items. The tests were distributed to a sample of 2000 students beginning upper secondary school so that each item was solved by approximately 200 students. It was observed that the solution frequencies strongly depended on minor changes, from a mathematical point of view, of the task characteristics. As an example, by rotating the same right angled triangle, the solution frequency to an area calculation task changed from 39% to 64%, or (in a simplification task) inverting the expression $\frac{(mv)^2}{mv^2}$ caused a change from 41% to 17%. Another remarkable observation was the big difference in difficulty between the simplification tasks $\frac{a^2}{a}$ and $\frac{a}{a^2}$, a difference that disappeared when visible coefficients were used: $\frac{3a^2}{6a}$ and $\frac{6a}{3a^2}$. The main conclusion from the study is that the mathematical skills of many students are based on the application of trained patterns rather than on understanding of what they are doing. A similar conclusion is made in recent interview studies by Lithner (1998, 1999) on undergraduate mathematics problem solving.

The effect of using electronic calculators

In Sweden there were some early studies on a broad basis investigating the effects of the use of the electronic calculator in elementary mathematics teaching. In a study during the years 1977-1983, called the RIMM project (Hedrén & Köhlin, 1983), 7 classes were studied during their years in the Swedish ”middle grades” (grades 4-6). Using an experimental design with experiment and control groups, it was clearly shown that the experimental groups, where the calculator was consequently used, showed the same ability as the control groups to do arithmetic calculations, mentally as well as with pencil and paper, but showed significantly better results on number estimation, choosing correct operation, and to use relevant information when solving word problems. The explanation given for the results was the increased amount of training on problem solving that was possible when the calculator reduced the time needed for performing the necessary calculations. This idea also formed the basis of the longitudinal project ADM, investigating the use of computers and later graphic calculators in secondary school (Björk & Brolin, 1995).

Connecting everyday knowledge to mathematics teaching

There has been a long tradition in Sweden to relate children’s mathematical work in school to their every day world outside school, something that is also visible in Swedish curriculum texts. However, this has caused a tension in school mathematics between every day mathematics (since ICME5 known as ethno-mathematics) and ”academic math”, i.e. mathematics as a scientific discipline. In a three years project ”Everyday knowledge and school mathematics”, the aspect of using everyday knowledge to learn mathematics was highlighted (Wistedt et al, 1992). The project was a co-work between a teacher, a pedagogue, and two mathematicians, the main material videotaped and audio-taped classroom activities, including group work, and discussions among teachers.

Using everyday knowledge in school mathematics means for students that their *reference world* (everyday experience) in some way must connect to the *world of (school) mathematics*. To do this connection the student has to create a *reference domain* that picks out those aspects from the reference world that come into play in the mathematics ”game” in school. This reference domain is a kind of model world (the terms are from Schoenfeld, 1986). To use everyday knowledge for learning mathematics the reference domain is the crucial link between the existing intuitive knowledge of the student (his/her reference world) and the new knowledge (of mathematics) he/she is trying to construct. The different modes of thinking and usage of words in these worlds create conflicts in the learning process, and one main conclusion is that instead of being a process of induction from the reference world via the reference domain to the world of mathematics, learning mathematics by connecting everyday knowledge means that the student is using knowledge and experiences from two worlds – the everyday world and mathematics – when solving the problems. The results show that students use knowledge from both worlds, the reference world and mathematics, when constructing their reference domain (from Wistedt et al, 1992, p. 123):

LEARNING TASK

REFERENCE DOMAIN

REFERENCE WORLD **MATHEMATICS**

This means that the learning paradox is coming into play, since it seems as the student needs an existing conception of mathematical abstractions to move from reference world to reference domain, a domain which is supposed to be the link to understanding the mathematical abstractions. To use some kinds of manipulatives for learning, for example, presupposes that the manipulatives be interpreted in a way that presupposes knowledge of the mathematics they are supposed to learn by the material. To come out of this paradox, and to learn mathematics by connecting to everyday knowledge, a dialectic view of learning in a cultural perspective is needed. Connecting everyday knowledge can work as an instrument in a mutual communication between a personal world of experience and a cultural tradition such as mathematical thinking. The main result from the study is the opening of a way to bridge the gap between everyday knowledge and mathematics, by showing the way students construct reference domains that build both from contexts of practice and of theory.

Young children's mathematical thinking (pre-school and first school years)

This problem area has attracted several Swedish researchers, with the most extensive studies by Ann Ahlberg and Dagmar Neuman. An increased interest in pre-school mathematical experience can be noticed the last years, with a number of recent Swedish publications.

As an example, Ahlberg's study ”Children's ways of handling and experiencing numbers” (1997) is showing the complexity of early number experience. The study is part of the project ”Numerosity and the development of arithmetic skills among visual impaired children, hearing impaired children and children without these impairments”. It is an interview study within the phenomenographic framework, where the interview is treated as a *conversation with a structure and a purpose*. 38 children from 3 different pre-schools (average age 6.7 years) were interviewed on 3 different kinds of tasks, *every day problems*, *decomposition problems* (cf Neuman, 1987), *and contextual problems* (cf Ekeblad, 1996). Children were not allowed to use any manipulatives. Interview outcomes were classified into a number of main categories, in line with the phenomenographic approach, and were analysed under the main headings ***Ways of handling numbers*** and ***Ways of experiencing numbers***. One of the main results is that there was not a one-to-one correspondence between the way children handle numbers and the way they experience them, as shown in the matrix below.

	WAYS OF EXPERIENCING NUMBERS			
Ways of Handling Numbers	**Number Words**	**Extents**	**Positions in Sequence**	**Composite Units**
SAYING NUMBERS				
Random Number Words	•			
Equal Numbers	•			
Successive Numbers	•			
ESTIMATING	•	•		
COUNTING				
Double Counting	•		•	
Counting and Tapping	•	•	•	•
Counting and Looking	•	•	•	•
Counting and Listening	•	•	•	•
Finger Counting				
Using Fingers; Counting All	•	•	•	•
Using Fingers; Touching	•	•	•	•
Using Fingers; Looking	•	•	•	•
STRUCTURING				
Seeing	•	•	•	•
Using Derived Facts	•	•	•	•
USING KNOWN FACTS	•	•	•	•

Table from Ahlberg, 1997, p 85

Five categories of handling numbers were identified, of which ***Saying numbers*** and ***Counting*** were the most frequent. The four identified categories of experiencing numbers all come into play, more or less, when children handle numbers by counting, structuring, and using known facts. This pairing of different dimensions deepens the picture of the complexity of early numerosity development, and Ahlberg concludes:

> When trying to grasp numerosity children handle numbers in a vast array of ways, and thereby experience different aspects of numbers. However, in spite of using different ways of handling numbers, the numbers may appear in the same way to them and they may experience the same meaning. Consequently, there is not only one pathway, but many pathways to numbers."
> /…/
> Understanding numbers and learning arithmetic skills is not only a question of the quantification of objects or fingers. Neither is it a matter of learning how to count on the number sequence or developing logical thinking. It is instead, a question of being able to explore and discern different aspects and possible qualities of the numbers - of experiencing numbers in the sense of sensuously and simultaneously perceiving different aspects of numbers. (Ahlberg 1997, p. 109)

The results have clear implications for teaching.

Problem solving

Jan Wyndhamn, together with Roger Säljö, has done extensive work within the socio-cultural and situated cognition framework. Wyndhamn, with Riesbeck and Schoultz, has recently finished a project called ”Problem solving as metaphor and practice”, where problem solving activities in classrooms and teachers' views on problem solving were scrutinized using qualitative data techniques (Wyndhamn, Riesbeck & Schoultz, 2000). Some of the results indicate that problem solving in practice most often reduces to solving word problems in class, and that group work activities become just another version of ordinary work with mathematical tasks. No transfer effects were found between everyday mathematics and academic math (cf. Wistedt et al, 1992). Problem solving in school mathematics seems to reduce itself to a metaphor *for* a practice, related more to the organization of teaching than to mathematical content.

One critical question is the dissemination of research results. It is not always easy to pick up a ”result” from a study and ask practitioners (e.g. teachers) to use it. It depends, among other things, strongly on the origin of the research question and the level of generality of the result. Bishop (1998) argues that researchers should become more aware of the fact that practitioners are the only actors for change:

> ”The research site should be the practitioners’ work situation, and the language, epistemologies, and theories of practitioners should help to shape the research questions, goals and approaches.” (p. 43).

Many of the persons in Sweden doing research and developmental studies in mathematics education have a background as teachers, keeping their research close to the teaching and learning practice. Maybe this is the ”Swedish tradition” in mathematics education research. The question of information and a common discourse still remains, however. The only Nordic research journal in mathematics education, *Nomad* (Nordic Studies in Mathematics Education) is still young and has not yet succeeded to reach a broader audience among practitioners. The journal *Nämnaren* for teachers of mathematics has been the most important source in this respect in Sweden for the last 25 years (though not a research journal), as well as the mathematics teacher congress *Matematik-biennalen* (every second year since 1980) and its regional follow-up meetings, and meetings arranged by mathematics teacher associations. In some recent publications Swedish research has been made available for a broader audience, primarily for use in teacher training (e.g. Ahlberg, 1995; Gran, 1998; Neuman, 1989). Hopefully, the present conference will contribute to an increased awareness and interest also among practitioners what research can offer.

Face 4: *Research communities*

Before the ”shift” after the new math most Swedish research in mathematics education took place at departments of education. At the end of the seventies some PhD programmes also involved departments of mathematics, but it is not until recently that departments of mathematics have begun to create research milieus in *matematikdidaktik*, as in Umeå. The first PhD thesis with a mathematics education content at a department of mathematics in Sweden was Dunkels (1996) in Luleå. Other sites in Sweden for research activities in the field of mathematics education are the national testing institutions in Stockholm (PRIM) and in Umeå (Nationella provgruppen). These institutions do test constructions and research on assessment in mathematics, providing long term descriptions of mathematical skills and attitudes of Swedish school children. In fact, results of such measurements often produce the strongest direct influence on practice. As an example, the studies by Lindblad (1978), followed up by Ljung (1987), caused a change of the prerequisites for entering teacher training colleges, and the second IEA study started huge efforts on a national level to educate Swedish teachers of mathematics (see Utbildningsdepartementet, 1986)

There are also networks and organisations outside universities that play an important role for research in mathematics education. The network *Women and mathematics* under the leadership of Barbro Grevholm has organised a number of international conferences in Sweden, with proceedings of research papers, one of which was the ICMI study conference on gender issues in mathematics education (Grevholm & Hanna, 1995; Hanna, 1996). The present conference was organised by the new and independent *Swedish society for research in mathematics education* (SMDF[11]).

Face 5: *Looking ahead*

With this history behind, what might the future of research in math education look like in a small country like Sweden? In fact, some opposite trends can be identified at present. On the national level teacher education seems to move towards establishing a more generalized educator profile, with less emphasis on teaching subject matter towards a teacher as an administrator and supervisor of learning. On the local level, at departments of mathematics and didactics, new research milieus for mathematics education are being established, and teacher training programmes include courses of a scientifically oriented *matematik-didaktik*. Again, on the government level, resources have been given for researching and educating teachers of mathematics in subject matter and *didaktik*.

The increased interest among teachers as researchers (as mentioned above), and the increased emphasis of a research based teacher training, are important backups for the future development of Swedish *matematikdidaktik*. For this we need people that can inspire the way Andrejs Dunkels did, and Gudrun Malmer is doing. I also believe that the expansion of research in *matematikdidaktik* into the departments of mathematics will be a necessary and important factor for a promising development of the range and quality, and in the search for an identity, of Swedish research in mathematics education. Today, measured in number of publications, Swedish research in mathematics education is hardly visible on the international scene in the increasing stream of articles and books. No doubt a change is on the way. Using a metaphor one could say that research in mathematics education in Sweden is a kettle of water heated up so that it, maybe, soon will start boiling.

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[11] See the web page of SMDF at www.mai.liu.se/SMDF/

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/ Christer Bergsten

New faces of Swedish research in mathematics education

Research in mathematics education in Sweden has increased in both scope and volume since the turn of the millenium[1], promoted by events such as the establishment of the MADIF conferences[2] and the National Graduate School[3]. A basis for this development was grounded during the 1990s by an increased activity and recognition of the need and importance of high quality research in this field. Since the year 2000 a total of 11 PhD theses within the field of mathematics education in Sweden have been presented, to be compared to a total of 30 during the whole last century[4]. A PhD is in Sweden the result of a 4 full years study programme of course work and writing of the thesis. It is also possible to earn an intermediate exam at ”half time”, called ”licentiatexamen”, including the writing of a smaller thesis[5]. Traditionally, academic research in Sweden within the field of mathematics education has been done at departments of education at universities, a situation that has changed with the National Graduate School where mathematics departments are actively involved. This means that the basis and the directions of the studies have broadened. To show some glimpses of what is going on, we will here offer short presentations of 5 theses recently presented for the ”licentiatexamen” within this National Graduate School.

The idea that history of mathematics and mathematics education research could both profit from each other is implicit in the licentiate thesis by Kajsa Bråting (2004). Based on a conceptional analysis of the history of a theorem by Cauchy on convergence of sums of functions, and its relation to work by the contemporary Swedish mathematician Björling, she uses the distinction between concept definition and concept image (Tall & Vinner, 1981) to illustrate limits of visualisation and intuitive thinking when dealing with advanced mathematical concepts

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[1] For an overview for the past century see Bergsten (this issue of the *SMDF Newsletter*).

[2] These international seminars, organised by SMDF in cooperation with the local universities and NCM, took place in Göteborg in 2000, in Norrköping in 2002 and in Malmö in 2004. See the proceedings in Bergsten et al (2002), Bergsten and Grevholm (2003; 2004).

[3] This graduate school started in 2001, with approximately 20 doctoral students of mathematics education at departments of mathematics nationwide. See web page www.msi.vxu.se/Forskarskolan/

[4] A complete list (with abstracts) of Swedish PhD studies within the field of mathematics education is found at the web page ncm.gu.se/index.php?name=forskning-litteratur-doktorsavhandlingar

[5] For a list (with abstracts) of recent ”licentiate” theses, see web page ncm.gu.se/index.php?name=forskning-litteratur-licavhandlingar

such as infinity. The need to formalise and base the arguments strictly on concept definitions is seen through the historical analysis, as well as by the (well known) problems that students encounter when trying to use their concept images that do not fully capture the scope of such formal definitions.

Monica Johansson's licentiate thesis has the title *Textbooks in mathematics education: a study of textbooks as the potentially implemented curriculum*. Earlier studies show that textbooks are an important tool in the teaching of mathematics in Swedish classrooms. Actually many teachers and pupils think that the textbooks determine what school mathematics is and what is going to be taught. Previous studies on textbooks and the use of textbooks in teaching and learning mathematics have raised important questions about textbooks as representations of the curriculum. What is their role as a link between curriculum and activities in the classroom? In her thesis Monica reviews and analyses some international investigations in connection to this question. To illustrate the textbook as the potentially implemented curriculum a content analysis of a textbooks series was conducted. The development of a commonly used textbooks series in Sweden is portrayed in the light of the curriculum development. Some findings from the analysis of textbooks show that the goals of mathematics teaching, as they are formulated in the national curriculum, are only partly realized.

In her thesis *Learning limits of functions, University students' development during a basic course in mathematics*, Kristina Juter presents four articles and an introduction including the theory that is the frame for the articles. The main question is: How do students deal with the concept of limit of a function at the basic university level in Sweden? This wide question is narrowed down into: what are the results of students' creations of mental representation of limits of functions? Do the representations change during the time of study? What changes if anything? Do high achievers representations of limits differ from low achievers? How do students solve problems with limits? How do they explain their solutions? Questionnaires, interviews and field observations were used to seek answers to the questions. The limit concept is an important part of the foundations of mathematical analysis and if the students do not understand clearly what it is about, they can get problems when they are dealing with concepts as continuity and derivatives. The representations students have are not necessarily coherent or true and they are not always aware of that. The formal definition is hard for the student to grasp and they fail because of lacking algebraic skills. No such study on limits has previously been done in Sweden so it is compared mostly to foreign results. It is important that people who work with mathematics education at university level are aware of the situation the students are in.

Per Nilsson studied grade 7 students' ways of treating the concept of probability in an experimental situation, based on problems given in relation to games using sums of dice, designed to highlight different aspects of the probability concept (Nilsson, 2003). A learning perspective was used, with the aim of describing students' ways of contextualising such probability problems, given those conceptual and situational/ cultural resources at their disposal. An intentional analysis of data was used, providing a basis and meaning to the students' actions. The results show the importance of relating students' conceptualising probability to their ways of creating meaning in a task situation.

The mathematical discourse between small groups of undergraduate students, working with concept maps during a linear algebra course, was studied by Andreas Ryve in his in the licentiate thesis (Ryve, 2003). Using video-tape recordings, a focal and preoccupational analysis, by means of interactive flowcharts to follow student engagement in the discourse (Sfard & Kieran, 2001), showed that the communication among students was effective and mathematically productive. The study also aimed at developing the methodological framework used, which was carried out by intentional analysis (see Downes, 1998) and an explicit categorisation of mathematical utterances and reasonings (Lithner, 2003). This way a more nuanced picture of the discourse could evolve.

These studies cover areas in mathematics education that have not been very much explored in previous Swedish research (cf. Bergsten, this issue of the SMDF Newsletter; Björkqvist, 2003), i.e. a focus on specific mathematical concepts, advanced mathematical thinking, and discourse and textbook analysis. Ongoing studies within the national graduate school, as well as in other settings (see e.g. Bergsten et al, 2002; Bergsten & Grevholm, 2003, 2004), add to this new course of development in Swedish research.

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/ Christer Bergsten, Barbro Grevholm

In Memoriam – Göte Dahland

We remember our dear friend and colleague Göthe Dahland

Vid juletid 2003 nåddes vi inom SMDF av beskedet att vår avhållne vän och kollega Göte Dahland drabbats av en obotlig sjukdom. Trots vetskapen om situationen fortsatte Göte med värdighet sitt arbete och medverkade i valberedningen för SMDF ännu en gång. Han orkade även med att organisera sitt stora matematikdidaktiska bibliotek och fatta beslut om att donera det till NCM i Göteborg. Slutligen segrade dock sjukdomen och vi fick ta farväl av Göte.

Göte har varit en central person inom utvecklingen av matematikdidaktik i Göteborg och Sverige genom att tidigt organisera kurser för blivande doktorander och seminarieserier och konferenser. Genom att själv gripa sig an en forskarutbildning i mogen ålder och slutföra den på ett gediget sätt föregick han som ett gott exempel för många yngre kollegor i lärarutbildningen i Göteborg. Han inspirerade och stöttade många att gå in i en forskarutbildning i matematikdidaktik. Efter sin pensionering var han fortfarande full av verksamhetslust och arbetade aktivt med matematikdidaktik för högskolan i Halmstad. Det blev dock en alltför kort tid som Göte fick njuta av att vara ledig från schemalagt arbete och disponera sin tid för familj och vänner.

För SMDF har Göte varit en viktig person. Han har varit aktiv i föreningen alltsedan dess start och verkat som kassör under de fyra första åren. Därefter har han ingått i valberedningen. Göte var med och organiserade det andra matematikdidaktiska seminariet i Göteborg i januari 2000. Han ingick i redaktionen för konferensboken, *Research and action in the mathematics classroom*. Med sin lugna, trygga uppenbarelse bidrog han alltid till att arbetet som utfördes blev seriöst och genomtänkt. Med sin långa erfarenhet av svensk gymnasieskola och lärarutbildning i matematik hade han mycket att tillföra i diskussioner och överläggningar. Hans doktorsavhandling, *Matematikundervisning i 1990-talets gymnasieskola*, är ett studium av hur en didaktisk tradition har påverkats av informationsteknologins verktyg. Avhandlingen försvarades i maj 1998 och Göte fick alltför få år på sig att föra ut sina forskningsresultat i Sverige och internationellt. De elektroniska hjälpmedlens inverkan på skolans matematikundervisning är och kommer att vara väsentlig även om vi kanske inte ser det när vi nu är mitt uppe i utvecklingen. Göte förstod betydelsen av dem och både hans licentiatavhandling från 1993 och senare doktorsavhandlingen kommer att vara en stadig grund för fortsatt forskning på området.

Vi minns Göte som en gentleman i ordets bästa bemärkelse, vänlig, glad och lugn i alla lägen. Med sin göteborgska humor hade han alltid ett gott ord till alla. Vi saknar vår vän och kollega Göte och hedrar hans minne.

/ Barbro Grevholm

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Future conferenses in the Nordic countries

ICME10 Köpenhamn, 4-11 July 2004
http://www.icme-10.dk

HPM 2004, Uppsala, 12-17 July 2004
http://www-conference.slu.se/hpm/index.html

PME 28, Bergen,14-18 July 2004
http://www.pme28.org/

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