SMDF

Svensk Förening för **M**atematik**D**idaktisk **F**orskning

MEDLEMSBLAD

Nr 8

December 2003

INNEHÅLL

Redaktör för Medlemsblad nr 8 har varit *Christer Bergsten*

Medlemsblad nr 8

Under rubriken *Några rader från...* berättar inledningsvis föreningens ordförande om vad som är på gång i SMDF och svensk matematikdidaktik i övrigt.

Medlemmarna i SMDF inbjuds att till medlemsbladet skicka in kortare artiklar eller berättelser, som kan vara av intresse för föreningens medlemmar att ta del av. Detta nummer speglar den ökande och varierande aktiviteten inom vårt område.

Ann-Sofi Röj-Lindberg rapporterar inledningsvis från en spännande studie om lärares arbete med att förändra sin undervisning. Semiotiken är ett område som fått allt större betydelse för matematikdidaktiken och Norma Presmeg visar i sitt bidrag hur den erbjuder intressanta analysinstrument. Tekniska hjälpmedel är ett annat sådant område och här berättar Lil Engström om sitt avhandlingsarbete kring dynamisk programvara.

Några av doktoranderna i den nationella forskarskolan i matematikdidaktisk forskning, Jesper Boesen, Linda Mattsson, Johanna Pejlare och Örjan Hansson, delar med sig av sina erfarenheter från verksamheten i forskarskolan, av vad studier i matematikens historia kan erbjuda, och från årets PME-konferens.

Thomas Lingefjärd lyfter fram ett i Sverige försummat område i matematik-didaktiken och informerar om det internationella nätverket ICTMA om matematisk modellering. Avslutningsvis ger Barbro Grevholm som vanligt en rapport om vad som händer i vår danska systerförening *Forum for Matematikkens Didaktik*.

/ Christer Bergsten

Några rader från…

Det fjärde matematikdidaktiska forskningsseminariet *MADIF 4* anordnat av SMDF närmar sig. Den 21 och 22 januari 2004 samlas en stor grupp matematikdidaktiskt intresserade lärare, forskare och forskarstuderande på Malmö högskola. Efter en review-process har cirka tjugo abstracts accepterats av programkommittén. Det ser alltså ut att bli ett om möjligt ännu mera innehållsrikt seminarium än de tidigare. De tre plenarföreläsarna Terezina Nunes, Willibald Dörfler och Astrid Pettersson presenterar sina föredrag i korta abstract på seminariets hemsida[1] (**XX**. Vi ser fram emot ett spännande program och många värdefulla diskussioner och samtal.

Onsdagen den 21 januari 2004 kl 17.00 kommer årsmötet för SMDF att äga rum i anslutning till programmet för MADIF4. Kallelsen har sänts ut till alla medlemmar omkring den 18 november. Om du inte har fått den kan det bero på att du glömt anmäla adressändring? Vi ber all medlemmar hålla kassören informerad om ändringar i e-postadress eller postadress. Det händer ibland att vi får medlemsavgifter utan avsändare. Om du känner på dig att du kan ha betalt utan att uppge namn ber vi dig kontakta kassören för att reda ut frågan. Adresser till styrelsens medlemmar finner du längre fram i medlemsbladet.

Direkt efter Madif4 följer så på Malmömässan den 13:e Matematikbiennalen med temat *Matematiken bygger broar mellan människor, kulturer, tidsepoker och vetenskaper*. Programmet, som är mycket varierat, finns tillgängligt på hemsidan[2]. Många av dem som deltar i MADIF 4 kommer även att medverka i Matematikbiennalen. På biennalen kommer senaste nytt om den stora internationella matematikdidaktiska kongressen ICME10 att presenteras. ICME10 är en världskongress med kanske 4000 deltagare och går av stapeln den 4–11 juli 2004 i Köpenhamn med alla de nordiska länderna tillsammans som ansvariga. Flera av SMDF:s medlemmar är engagerade i organisationen av ICME10 och aktiva i det vetenskapliga programmet. Detta och kongressens alla aktiviteter kan du studera vidare på hemsidan[3].

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[1] www.mai.liu.se/SMDF/madif4.htm
[2] www.lut.mah.se/nms/matematik/BIENNAL2004.htm
[3] www.icme-10.dk

Det är första gången kongressen äger rum i Norden. Och förmodligen ska vi inse att det är ”once in a lifetime” för de flesta av oss som vi finner en ICME på så nära håll. Därför bör du inte missa tillfället att delta eller medverka i denna kongress. Det är viktigt att svenska lärare syns och hörs när vi nu är medarrangörer i ICME10. Det finns ”topic study groups”, diskussionsgrupper och workshops med många olika intresseinriktningar och säkert finner du något område som är särskilt lockande för dig. Det skulle vara mycket värdefullt om svenska lärare och forskarstuderande i hög utsträckning kunde medverka i de olika programinslagen. Ta gärna personlig kontakt med någon av dem som står angivna som ansvariga för respektive programpunkt. Om du själv skulle vilja informera på din skola eller i din kommun om ICME10 finns det bra informationsmaterial, broschyrer, affischer, en CD med presentation av kongressen samt ett Power Point program som presenterar aktiviteterna under ICME10. Detta material kan du beställa från Congress Consultants via hemsidan. Det är viktigt att vi alla bidrar till att Sverige är representerat med många deltagare på ICME10.

SMDF kommer tillsammans med vår danska systerförening *Forum* att ha en utställning och en föreläsning om våra organisationer, där vi presenterar deras syften och verksamheter. Det kommer bland annat att ske under den Nordiska presentationen, som är ett speciellt inslag i programmet.

Symposiet i Malmö den 5-7 maj med rubriken *Morgondagens matematiklärare* genomfördes med stort deltagande både i den öppna dagen och under den slutna delen av programmet med internationella gäster. Alla bidragen till den bok som ska dokumentera konferensen har nu kommit in och boken är under redigering. Vi hoppas att den föreligger klar i manusform till MADIF4 och kan visas där. Avsikten är att boken ska vara tryckt i god tid före ICME10 och kunna användas i det sammanhanget.

Säkert har alla sett rapporter i massmedia från den i Örebro genomförda andra nordiska konferensen om matematiksvårigheter. Den har fått goda vitsord från deltagare och vi ser fram emot en bok som delger oss alla vad som förevar i programmet. I slutet av november ordnas ännu en konferens om *Matematik-svårigheter i skolan* (av en kommersiell arrangör), så det verkar som om det finns ett stort behov bland matematiklärare att fokusera frågor om matematiksvårigheter.

Doktoranderna i forskarskolan i matematik med ämnesdidaktisk inriktning har nu varit igång i fem terminer. Det betyder att de första doktoranderna har nått fram till en licentiatexamen. I december och januari kommer Per Nilsson, Andreas Ryve, Monica Johansson och Kristina Juter att försvara sina licentiatuppsatser. Vi gratulerar dessa första svalor på forskarskolans sommarhimmel. Ytterligare fler kommer att bli klara med en licentiatexamen senare under våren 2004.

Maria Thunholm och Annika Bergehed har sänt mig en rapport från ett förändringsarbete som de genomfört på Folkparksskolan i Linköping med stöd av ett stipendium från Gudrun Malmers stiftelse. Avsikten med arbetet var att sluta med den tysta räkningen i klassrummet och försöka arbeta mer med problemlösning och kommunikation. De ville även arbeta med att nå alla elever, också de som saknar förmåga att arbeta i smågrupper (social kompetens). Fokus var även på frågan om eleverna utvecklas enskilt när de arbetar i grupp och inte bara arbetar med sådant de redan behärskar. De har även frågat sig hur lärare ska agera vid problemlösning för att eleverna ska utveckla en bra matematik. För andra som vill försöka förändra sin undervisning i matematik erbjuder rapporten intressant läsning. Den kan beställas från Maria Thunholm. Vi hoppas kunna presentera en sammanfattande artikel om arbetet i nästa medlemsblad.

En nordisk forskarskola i matematikdidaktik kommer att starta 2004. Nordiska forskarutbildningsakademin (NorFA) har beviljat ett anslag på fem miljoner norska kronor till Högskolen i Agder för att driva forskarskolan. Mer om det berättar vi i en särskild artikel.

Vetenskapsrådet har gett professor Rudolf Strässer i Luleå i uppdrag att skriva en rapport under 2004 om forskningen i matematikdidaktik i Sverige. Det arbetet ska ske under två månader och komplettera den rapport som Ole Björkqvist publicerade nyligen på uppdrag av Svenska kommittén för matematikutbildning och NCM. Den expanderande forskningen i matematikdidaktik i Sverige tilldrar sig således stort intresse från vetenskapssamhället.

Det återstår att se om den forskning som nu bedrivs kan påverka matematikundervisningen i positiv riktning. Forskning innebär ett samhällsuppdrag och i fallet matematikdidaktik är det uppenbart att det finns en förväntan att skattemedlen ska ge resultat som kommer lärare och elever i matematik till godo. Det finns alltså en starkt önskan att forskarna ska kunna visa på vilka implikationer deras forskning kan ha för undervisningen. Det kräver utåtriktat arbete från forskarnas sida och förmåga till inlevelse i den situation elever och lärare har i skolan. Vi hoppas att SMDF i någon mån kan medverka till denna påverkansprocess genom att erbjuda mötesplatser och diskussionsfora för lärare och forskare.

… Barbro Grevholm, ordförande i SMDF

”I was trapped in a pattern”: The dilemma of change

Abstract

This paper presents six mathematics teachers' thinking and understanding of teaching while they were involved, more or less intensely, in an action research process in connection to their teaching at the lower secondary level of a teacher training school in Finland. I followed the teachers for three years as a co-manager, observer, recorder and researcher, but in this paper I concentrate on findings from discussions with the teachers in the autumn the first year. In the discussions the teachers described their mathematics teaching, their beliefs about learning, attitudes to change, frustrations and shortcomings and also how they conceived their roles within the collaborative action research process. The metaphorical title of the paper mirrors the feelings of one of the teachers as we discussed his reasons to bring about changes in his teaching. His experiential insights from a long teacher career acted as a safe but professionally alienating shelter.

Introduction

One day a professor from the Faculty of Education rang me and asked: Would you like to leave your job as a mathematics teacher for half a year and find out what the PUMA-project is all about? I did not hesitate for two reasons. Firstly this was a good opportunity for me to get my postgraduate studies started and secondly my curiosity awoke: a PUMA in the mathematics classroom must mean a very active and different teaching approach! This happened eight years ago. Since my engagement later eventually turned into a research study, I never went back to my job as a teacher in mathematics, physics and chemistry at a lower secondary school.

During the three years that followed the phone-call I accompanied a group of mathematics teachers as they were trying to change and improve their instructional approach. The teachers had given their development project the name "PUMA" which is an acronym for the Swedish version of the phrase "processes and assessment in mathematics teaching". This phrase reflects two very broad aims for the teachers' change process: to find ways to improve the teaching and learning of mathematics and to renew the assessment structure; the latter was considered to act as an impetus for the desired changes in teaching and learning. Before I entered the scene the teachers had already decided to follow a cyclical action research pattern to manage the change and development process. My role unfolded in two directions: to act as a co-manager of the process and to act as a researcher. In my role as co-manager I called the teachers to meetings, wrote protocols of the meetings, distributed the protocols to the teachers for comments etc.. Together with other personnel from the Faculty of Education I also introduced topics for discussions during the meetings. In the role as a researcher I followed the whole development process during three years with my main research focus on how the students' conceptions of teaching and learning mathematics were influenced by what the "PUMA-teachers" did in their classrooms. I observed lessons and gathered material like tests and project-works done by the students, results from questionnaires etc. I also did individual interviews with the teachers at Christmas-time the first year and with four students from each of the six classes involved at several occasions during the three years. The research study will be presented in a doctoral thesis within the field of mathematics education.

In this paper I will offer a brief presentation of the context and then move on to a discussion of some results from the teacher interviews. I will sum up with some comments on the PUMA-project and its action research format.

The context

Six teachers of mathematics at a teacher training school[1]

The PUMA-project was launched already a year before I joined the group. PUMA was essentially the work of the three teachers Tom, Per and Kaj, with Tom as the main initiator. When the managing of the puma-project was formalised into a collaborative action research process the group of teachers expanded with Ove, Åke and Alf.

At the time of the interviews these six teachers represented a wide range of experience of mathematics teaching at the lower secondary level, from two years of teaching up to 24 years. All the teachers were male simply because there were no female mathematics teacher teaching in the school at this time. One teacher, Ove, had majored in chemistry and the others in mathematics. Kaj, Åke, Alf and Ove were teaching full-time with teaching duties at both the lower and the upper secondary section. Tom had recently become headmaster of the upper secondary

[1] "Teacher training school" is a translation from the Swedish words "övningsskola".- a more verbatim translation would be "a school where you practice to become a teacher".

section of the school. His teaching duty was therefore reduced to a minimum and he had no class at the lower secondary level. Since Tom was one of the three founders of the PUMA-project I decided to interview him as well even though he did not take part in the collaborative action research process. Per was an outsider in the sense that his primary profession was to be a lecturer in the didactics of mathematics at the Faculty of Education. At the time of the interview he was responsible for teaching just one class at the lower secondary level.

Another common ground in the professional lives of these six teachers was a duty to supervise the teaching practice of subject teacher students. In Finland each teacher education unit has its teacher training school and the school considered here was a school of this type. The teachers at the teacher training schools are all expected to supervise teaching practice, and to be active developers of their teaching. Consequently I can state that these six teachers were no ordinary mathematics teachers. They were all more or less used to guiding and supervising teacher students and, hence, they had a more developed skill in discussing and reflecting upon epistemological and pedagogical issues than is usually the case with teachers.

According to the regulations, teachers at teacher training schools are required to have shown good teaching skills, to be good teachers. In spite of this fact, Tom characterized the general style of teaching in the school as teacher-centred including a fairly passive student role[2]. But he was satisfied with the teamwork and with the teachers' collaborative planning and support of each other in the beginning of the change-process. He pointed at one attribute of good teaching, which he referred to as being an especially important aim of the change process: to make the teacher a listener. He said that "*in our school the teacher is the talking party, the students are, at the very most, the answering party. In the change-process we try to reverse this situation, and it is difficult*". Furthermore, he described the teaching as unilateral with teacher questions followed by very short answers from the students and he referred to the students' prevalent view

[2] The pedagogical situation in this school was by no means exceptional. In Finland, as in very many countries all around the world, classroom instruction in mathematics has tended to rely heavily on teacher demonstrations and routine drill and practice (e.g. Boaler 2000, Bodin & Capponi 1996, Fennema & Nelson 1997, Kupari 1999, Röj-Lindberg 1999). A more progressive, open and student-active approach to learning is very rarely taken even though evidence of its positive implications has been verified by research (e.g. Boaler 1997, Kupari 1999, Röj-Lindberg 2001).)

of studying mathematics at the upper secondary level using the words “*you don't read mathematics, you listen to it and write it down*”.

The PUMA-project

As the teachers decided to pursue a collaborative enquiry in the action research format, their overall aim was to accomplish improvements in their teaching of mathematics at the lower secondary level. They had reached a critical juncture in their professional lives as mathematics teachers, a situation which according to Raymond & Leinenbach (2000, p. 305) often is a starting point for action research.

> *Teachers who engage in action research are generally teachers who are at a critical juncture in their teaching practice and are in a state of mind where they are open to change. The state of mind might be that one's philosophy of teaching may be in a state of disequilibrium or perhaps the teacher is faced with a dilemma*.

The big question for them was how to make the improvements happen. The desired mathematics classroom was to be different from the one with which the teachers and the students had been familiar so far (see also Tom's comments above) and, hence, there was a demand for transformation not only in the teachers' beliefs and understanding of the teaching and learning process but also in the tacit agreement about how the teachers and the students were supposed to act in the classroom (also called the didactical contract[3]). Many studies of mathematics teaching have showed that these changes are very hard to accomplish (see e.g. Bodin & Capponi 1996, Ernest 1989, Franke, Fennema & Carpenter 1997)

At the time of the interview 106 students at grades 7 and 8 divided into six heterogeneous classes with between 13 and 22 students in each class, were involved in the PUMA-project. Kaj, Åke, Per and Ove were teaching one class each and Alf was teaching two classes, one class in each grade[4]. Hence the teachers did not include all their mathematics teaching into the collaborative enquiry. In the conclusion of the paper I will discuss how this division of

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[3] Brousseau has coined the concept "didactical contract" to describe the invisible agreement that exists in the classroom and deals with what the teachers and the students are supposed to do. (Brousseau 1990).

[4] Setting of students in comprehensive school according to ability was abandoned by law in Finland in 1985.

teaching practice into PUMA and non-PUMA might have affected the change process.

In the improvement process the action research model by Kemmis & McTaggart (1988) was followed with its spiral of cycles of reconnaissance, planning action, enacting and observing the planned action, reflecting on the implementation of the plan using the observation data collected, replanning (developing a changed or modified plan), further action and observation, further reflection, and so on. Kemmis & Taggart (1988, p.5) provide the following definition of action research, which emphasises its participatory, collaborative and self-reflective nature.

> *Action research is a form of collective self-reflective enquiry undertaken by participants in social situations in order to improve the rationality and justice of their own social or educational practices, as well as their understanding of these practices and the situations in which these practices are carried out.*

As I already mentioned, the goals of the PUMA-project were already more or less established by three of the teachers during the preceding school-year and only some minor changes were made as a result of the first reconnaissance and planning meeting as the action research process started.
In short, the goals for the PUMA-project were the following:

- To increase the amount of student-talk in the classroom
- To spend more teaching time on problem-solving and strategies for problem-solving
- To make the students more aware of their own responsibility for learning
- To put a stronger focus on the individual student
- To introduce (teacher-initiated) project-work as an assessment and learning tool
- To introduce new components in the formal assessment of the students' learning

Assessments and teaching strategies were presented and discussed by the teachers during meetings organised according to the action research model. Each teacher tried the agreeds teaching strategies in conjunction with using the ordinary textbook. Of the new assessment components the teacher-initiated project-work differed most from the former routines and was most time-consuming for both teachers and students. The project-works were handed out to the students as individual homework to be completed over a period of two weeks. During the term before the interviews one project-work each was completed by the students in grade 7 and 8: in grade 7 the project dealt with number theory, and in grade 8 with functions. Another totally new assessment component called "explanation-tasks" was added to the formal tests. One aim of these tasks was to evaluate the students' ability to explain strategies for solving problems.

The collection and processing of data

I interviewed the teachers in December, midway into the first year of action research. My intention was to get a deeper insight into their commitment to change and into their beliefs of teaching and learning of mathematics. Because I was a quite inexperienced interviewer at the time, I used an interview format with pre-formulated questions. Thus the interviews could be characterized as semi-structured. In the interviews the same questions were asked of the interviewees and I asked follow-up questions to issues that were raised as important. Each interview was about 30 minutes long. The interviews were transcribed word by word and comprised a total of 76 pages of text.

Before the interviews I had followed the action research process for five months and a preliminary and informal interpretation of the teachers' beliefs was done during these early stages as I listened to and discussed pedagogical issues with the teachers. My preliminary interpretations were refined during the interviews and acted as a cognitive framework for further discussions within the subsequent action research process.

In this paper I base my arguments mainly upon interpretations arising from the interviews as texts. This stage of the interpretation process took place quite a few years after the interviews and is therefore coloured by my knowledge and understanding of the completed action research process. As I started working with the interview-texts one of my problems was to decide which form of presentation would be the best in order to capture what was said by the words being said by the interviewees. Would I tell their stories by using the narrative format or would I try to implement a process of comparing and contrasting their answers in order to let themes emerge out of the texts? I decided to try out the latter form of working and thus use a variation of the grounded theory approach (Glaser & Strauss, 1967). A number of broader themes and particular issues emerged, issues that the teachers described as significant to their personal commitment to the PUMA-project and to their beliefs about mathematics teaching and learning.

Results

Insufficiency and safety of the known

I asked all the teachers to tell me about their rationale to be involved in the action research project. A common theme in the answers was some kind of disaffection towards their own teaching style and the results of teaching. The "old" teaching practice was considered as insufficient. But leaving it was hard: there was also a feeling of safety of the known. The teachers wanted to be better teachers and they asked for more knowledge about other approaches to teaching. On the other hand they were worried about negative reactions from the students and about unknown effects on learning, especially for the low achievers.

The reasons the teachers gave for changing their teaching practice were many, and related to both their professional needs and to the needs of the students. The teachers referred to the monotony of the traditional teaching approach and considered their teaching as being too theoretical and with far too little practical mathematics. Bumping around with rules, too little time to think for the students and too much reliance on traditional tests were other statements about their teaching. The teachers also referred to bad learning results, many uninterested and passive students with no or very little ability to think mathematically and to students with no ability to put their mathematical knowledge into words or to read mathematical textbooks on their own. To put a stronger focus on the individual student and on fostering the students' responsibility for learning were goals for the PUMA project but caveats against too much reliance on individualized instruction were also expressed, the effect of which could, according to Kaj, be "*a kind of surface level knowledge when the students know things just for the moment*".

Tom, who was one of the initiators of the PUMA project, described his feeling of meeting an impasse after over 20 years of teaching:

> *I came to a point where I felt that I was stuck, I felt that I was trapped in patterns that I didn't find successful. All my teacher life I have tried to improve myself, but in a way I felt that I was empty and repeating myself.*

But, again, as the students obviously wanted Tom to act in the same way as usual, it was also easy for him to find motives to stop the struggle for changes in his instruction:

> *The students want teacher-centred teaching, that is what they want and that is what I am good at, I am such a person who is good at explaining and it is the most convenient way for the students, to get it explained by me, they understand and it goes well and it is a simple way to take. As soon as I slipped back into my*

old teaching style they (students) started to look well fed because I looked the way they were used to.

Despite of this expressed means-end connection between teacher explanations and students' satisfaction and understanding, some minutes later in the interview Tom declared bad learning results from his traditional mathematics teaching as one of main motives for the PUMA project. Unfortunately this contradiction was not dealt with any further during the interview.

However, resistance to change is understandable and this was a topic that had to be dealt with within the action research process. When a teacher has used the same approach for years, it has become very self-explanatory and convenient to follow. The teacher feels safe in the instructional pattern that he has built during the years, a pattern that the students also have become used to. The words of Åke as we were discussing changes in the role of the teacher, may characterize this situation: "*you take the book, go into the classroom without thinking, especially in grade 7, you give the answers to keep the students satisfied*".

Resistance to instructional change among other colleagues was also touched upon by Åke, who said in the interview that those teachers "*who like to walk in the same old rut*" can turn out to feel threatened or jealous and to be very negative towards those teachers who "*have the ability or the strength*" to commit themselves to change. His remark opens up a very important issue, namely, in order to understand teacher change one has to look at the whole educational culture wherein the teacher is just one of the components. This issue transcends the scope of the paper and will not be dealt with any further.

Beliefs and doubts, successes and shortcomings

Since introduction of new components in the formal assessment of students' learning was one of the aims of the PUMA-project, it was no surprise that the belief in assessment as a key device in changing students' working and learning habits was very explicitly expressed by the teachers.

Especially Tom who, now in retrospect, discussed the assessment structure introduced by him and his two colleagues during the preceding school year, was very clear in his stated belief: if you change the objectives of the assessment this will inevitably be followed by a associated change in student learning.

In the long run I think that it is the assessment, the pieces of evaluation that are important, because it is with them we can change the behaviour of the students. They do those things that they know are evaluated… it is as simple as that. If *you say that this will appear in the test, whatever type of test, that is what they learn. That's the way they function.*

The new components of assessment were also thought of as having the most viability and to be one of the lasting outcomes of the action research project.

When PUMA ends, I think the assessment components, like the new test-structure, the projects, the monthly problems, perhaps the written homework assessment, will stay in some form, but I am not sure to which extent, these are as we know quite heavy now, but I think they could be something that perhaps have got the strongest foothold. (Alf)

Even though Alf believed in the lasting power of the new assessment components, we can see from the quotation above that he nevertheless was somewhat critical to their relative importance within the PUMA project. In the quotation he describes the assessment structure as "*quite heavy*". Later, when I asked him to comment on and predict the eventual learning differences between "PUMA students" and other students at the time they leave lower secondary school, he also expressed a sincere worry about the possible negative impact on the low-achieving students and stated that "*it is a little bit alarming, for the weak students, it seems like it is not to their advantage and PUMA teaching might even be worse for them than traditional teaching*".

During the early stages of the action research process I had become aware of contradictions between aims of the project regarding the students and the way these aims were construed and implemented. For instance, even though more student-talk and communication was considered as an important aim, there was still a tendency among some of the teachers to regard silent classrooms and concentration on individual seatwork as good norms. These teachers described a teacher-centred style of teaching. They were relying more on bi-lateral teacher-student communication than on small-group arrangements and discussions among the students.

If you are going to reason with the students and build on their talking mathematics, you demand that they concentrate on that and this is kind of hard for them, perhaps we have more of the type of teaching that demands concentration and that they don't talk to the class-mates (Per).

But, on the other hand, the descriptions of implementation also indicate that the teachers really tried out ways to increase the amount of student-talk in the classroom. To withhold from temptation to give answers too quickly, to ask for different viewpoints and explanations, to be more patient and to give the students more time to think, these were all methods used by the teachers in order to support student-talk. One teacher also described explicitly how he empowered the students by letting them work in pairs.

> *More this pair-work type, when you work with the tasks, I urge them all the time to discuss with each other, to help each other, they have to take responsibility for each other. I even become angry if they don't take the responsibility. This may be a result of PUMA.* (Ove)

There were obviously several stumbling blocks in the process to increase the amount of student-talk. One was the felt pressure of delivering the curriculum in conjunction with lack of teaching time. This made the teachers use a more didactic, teacher-centred approach than they wanted to.

> *In the beginning I thought more of the process and getting the students to talk mathematics and find out the answers, but there is lack of time and I notice how I slip back to my old teaching style … I have never been really didactic but … yet I think I am not enough patient to pull out the answers from them, don't give them the time to think, to think things through* (Åke).

Another stumbling block can be found within the totality of the instructional situation. Some students did not accept the rules of the new didactical contract in the way the teachers wanted it to be established in the classroom, and from the teacher's point of view the students did not behave properly because of lack of motivation. Per described the situation as "*some students are playing the game, which was expected, some are motivated, then there are others who might tear it down and if there are enough of these kids in a classroom all teaching, not only PUMA teaching, gets difficult*"

In the way the teachers commented on effects of the PUMA project I could notice a tendency to see the new teaching strategies and assessment components as external and time-consuming "bits and pieces" that disturbed the delivery of the ordinary content.

> Kaj: *You have to be a little bit careful not to split up the normal topics, they might fall apart by all this other stuff.*
>
> A-S: *What do you mean with normal topics?*
>
> Kaj: *If we take, say, calculation of percentages, and you are working with that, and then, in the middle, you jump out and do something else…*

Before the part of the interview quoted above, Kaj was commenting on possible negative side-effects of project-work, especially for the low achievers. According to him traditional teaching and a stronger concentration on basics would enhance their learning more than the "*other stuff*". Obviously one solution to this dilemma, from Kaj's point of view, would be to integrate the project-work into the normal curriculum and hence have the projects subordinated to the pace of the textbook, instead of having them separated and done by the students outside ordinary lessons.

When we discussed the teaching strategies that were used to implement the goals of the PUMA project the teachers talked about the nice sense of success from giving the students more time to think and from devoting more time to class-discussions. According to one of the teachers this also gave the boys a more honest chance, boys that were innovative in the discussions but didn't succeed so well in the formal tests. But the sense of success was whittled down by a devastating feeling of not being able to gain and hold the student interest and getting all the students motivated to contribute to the teacher-led class-discussions.

Besides student motivation, another area of concern was how to cope with the academic and motivational heterogeneity among students. The teachers felt quite strong tensions between their concern for scaffolding the learning of individual students and managing the whole class. Even though these problems were not new and not directly outcomes of the new teaching components, there seemed to be connections. A common opinion among the teachers was that the good students were the winners and that the gap between low- and high achievers was widening. Good students work well within any teaching approach and now the many project-works, problem-solving sessions, class-discussions etc. raised their standard of knowing and thinking more than ever before. There were concerns among the teachers that the low achievers and unmotivated students didn't get to practice those things they needed. Traditional teaching, where the teacher is pushing the low achieving student to practice one, preferably practical, task at a time would perhaps be better? One teacher suggested the introduction of two types of project-works, one for those who need to practice the basics and another type for those who are high achievers. The teacher could then direct the low achievers to the former type where *"you more work with something and where the mathematical connections and patterns and such isn't a salient feature*". Students, who wanted to achieve more and get higher grades, could then be directed to project-works where finding connections and patterns was a goal. Another type of solution suggested was to form two or three homogeneous groups according to ability and then give the students an opportunity to choose between the ability groups, at least occasionally.

I think it would be ok to let the students choose according to ability, in my class two groups of students suffer, the very best and the very weakest. And I think the best students suffer most as I have so many weak students in my class, so I concentrate on them (Åke).

Finally, when I looked at how the teachers described their teaching situation I found very many metaphorical expressions with an essence that the teacher is a kind of coach whose duty is to stop students from falling behind and to lift weak students up, to lift all students to certain knowledge levels and to guide the students or lead them step by step. The teachers described helping, forcing, whipping, pushing, pulling, steering and paving the way for students learning and to make the students work, discover or think. Some teachers expressed concerns about white spots or big gaps in the students' mathematical knowledge and talked about the need of giving tools to the students, about tracking the thoughts of students and students tracking the thoughts of the teacher.

Moreover, the teachers also used metaphors to describe and define their professional reality. In this connection they used phrases like:

- to take a more simple road,
- to follow the stream or groping one's way,
- to be trapped in patterns,
- to be in ten fathoms of water,
- to walk in the same old wheel-tracks,
- to grope one's way,
- to drive according to your beliefs,
- to jump on something that moves and to switch on the brakes.

As I continued to read the interviews over and over again and compared and contrasted texts in order to understand and find meanings behind the statements, I found two main metaphors emerging out of the teachers’ stories: mathematics teaching as transport along tracks and mathematical ideas as commodities stored in individual containers.

The transport-track and commodity--container metaphors

The mathematics teacher is responsible for the transport of every student from the beginning to the end of lessons, through the school year, from one form to another and from one school to the next or into the life outside school. During the transport the teacher selects different strategies and teaching styles, tracks, in order to help individual students to get a certain amount and type of mathematical knowledge. The students' minds are seen as containers where the knowledge is stored. The teachers should change track, use several tracks simultaneously and speed up or slow down the transport according to the needs of the students and the social situation in the classroom. Some students follow keenly the teacher along a more demanding track while some students need to pushed, pulled or lead by the hand along the track because these students might have another goal for the transport than the teacher.

The teachers store the mathematical ideas in each student's container in various ways and in various compositions depending on the quality and type of the followed track. The students themselves might be active to find out what the teacher thinks should be stored in each student's container. Also, there might be some negotiation between the teacher and the student about the amount and structure of the selected content and the pace of the transport. But it is the teacher who has the last say. It is the teacher who has the responsibility to open up all the individual containers, to connect them to the system and make them ready for the transport and who also should decide which type of content each individual container should have when it leaves the school.

> *Mathematics teaching should be more student-friendly, I mean, useful mathematics, student-friendly also in the sense that that they are the ones who should have much to say about the pace, I think they want variation in the methods of teaching, that makes teaching more student-friendly, compared to being one-sided.* (Ove)

> *New methods, like inductive methods, give the students possibilities to discover the new, you reason your way through it but the students come up with the final statements… they will remember more… you give them a more mathematical eye* (Per).

> *I always try to get the students to connect their brains, to think for themselves, but this doesn't always happen, especially not with all of the students. The most effective way to do the connection, to start their thinking about a task, is to walk around in the classroom* (Kaj)

When deciding about the transport and the track it is an advantage for the teacher to know under which circumstances the student will need the stored mathematics (demands of the next school and of life), and also to know how to adjust the transport and the track according to the types and capacities of the different containers (ability, interest, motivation of student). When the students are transported to the next school or out in the real life, the students need their containers to be filled with "*a rich store of mathematical tools*". For instance, a student aiming at upper secondary must know the basic rules by heart otherwise he will get problems to get his container filled later on with more sophisticated mathematical stuff and reach higher levels of understanding. But all students should have more mathematical tools stored in their containers than they will need later on.

> *You should demand that most of the students learn more than they need.* (Alf)

On some tracks the students are offered responsibility for filling up and also deciding on the content of their own containers and also the containers of their friends, but the students don't always have motivation or interest in doing this without the teacher by their side.

> *I transfer a part of the responsibility to them, that they must be responsible for their friends in the group.* (Ove)

> *I said, like, read this, do that, look at those examples and to the tasks … no one did anything, when I came they had been sitting for an hour doing nothing, I think it is really too bad.* (Tom)

Students might also fill their containers together and decide about the content all by themselves or together with their parents. Sometimes students copy work of others and are thus some kind of freeloaders. Some students need be put on remedial transport with some other teacher and this makes it more difficult for the ordinary teacher to do or supervise the filling of the container of these students. It happens that a student is neither aware of nor interested in which transport he is on and what type of ideas he should get in his container. "*There are students who are not aware of that they need this type of knowledge*" (Ove). Some students have a tendency to fall off the transport and some have holes in their containers.

Some final comments

Comments on the PUMA project

Before I did the interviews in December one cycle of the action research process had recently been closed. A reconnaissance and planning meeting was held in August, followed by reflection meetings in October and early December and finally a meeting with evaluation and replanning just before the interviews. In between these more formal meetings the teachers met rather regularly to discuss lesson plans and other curricular issues. Here I will comment on some aspects of the PUMA project on the basis of my insights from the whole three-year process and information from the interviews with the teachers.

To the most powerful aspects of the PUMA project I absolutely count the collaborative planning and the regular group-discussions. In the interviews all the five teachers that were active in the action research process emphasized the collaborative dimension as being the most valuable. The meetings were also platforms where the teachers had the possibility to “*formulate things as thoughts that you perhaps wouldn't have done otherwise, you would just go on*”. These words said by Kaj point at what I want to call the *zoom in - effect* of the collaborative dimension: knowing that you will meet colleagues and discuss certain issues with them, makes you more sensitive and aware of components in relation to these issues as they appear in the classroom.

The younger teachers in the group especially appreciated the apprentice-mentor dimension. To work with more experienced colleagues and learn from their experience was of very great value to them even though this dimension also obviously included a power component that possibly put a check on some critical points during the first action research cycle. This is reflected in the discussion with Åke when he tells me that as he did join something that was already settled by some of his colleagues he just followed the plans without any overt critical reflection.

A-S: *Is there something you miss?*

Åke: *Yes, maybe freedom, I have felt rather tied up. We have discussed, I mean taken a week at a time, and this system makes you rather tied up…*

A-S: *How has this affected you?*

Åke: *I don't know… maybe you haven't done things the way you would have wanted to. Sometimes you want to do an odd thing. With a good class it would have been ok to do the odd things anyhow, but I have a rather weak class, they are completely busy doing this even though they have all the time. Actually they would need more time, I have decided not to feel tied up during the spring, I'll try do take more freedom… perhaps the freedom have suffered because you had to come on now and jump right into the middle of this type of project, and you perhaps didn't want to be bossy right away…And this is not only PUMA, this is if you plan to have the same tests …rather tied up systems.*

The PUMA classes wrote the same tests and did the same project-works according to a timetable that was set in the beginning of the autumn. From the teachers' point of view this uniformity saved time but as we can see from the above excerpt it might have had a detrimental influence on other goals of the project. This type of uniformity reflects a tacit agreement that the same teaching-content is delivered at the same pace in all classrooms and doesn't give enough room for class-bound solutions and the needs of individual students. The commodity-container metaphor discussed earlier helps us to further understand this facet of the instructional practice.

Another problematic aspect was that the set up of the PUMA project did not focus on all the teaching of one teacher, just on his practice in selected mathematics classrooms. This created some kind of a professional schizophrenia. Stepping into a PUMA class meant taking on a PUMA teacher role while being the ordinary mathematics teacher in other classes. This tendency was enhanced by the strong emphasis on special instructional techniques like project-works and other special assessment components that were implemented only in the PUMA classes.

The PUMA project was a process of improvement that focused upon a team of mathematics teachers and restructuring of their pedagogical practices in certain classrooms. The action research format did provide a strategy for the process but, from the point of view of the teachers, the research dimension was a difficult endeavour. The excerpt below points at one reason why a combination of action AND research really was problematic.

> *I don't think before I implement things, I mean, I think when I implement, I mean, when I am faced with a problem the answers come up, I don't sit and think in a vacuum and create solutions, they come up when they are needed* (Tom).

In the excerpt the teacher describes how *he doesn't sit and think in a vacuum* within the classroom context, meaning that the pedagogical transport is on its way and he cannot stop it momentarily to evaluate and reflect, he has to make decisions on the spot. Tom used the words *the answers come up* to describe his decision making in a moment like this. His decision is rapid, pragmatic and related to the totality of the problematic classroom situation. In this instant he acts upon his experience from similar situations and also upon his knowledge about the individual students. The thinking and reflection may come afterwards, but in this occasion new problems might already have entered on the scene.

The complexity of the classroom context makes it difficult for the teacher to screen off the rest in order to sustain concentration and reflect upon aspects related to the facilitation of mathematical learning in order to document them and put them forward within an action research meeting. Also, the teacher's decision making in the mathematics classroom is influenced by the simultaneous existence of mutually competing motives (Skott 1999). For instance, in the PUMA project more student talk about mathematical issues was an espoused principle but organisational priorities made the teachers put constraints on the students' communication in the actual practice (Mellin-Olsen, 1991).

Summary

In the paper I have briefly presented a school-based project, the PUMA project, where five mathematics teachers for three years were developing their instructional practices at the lower secondary level of a teacher training school. Action research was used as collaborative and supportive framework for the change process. To illustrate the dilemma of professional change I have used the teachers' own voices that were captured during interviews half a year after the start of the project. Using the teachers' descriptions of their professional reality I have showed their struggle to implement the espoused principles of the project and thus leave the safety of the known and established classroom routines. For instance the teachers realized that the implementation of some of the espoused principles of the PUMA project conflicted with the expectations of many students and also with what they themselves considered to be the needs of the low-achieving students.

From my own experience as a mathematics teacher and from listening to Tom, Åke, Kaj, Per, Alf and Ove I can assert that improvement and change of mathematics teaching is not easily done by a linear plan-act-reflect-evaluate-plan etc -process. Mathematics teaching, as all teaching, can be compared to a chaotic situation where many disparate demands are hanging over its executors. Dispirited, uninterested, anxious or confused students spell management problems. An uncovered syllabus might spell negative reactions from colleagues in the team and on the next level of schooling. It might perhaps also spell official and parental disapproval. According to Goldsmith & Schifter (1997) teachers can neither realize the new insights of a progressive not-traditional instruction themselves nor rearrange this view of school-mathematics by simply adjusting a bit of practice here and there or by importing a teaching technique or curriculum practice (p. 20). In addition to the changes that have to occur in the teachers' own beliefs, understandings and style of mathematics teaching, there also has to be a continuous support from colleagues and from researchers. The action research format can act as a framework for such a support structure if only the teachers themselves are willing to let go of the safety of the known.

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/ Ann-Sofi Röj-Lindberg

Ancient areas: A retrospective analysis of early history of geometry in light of Peirce's "Commens"[1]

Abstract

In his writings, Peirce indicated that he was well aware of the role of individuals, with their thoughts and feelings, and of their social and cultural milieu (not his terminology – he preferred to refer to community*) in the construction and communication of knowledge, including mathematical knowledge. In this sense he bridged the dualisms of Descartes – including the split between imagination and reason – and antedated the present trend of acknowledging both psychological and sociological aspects as important in studying the teaching and learning of mathematics. In this paper some questions involving the early history of geometry are explored in the light of Peirce's construct,* commens*, which he defined as "that mind into which the minds of utterer and interpreter have to be fused in order that any communication should take place" (Peirce, 1998, p. 478). How this construct and others related to it in a semiotic system can inform a retrospective look at geometry with an eye on classroom learning of geometry today, is the topic of this paper.*

I shall start in the first section by identifying some of the notions in Peirce's philosophical writings that have relevance to the historical development of mathematical thought. These notions include the construct of *community* – which he used without definition – and his definitions of *commens* and *synechism*, including his "law of mind". The structure of this paper is then continued in a section showing how these constructs have bearing on some aspects of the history of geometry. Finally, a third section traces the relevance of Peirce's insights in this regard in the present-day teaching and learning of geometry.

Peirce's views on the community's role in knowing

Many of Peirce's thoughts concerning the community and its role in human knowledge were expressed in his earlier writings in the context of the relative

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[1] This draft paper was presented in the Discussion Group 7, *Semiotic and socio-cultural evolution of mathematical concepts*, at PME 27, Hawaii July 13-18, 2003.

merits of nominalism and realism. On the one hand, he defined nominalism as a form of individualism, “the doctrine that nothing is general but names; …common nouns represent in their generality nothing in the real things, but are mere conveniences for speaking of many things at once” (Peirce, 1992, p. xxiv). Realism, on the other hand, holds that “the essences of natural classes have some mode of being in the real things” (ibid.). These modes of thought, and the *ideal-realism* that Peirce espoused in the later evolution of his thought, will not be interrogated in this paper except insofar as they relate to Peirce’s views on the community:

> Though the question of realism and nominalism has its roots in the technicalities of logic, its branches reach about our life. The question whether the *genus homo* has any existence except as individuals, is the question whether there is anything of any more dignity, worth, and importance than individual happiness, individual aspirations, and individual life. Whether men really have anything in common, so that the *community* is to be considered as an end in itself, and if so, what the relative value of the two factors is, is the most fundamental practical question in regard to every public institution the constitution of which we have it in our power to influence (Peirce, 1992, p. 105; his emphasis).

In this view, the role of community in the public institution of mathematics education is an issue of fundamental practical importance. The significance of the community of thinkers in the evolution of mathematical knowledge is indicated in Peirce’s somewhat negative designation of the individual – uninformed by the sociocultural milieu – as ignorant and in error:

> The individual man, since his separate existence is manifested only by ignorance and error, so far as he is anything apart from his fellows, and from what he and they are to be, is only a negation (Peirce, 1992, p. 55).

With regard to the genesis and evolution of mathematics, a point that has relevance in Peirce’s epistemology is the continuity of past, present, and future. Continuity is central in Peirce’s definition of *synechism* as “the tendency to regard continuity … as an idea of prime importance in philosophy” (Peirce, 1992, p. 313). Synechism involves the startling notion that knowledge in its real essence depends on *future* thought and how it will evolve in the community of thinkers:

> Finally, as what anything really is, is what it may finally be come to be known to be in the ideal state of complete information, so that reality depends on the ultimate decision of the community; so thought is what it is, only by virtue of its addressing a future thought which is in its value as thought identical with it,

> though more developed. In this way, the existence of thought now, depends on what is to be hereafter; so that it has only a potential existence, dependent on the future thought of the community (Peirce, 1992, pp. 54-55).

Whether “the ideal state of complete information” is ever an attainable goal, is a matter of doubt, but the relevance of synechism for the history of mathematics lies in the role attributed to future generations of thinkers in assessing the achievements of the past and present. The notion of synechism is further explicated as follows, in connecting individual and community ideation through the role of convention in semiosis.

Individual semiosis in relation to the community.

Notwithstanding Peirce’s early negative characterization of the “ignorant” individual, his writings express an appreciation of the importance of individual interpretation in semiosis, the use of signs in communicating ideas. This individual interpretation is balanced by the role of convention (and hence the community of thinkers) in symbolic representation:

> Peirce’s inclusion of the interpretant as fundamental in the sign relation shows that all thought is *to some degree* a matter of interpretation. All advanced thought uses symbols of one kind or another, and thus rests on convention. On Peirce’s view, then, all advanced thinking depends on one’s participation in a linguistic or semiotic *community*. Peirce’s stress on the importance of community was a common theme throughout his work, and it increased as he came to understand more fully the importance of convention for semiosis. Peirce appealed to a community of inquirers for his theory of truth, and he regarded the *identification with community* as fundamental for the advancement of knowledge (the end of the highest semiosis) and, also, for the advancement of human relations (Peirce, 1992, p. xl: introduction by the editors; their emphasis).

Because the semiosis of the individual is mediated by the community through the adoption of certain ways of thinking and representing ideas as conventional, the growth of (mathematical) knowledge manifests continuity. He cast further light on what he meant by continuity in his *law of mind*:

> Logical analysis applied to mental phenomena shows that there is but one law of mind, namely, that ideas tend to spread continuously and to affect certain others which stand to them in a peculiar relation of affectability. In this spreading they lose intensity, and especially the power of affecting others, but gain generality and become welded with other ideas (Peirce, 1992, p. 313).

Because of the importance of personal interpretations in forging a community of thinkers with its conventions, and thus in the continuity of ideas, Peirce formulated three kinds of interpretant in his semiotic model. But why do triads keep on appearing in his philosophical writings?

Trichotomic and communication.

According to Peirce (1992), *trichotomic* is the art of making three-fold divisions. By his own admission, he showed a proclivity for the number three in his philosophical thinking. “But it will be asked, why stop at three?” he wrote (Peirce, 1992, p.251), and his reply to the question is as follows:

> [W]hile it is impossible to form a genuine three by any modification of the pair, without introducing something of a different nature from the unit and the pair, four, five, and every higher number can be formed by mere complications of threes (ibid.).

Accordingly, he used triads not only in his semiotic model including object, sign (sometimes called the representamen), and interpretant, but also in the types of each of these components. This model includes the need for expression or communication: “Expression is a kind of representation or signification. A sign is a third mediating between the mind addressed and the object represented” (Peirce, 1992, p. 281). In an act of communication, then, there are three kinds of interpretant, as follows:

- the “*Intensional* Interpretant, which is a determination of the mind of the utterer”;
- the “*Effectual* Interpretant, which is a determination of the mind of the interpreter”; and
- the “*Communicational* Interpretant, or say the *Cominterpretant*, which is a determination of that mind into which the minds of utterer and interpreter have to be fused in order that any communication should take place” (Peirce, 1998, p. 478, his emphasis).

It is the latter fused mind that Peirce designated the *commens*.

For the continuity of mathematical ideas and their evolution in the history of mathematics, a central requirement is that there be a community of thinkers who share a “fused mind” sufficiently to communicate effectively with one another –

and with posterity through their artifacts – through this commens. While acknowledging the importance of both the intensional and effectual interpretants for communication (and the implicit potential for miscommunication), in this paper I shall concentrate on the third member of this triad, the interpretant generated by the commens, because it is this interpretant that leads to the adoption of conventional signs by an intellectual community.

In the following section these theoretical notions will be used as lenses in interrogating some issues in the early history of geometry.

Ancient areas: Some questions regarding early history of geometry

In any age, the commens plays a part in forging the community of thinkers in a particular field, and geometry is no exception. Perhaps influenced by economical, technological, or intellectual needs current at the time, some ideas resonate in this "fused mind," are hence are adopted according to Peirce's *law of mind* (quoted earlier), in which "ideas tend to spread continuously and to affect certain others which stand to them in a peculiar relation of affectability." In the continuity of this commens, a community is created. Such a community is illustrated by the geometers of ancient Egypt's University of Alexandria, which was named after Alexander the Great who established Alexandria as a capital city, and which was founded by the first Ptolemy, then ruler of Egypt, in 306 BCE. Members of this community included such illustrious geometers as Euclid, who was a professor of mathematics at the University of Alexandria, Archimedes of Syracuse in Sicily (who is reported to have spent time as a scholar at Alexandria), Eratosthenes of Cyrene who became the chief librarian at this University, and Apollonius who studied there under the successors of Euclid (Eves, 1992, p. 171). Euclid, Archimedes, and Apollonius are the three mathematical giants of the third century BCE, but the Alexandrian community included many other illustrious scholars some of whose works are still known today (e.g., Diophantus).

The community of mathematicians of Alexandria flourished for more than 500 years, but gradually faded during Roman occupation after 31 BCE, the decline being caused by a combination of technological, political, economical, and social factors (Eves, 1992, p. 137). It is possible that the decline was in accord with the second part of Peirce's law of mind, in which the continuous spreading of ideas fades but becomes absorbed into the general tone of the community's thinking: "In this spreading [ideas] lose intensity, and especially the power of affecting others, but gain generality and become welded with other ideas."

Recall that Peirce considered that "reality depends on the ultimate decision of the community;" and that "the existence of thought now, depends on what is to be hereafter; so that it has only a potential existence, dependent on the future thought of the community." In light of hindsight, and the *future's* characterization of the geometrical though of this age as extremely powerful, some specific questions come to mind.

- More than 2000 years ago, Archimedes employed a method of exhaustion (also known to Eudoxus and used by him and by others in that community) to calculate the area enclosed by a parabola and a line segment. Why was it only in the 17th century that the development of such methods became widespread with the advent of integral calculus?
- Hipparchus of Crete generated some excitement when he figured out that the area of the "lune" was the same as that of a right triangle whose hypotenuse was the diameter of the lune. Why was this discovery important in the geometry of the time?
- Why did it take two millennia for the consequences of challenging Euclid's parallel postulate to be brought to fruition in systems of hyperbolic and spherical geometries (or hyperbolic, parabolic, and elliptic geometries, as Klein called the three forms of geometry in 1871)?

In considering these questions it is necessary to recall that there were certain conventions associated with the geometrical *rules of the game* in the school at Alexandria. The "fused mind" of the community, even prior to Euclid, had adopted the convention that the only acceptable geometrical constructions were those performed using the Euclidean tools of compasses and straightedge. With these collapsing compasses it was possible to draw a circle with any given point as center and passing through any given second point, but the compasses would collapse if lifted from the drawing medium. With the unmarked straightedge, according to the *commens*, one could draw a straight line of indefinite length through any two given distinct points – but measurement was not permitted.

The self-imposed restrictions of the Alexandrian school, as it turned out, generated three unsolvable problems, namely, the well-known problems of antiquity: the trisection of an angle, the duplication of the cube, and the quadrature of the circle. In the commens of the age, it seemed reasonable to expect that these problems were capable of solution under the accepted conventions. After all, it was an easy construction to bisect an angle using Euclidean tools. And the lune of Hipparchus (to answer the second question in the foregoing) seemed to suggest a method that might possibly lead to the quadrature of the circle. It was only in the 19th century that the impossibility of solution of these problems with Euclidean tools was finally established. But the energetic search during many centuries was fruitful in that it led to analysis of the conic sections (by Apollonius), knowledge of many cubic and quartic curves and several transcendental curves, and much later, to insights regarding domains of rationality of equations, algebraic numbers, and group theory. The search for solutions to the three problems of antiquity thus not only profoundly influenced classical Greek geometry, but through the continuity inherent in synechism and the role of the commens in perpetuating the knowledge of this ancient community, generated a legacy for mathematics today.

In interrogating the third question in the foregoing (regarding why it took two millennia), the notion of a community of minds fused by the commens is again informative. Although many of the ancient manuscripts (including Euclid's original writings) were lost, the signs that were preserved of this community's achievements by Arabian and Latin scholars in the 8th and 12th centuries respectively were sufficient to perpetuate a commens that resulted much later in a new community of scholars, sufficiently removed from the old community at last to take the ideas "where they wanted to go" without stifling the burgeoning absolute geometry that was the logical outcome of challenging Euclid's parallel postulate. Even Gauss, eminent mathematician of the 19th century who did not need to prove his credentials, elected not to publish the results of his ponderings concerning this subject. But the commens was ready for this "strange new universe," as Janos Bolyai characterized it in a letter early in that century (Eves, 1992, p. 499). Thus many scholars, including Bolyai and Lobachevsky independently in Hungary and Russia respectively, and later in a different formulation, Riemann in Germany, developed the ideas in new directions and established a new commens, with new conventions, more than two millennia after the old. Others who developed the ideas and joined this new community included Beltrami, Arthur Cayley, Felix Klein, and Henri Poincaré (Eves, 1992, p. 500).

Thus the consideration of issues surrounding the three ancient problems of antiquity, and the birth of non-Euclidean geometries, illustrates that in a mathematical community the acceptance of conventions may both enable and constrain the development of mathematical ideas in that particular community. The continuity of ideas relies on the use of conventional signs, according to Peirce. But according to his law of mind, the establishment of a community through the commens may not guarantee the continuity of the creative energy that first originated these ideas. Once again, Peirce's law of mind is apposite: "ideas tend to spread continuously and to affect certain others … [but] in this spreading they lose intensity, and especially the power of affecting others, but gain generality and become welded with other ideas" (Peirce, 1992, p. 313). Thus the importance of future thought in synechism is illustrated.

Of the three questions raised at the start of this section, it remains to consider the first, and ask why the method of exhaustion used by Archimedes and others in classical geometry did not result in the development of integral calculus in that age. Apart from the constraining effect of the conventions and the loss of intensity caused by the spreading of ideas, there is another aspect that may have had a bearing on the decline of the Alexandrian school, and that may cast light on the limitation of the development of the method of exhaustion during that age, and that is the sociocultural milieu. When similar ideas again started to enter the forefront of mathematical thinking in the mid-17th century, intellectual society had different preoccupations, a different milieu. Sailing ships from Spain, Portugal, and The Netherlands were expanding the horizons of the ancient world, navigation was an important and developing science, and there was a need to understand the movements of planets and stars in a precise and systematic way. Newton's interest in optics and telescopes developed from the navigational needs of the age. Based on measurements that were precise for those times, Kepler and Tycho Brahe had formulated new theories concerning the movements of stars and planets. Problems concerning rates of change, and areas swept out by arcs of curved paths, created the right soil for the development of both integral and differential calculus in the early forms in which Newton and Leibniz, again independently and in two different countries, conceived them. With the acerbity that resulted from the bitter war over first authorship, it is a stretch to say that Newton and Leibniz shared a commens. However, the resonance of their ideas does suggest that they belonged to a common intellectual community, and the method of exhaustion of the ancient geometers found future fruition in the mammoth intellectual developments of 17th century mathematics.

In these illustrations of how some of Peirce's philosophical constructs may be used as lenses in viewing ancient geometry and its intellectual offshoots, there are possible implications for the teaching and learning of geometry today.

Relevance of Peirce's views in classroom learning of geometry today

In Peirce's later writings particularly, the importance of a sign as a medium of communication is stressed. The notion of the commens or fused mind of utterer and interpreter (taken broadly to mean the one who originates the communicative sign, either by the spoken word or in written text, and the one who interprets this sign) have been shown to have relevance in the forging of historical communities of mathematics scholars, whether in the ancient School of Alexandria, or in the originators and developers of the calculus and of non-Euclidean geometries in the 17th, 18th, and 19th centuries. These Peircean ideas – the commens and its role in forging communities of thinkers through communication – are quite current, and have bearing on the classroom teaching and learning of mathematics today. The importance of communication of many kinds, between teacher and learners, and amongst learners themselves, has been promoted in all the National Council of Teachers of Mathematics recent documents for reform in the teaching and learning of mathematics (NCTM, 1989, 1991, 2000). The role of discourse and its reflexive relationship with the creation of mathematical objects by individual learners in a discourse community is highly current (Cobb et al., 1997, 2000; Dörfler, 2000; Sfard, 2000). So too is the perception that the mathematical constructions of the individual learner – in a psychological approach to these issues – are in a reflexive relationship with a sociocultural approach, including the classroom negotiation of social norms and socio-mathematical norms, and hence with classroom practices (Cobb et al., 2000). A learning community is at the heart of these theoretical formulations. The terminology of Peirce may be different, but many of the underlying theoretical constructs are in resonance. Whether a community of research mathematicians or a community of teenagers in a high school geometry class are the focus of attention, it is the commens, the fused mind and its communicative interpretant, that results in the forging of social and socio-mathematical norms and the resulting conventions of the group. Geometry classroom practices thus rely heavily on the commens.

More specifically, in the semiosis of learning in a school geometry community, the sign's function of representing and communicating geometrical ideas is crucial. Recognition of this centrality is manifested in the NCTM's inclusion of representation (along with communication, reasoning, problem solving, and connections) in the *Principles and Standards for School Mathematics* (2000). The heart of representation is semiosis: thus it is likely that Peirce's theoretical constructs in these areas will continue to be relevant to research and to have bearing on the teaching and learning of mathematics today.

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/ Norma Presmeg

Dynamisk programvara, ett didaktiskt verktyg?

På SMDF:s årsmöte 24 jan 2003 höll Sveriges första professor i matematikdidaktik, Rudolf Strässer, ett föredrag rubricerat *Learning Geometry in Secondary Schools. Aims, Concepts, Computers.* Strässer menade att på tidigare skolstadier används geometri fortfarande som ett verktyg att utforska och modellera världen runt omkring oss med mening att bättre förstå den (SMDF:s Medlemsblad, nr 7). Han beskrev i samma artikel delar av datorprogrammet Cabri Géomètre och också nackdelar som kan uppstå i undervisningen med programmet.

Min erfarenhet är att geometri är ett mycket litet område i en skolelevs tolvåriga utbildning och att datorprogram används relativt lite i matematikundervisningen. Om man nu inte betraktar miniräknaren som en liten dator, för dagens miniräknare kan utföra komplicerade beräkningar och konstruera diagram mycket snabbt. I maj innevarande år deltog jag i en konferens med deltagare från Norge, Finland, Danmark, Nederländerna, Belgien, Tyskland och USA. Denna konferens arrangerades av Texas Instruments där vi skulle diskutera geometrins ställning och den dynamiska programvaran Cabri Junior, som skulle lanseras på en av deras miniräknare. Vi konstaterade att geometrin i samtliga deltagares hemländer förde en tynande tillvaro. Dock angavs det i två tyska delstaters läroplaner att det var obligatoriskt att använda dynamisk programvara i undervisningen av matematik. Möjligen fanns detta angivet i fler stater, men kunde inte verifieras. I Nederländerna ingick dynamisk programvara som obligatoriskt del i deras läromedel.

Många lärare i Sverige använder datorn som presentationsverktyg, ordbehandling, faktabas (Internet), kommunikationsverktyg och för att rita diagram. I matematikundervisningen används många gånger, av de få tillfällen då datorn används, program där man skall ange ett svar och få det kontrollerat av datorprogrammet, eventuellt kopplat till ett visst händelseförlopp.

Dynamisk programvara bygger på begreppen undersök, upptäck och generalisera. Det finns inga rätta svar som man hittar i ett datorbaserat ”facit”. Bland annat därför har denna typ av program intresserat mig sedan 1993 och jag har med tiden blivit alltmer intresserad av hur dynamisk programvara används och vilka strategier lärarna har för att utnyttja denna programtyps olika möjligheter för att studera och lära sig matematik. Många undersökningar har genomförts på elevers matematik-lärande på lägre stadier. Min uppfattning är att läraren är en stark påverkansfaktor bland andra, som påverkar och inverkar på en elevs lärande. Detta var bakgrunden till mitt intresse att påbörja ett avhandlingsarbete om lärares undervisningsstrategier då de använder dynamisk programvara på gymnasienivå. I denna artikel ger jag ett litet exempel på vad jag funnit vid första anblicken av min analys av en lärare i Schweiz. Det är viktigt att påpeka att analysen är långt ifrån färdig och än så länge endast en ansats.

Under en veckas tid besökte jag ett schweiziskt gymnasium (eleverna var ca 14 –19 år) i kantonen Valais. Deras lärare hade använt dynamisk programvara i 10 år. Det står ingenting explicit i deras läroplan att datorer skall användas i matematiken. Läraren i fråga angav att han hade sin rektors fulla stöd att konstruera sina egna matematikkurser. Det finns en läroplan för kantonen som han använder som underlag. Han undervisar i gymnasieår 1, 2, 4 och 5 och har dessutom en mate-matikkurs med dynamisk programvara i en klass två timmar i veckan i år 4. Denna kurs är en tillvalskurs utöver den ordinarie undervisningen. Detta besök var en del av min empiriska undersökning. Jag har även besökt två svenska lärare på gymnasiet. (Se tabell nedan)

Mitt urval av lärare framgår av tabellen nedan, där ”ch” betyder Schweiz och ”se” Sverige.

	Ålder	Kön	Ex år	Datorn i matematik	Undervisnings-ämnen	Und. program
A	**44**	**M, ch**	**1982**	**Ca 10 år**	**Ma**	**Cabri**
B	**34**	**F, se**	**1994**	**Ca 10 lekt**	**Ma, datakunskap**	**Cabri**
C	**50**	**F, se**	**1996**	**Ca 150 lekt**	**Ma, datakunskap**	**Cabri**

Följande kriterier användes vid val av lärare:

- Lärarna skulle vara kunniga i matematik
- Lärarna skulle ha erfarenhet av undervisning i matematik
- Lärarna skulle vara förtrogna med datoranvändning i allmänhet
- Lärarna skulle vara intresserade av att använda datorn i matematik

Önskvärt var att de hade kommit i kontakt med dynamisk programvara. De svenska lärarna har inte samma långa erfarenhet av dynamisk programvara i undervisningen som schweizaren. Det visade sig att lärare A och C hade ca 5 års studier och lärare B har 1,5 år i matematik på universitetsnivå.

Det är inte endast geometri som är ett användbart område då man använder Cabri. Även i funktionslära kan det vara lämpligt att använda denna sorts programvara.

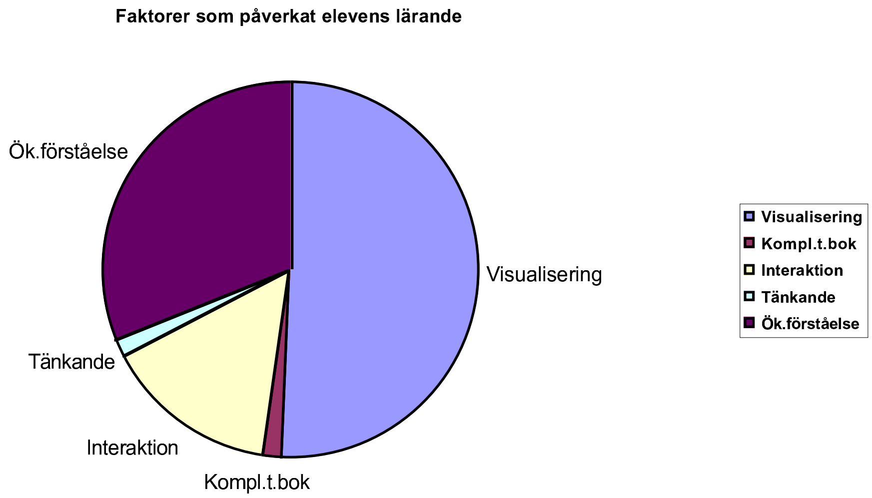
Mina övergripande frågeställningar är

- Varför har dessa lärare valt att använda datorer i sin matematikundervisning?
- På vilket sätt anser de att datorn kan användas som didaktiskt verktyg?
- På vilket sätt kan elevernas lärande i matematik stärkas genom datoraktiviteter med dynamisk programvara?

Dessa kan sammanfattas i följande: Vilka är lärarens underliggande undervisningsstrategier när dator används i matematikundervisningen?

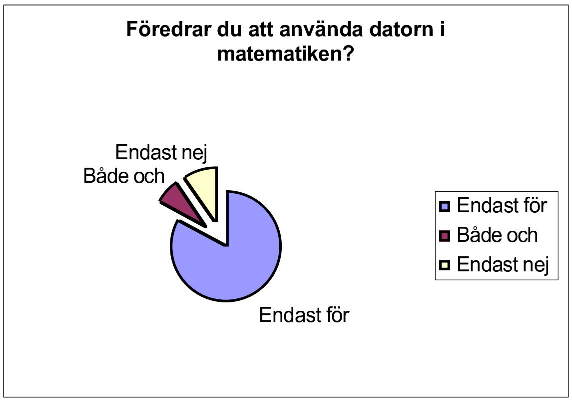
Under mitt veckolånga besök i kantonen Valais, besvarade 53 elever från år 1 till år 5 en enkät med följande frågor. Vad har du lärt dig denna vecka? Ge exempel på hur du tror att datorn interagerat i ditt lärande. Föredrar du att arbeta med eller utan dator när du studerar matematik? Motivera ditt svar.

Efter att ha analyserat och kodat svaren fick jag bland annat det resultat som visas i diagrammet.



På frågan hur datorn hade interagerat i elevens lärande framgick att visualiseringen var mest betydelsefull, vilket ansågs vara kopplat till ökad förståelse. År 2 var tyvärr inte med i denna enkät. Eleverna hade bland annat lärt sig att använda grundläggande kommandon och undersökningar, formulera matematiska satser och definitionerna på grundläggande geometriska axiom (under gymnasieår 1), under gymnasieår 4 hade de undersökt bland annat asymptoter, under gymnasieår 5 hade undervisningen bland annat gått ut på att upptäcka Bernoullis lemniscata och att finna olika sätt att lösa ett problem. Under gymnasieår 2 skulle eleverna finna en skatt genom att tolka en text och översätta den till geometriska figurer och därigenom finna denna skatt. Det var deras tredje lektion med programmet och det som de upptäckte hade varit svårt att se utan användningen av dynamisk programvara. Detta skulle senare formellt bevisas. Det ingick i lärarens undervisningsstrategi att det man ser på skärmen, inte måste vara sant generellt, utan måste också bevisas formellt. År 3 besökte jag inte klassen, då läraren inte hade någon undervisning i denna årskurs.

Elevernas svar med motiveringar på frågan om de föredrog att använda datorn i matematiken framgår av diagrammet och kommentarerna nedan. Den grupp som valt en kurs i matematik med dator bestod av sju elever och de svarade att det var på grund av datorstödet som de valt kursen.

	Anledningarna för *ja* var visualisering, möjligheten till *drag mode* egenskapen, intressant, snabbt, lättare att förstå och att man kunde samarbeta. *Drag mode* innebär att man kan förändra figurens form genom att dra i en punkt. Alla konstruktioner bibehålls och mätvärden anpassas. Anledningen till *nej* var framför allt att de var dåliga på att hantera datorn eller inte tyckte om datorer över huvud taget. *Både och* motiverades med att det berodde på vad man sysslade med i matematiken

Läraren använde olika sätt att undervisa beroende på innehållet. De tre övningar som behandlades i år 1 innehöll en tydlig progression: Första övningen var mycket utförlig med anvisningar om vilka kommandon eleverna skulle använda. Den andra var uppbyggd på samma sätt, men uppgifterna blev mer avancerade. I tredje övningen fanns inga anvisningar beträffande programmet men komplexiteten i uppgifterna ökade än mer. Lektion två till fyra ägde rum i klassrummet, med tillgång till lärardator och ägnades bland annat till formell bevisföring. Läraren var mycket noga med, som tidigare nämnts, att eleverna skulle förstå att det man såg på skärmen och hade generaliserat inte nödvändigtvis behövde vara sant för alla fall och därför var de tvungna att formellt bevisa den sats de arbetade med.

Jag ämnar inte göra någon övergripande jämförelse mellan lärarna i Sverige och läraren i Schweiz. Dessa tre lärare är intresserade att använda dynamisk programvara i matematiken. De har emellertid helt olika sorts elever. Eleverna i Schweiz är ett urval av elever som ämnar studera vidare på universitetet. För eleverna i Sverige är matematik inte deras primära ämne på samma sätt som i Schweiz.

Jag ser fram emot att analysera mitt material angående de svenska lärarna. Förhoppningsvis skall mitt avhandlingsarbete kunna avslutas under 2004.

/ Lil Engström

Två år i forskarskolan

Nu har de två första åren av forskarutbildningen förflutit och flera av oss närmar sig eller har kommit förbi ’första halvlek’. En tid av många lästa kurser, många av dem gemensamt i Riksbankens jubileumsfonds regi och många på våra respektive hemuniversitet. För min del innebär halvtiden att de flesta kurserna avverkats och att den kvarvarande tiden mer exklusivt kommer att ägnas åt avhandlingsarbete. Det känns både tillfredsställande och lite oroande att på pappret hunnit halvvägs. Skönt på det sättet att halvvägs innebär en sorts bokslut, men också oroande då halvvägs för min del inte innebär att jag hunnit halvvägs in i själva avhandlingsarbetet. Kursdelen har den fördelen att det går att ”bocka av” kurser då man läst färdigt dem, man kan lämna dem bakom sig och gå vidare. Avhandlingsarbetet ter sig mer iterativt. När man arbetat en lång tid med sin problemformulering och tror att man nått den ’slutliga’ formuleringen, kommer man obönhörligt fram till nya frågor som kräver andra metoder osv., dock med förbehållet att man ändå i bästa fall kommit en bit framåt.

En speciell sida av forskarskolan är, och har sedan starten varit, nätverkstanken. Oavsett om vi träffats för någon gemensam kurs eller den årliga sammankomsten har en viktig funktion varit att just få träffas, utbyta tankar, projektidéer och prata om vår aktuella och framtida forskning. Detta ser jag som en av de absolut bästa ingredienserna av själva forskarskolan. De gemensamt anordnade kurserna har också gjort att vi har på nära håll fått bekanta oss med våra kollegors hemmiljöer och har på ett naturligt sätt givit oss möjligheten att se hur olika institutioner arbetar med och organiserar i huvudsak samma verksamhet. Jag tror att få andra doktorander ges den möjligheten. Vid just en av de årliga sammankomsterna, i Sigtuna i våras, fick vi även förmånen att i ett vidare samhälls- och utbildningspolitisk perspektiv få höra om de förväntningar och de olika roller beslutsfattare på olika nivåer tror att vi kan tänkas få för framtida utbildning i Sverige.

Ett återkommande syfte med de årliga mötena är också att ta upp och belysa sådana moment som är viktiga i ’allmänbildningen’ hos en forskare, men som inte ryms inom ramen för traditionella kurser. Exempel på teman som tagits upp är handledarrollen, olika forskningsetiska frågor och kopplingen mellan forskning och praxis. Dessa teman gör att man lyfter blicken från den ibland ganska ensidiga fokuseringen på kursläsandet och den egna forskningen och skapar förutsättningar för att vidga perspektiven lite grand.

Avslutningsvis vill forskarskolans doktorander genom undertecknad rikta ett stort tack till ledningsgruppen och ett än större tack till Gerd Brandell för att ha skapat ett unikt nätverk. Här skapas förutsättningar för att matematikdidaktiken skall kunna växa och att detta kommer att kunna ske genom samverkan.

/ Jesper Boesen

Matematikens historia – ett äventyr

För alla matematikdidaktiker är det viktigt att aldrig tappa kontakten med kärnan i verksamheten – nämligen matematiken själv. Ett sätt att närma sig matematiken såväl inifrån som utifrån är att studera dess historia. Som doktorander i den nationella forskarskolan i matematikdidaktik har vi under läsåret 02/03 haft möjligheten att gå två kurser i just matematikhistoria. Under hösten deltog vi i en inledande kurs i Anders Tengstrands regi vid Växjö universitet och under våren gick vi en kurs ledd av forskarskolan genom Uppsala universitet och Sten Kaijser. Efter denna engagerande och inspirerande tid vill vi här dela med oss av våra tankar om matematikhistorien i relation till undervisning i matematik och upplevelsen av matematiken som ämne.

Matematikdidaktiska studier av matematikhistoriens relation till elevers och studenters lärande i matematik är ett ständigt växande forskningsfält. Exempelvis behandlas frågor kring hur kognitiva språng, som har uppkommit under matematikens utveckling, kan relateras till de eventuella kognitiva hinder som elever och studenter upplever i dagens undervisning. Dessa och andra frågor kan vara bra att ha i bakhuvudet då vi studerar matematikens historia och dess eventuella möjligheter för oss som matematiker, matematikdidaktiker, lärare eller matematiklärande.

En vanlig uppfattning om matematiken är att den är ett fixt och färdigt ”paket” som saknar själ och identitet. Denna fyrkantiga bild av ämnet tror vi kan transformeras med hjälp av matematikhistoriens funktioner. Till exempel kan studiet av framväxten av matematiska begrepp visa oss den vidd av insikter och olika idéer som ligger till grund för de begrepp vi har idag. Därmed kan man även peka på att den matematiska utvidgningen knappast har varit friktionsfri och linjär utan har kantats av ständiga kontroverser och motgångar. Vidare kan de olika föreställningarna ofta relateras till upphovsmännens samtid. Detta gör att individerna bakom matematiken kan få träda fram och ge matematiken en mer personlig karaktär. Vår uppfattning är att detta kan leda till en djupare förståelse för matematiken som ett levande ämne.

Har det funnits revolutioner i matematiken? Denna fråga väcktes under den ena kursen och oavsett svar på frågan kan man tveklöst säga att människans tankar kring matematiken, vårt sätt att se på vad matematik är, har förändrats med historien. Med matematikens utveckling har vi inte bara fått ytterligare substans att arbeta med, utan även vårt sätt att tänka metamatematiskt har förändrats. Med andra ord kan vi genom historiska betraktelser tydliggöra förändringar *i* matematiken såväl som i vårt tankesätt *om* matematiken. Detta torde utgöra intressanta och värdefulla grunder för alla som arbetar med matematik.

Ett naturligt sätt att närma sig matematikhistorien är att läsa någon av de matematikhistoriska sammanställningarna som skrivits genom åren. Det finns också bra exempel på böcker som lyckas balansera presentationer *om* matematiken med framställningar *i* ämnet. Vi kan exempelvis rekommendera *A History of Mathematics* av Katz. I detta sammanhang vill vi dock betona att vi hellre uppmanar läsaren till att själv ta del av originaltexter i matematik istället för att läsa om dem i andra hand. För oss är det framför allt denna erfarenhet, från kursen i Växjö, som har väckt vårt engagemang och vår lust inom ämnet. Att försöka läsa och förstå texterna, med den äldre notationen och de från vårt perspektiv begränsade verktygen, är otroligt spännande för att inte säga äventyrligt. Sätten att angripa matematiska problem hade ofta fokus på andra metoder eller moment jämfört med hur vi skulle hantera motsvarande problem i vår tid. Förutom att lära oss att se andra möjligheter och idéer präglade av författarens samtid, så lär vi oss också förstå värdet i de matematiska verktyg som till exempel ett välutvecklat beteckningssystem ger.

När vi talar om matematikens utveckling vill vi betona att den inte nödvändigtvis alltid innebär förbättringar. Exempel på detta är själva presentationen av matematiken i de skrivna verken. Vi vill påstå att vi idag inte är bättre pedagoger än flera av de tidigare matematikerna, något vi får belägg för om vi till exempel läser *Introductio in Analysin Infinitorum* eller *Vollständige Anleitung zur Algebra* av Euler. Dessa verk tar på ett underbart tydligt sätt upp grunderna inom respektive område och den löpande texten, där exempel och teoriframställning varvas, är förhållandevis lättläst. (Vi jämför då inte med verkets samtida texter utan dagens kursböcker för gymnasie- och högskolor!) Vi tror att det kan finnas stora förtjänster i att använda sig av dessa texter som grund i undervisningen och använda kompletterande inslag från dagens moderna kurslitteratur. Detta kan ha stort värde för kunskapsutvecklingen såväl i som om matematik och dess utveckling.

Språket kan ibland vara ett hinder då vi försöker läsa och förstå mästarnas originaltexter, men sök då efter en översättning som ligger så nära ursprungsverket i tid och struktur som möjligt. Enligt vår erfarenhet är det inte alltför svårt att få tag i kopior av sådana dokument. Men liksom arbetet med själva ursprungsdokumenten så kan även sökandet efter dem vara början till upptäckter av andra äldre matematiktexter och nya äventyr.

Matematikhistoriens möjliga bidrag till undervisningen i matematik diskuterades på Uppsalakursen under ledning av matematikern David Pengelley från New Mexico State University. Pengelley har sedan flera år tillbaka använt matematikens historia som sitt viktigaste verktyg i matematikundervisningen och funnit att studenterna förutom ett matematiskt kunnande också fått en mer levande bild av matematikämnet. Genom att ta del av Pengelleys erfarenhet av användning av historiska originaltexter (eller så nära originalen som språket tillåter) i matematikundervisningen fick vi ytterligare stöd för vår uppfattning om värdet av användningen av ursprungslitteraturen vid matematiklärande. Alltså är det inte bara vi som har blivit ovanligt nyfikna av matematiken och dess historia genom liknande arbete.

Tillsammans med Robert Laubenbacher har Pengelley skrivit *Mathematical Expeditions,* där de tar upp utvecklingen av olika matematiska områden genom att spränga in originaltexter i framställningen. Här uppmanas läsaren inte bara till att försöka förstå vad som skrivits utan utmanas också till att finna misstag eller tillkortakommanden i mästarnas dokument. Ett exempel på detta finns redan i bokens första kapitel som behandlar parallellaxiomet. I sin strävan att bevisa att parallellaxiomet följer ur de övriga axiomen härledde Legendre felaktigt ett ekvivalent påstående. I hans ”bevis” döljer sig nämligen ett misstag då han förutsätter den Euklidiska geometrin. Vad vi fann så inspirerande var inte bara att finna felet i beviset, utan även insikten att också de gamla mästarna kunde vara begränsade i sina tankar kring matematikens möjligheter. I det här fallet förblindades Legendre av den värld, det rum, vi lever i.

Allmänt när det gäller studier av originaltexter är det inte minst fängslande att få ta del av de beskrivningar av ”aha-upplevelser” som författarna uttrycker emellanåt, och vi kan inte annat än instämma i de inledande raderna i *Mathematical Expeditions*:

> Nothing captures the excitement of discovery as authentically as a description by the discoverers themselves.

Innan vi avrundar vill vi delge er ett av våra mest fascinerande minnen från det senaste året - besöket vid *Carolina Rediviva*. Att gå fram längs med de högresta och verkligt mäktiga hyllorna, fyllda av verk med anor, och låta sig fyllas av skönheten från böckernas ryggar är magnifikt. Ändå är det än större att få bläddra och läsa i böcker som Descartes *Discours de la méthode*, tryckt 1637 och

Euklides *Elementa geometriae*, tryckt 1482. Att det senare verket dessutom är fyllt med marginalanteckningar från en tidigare ägare gör upplevelsen än mer speciell, särskilt som ägaren var den inte alltför okände Copernicus. Då det är svårt att förmedla känslan vi fick i närheten av mästarnas verk föreslår vi att ni gör egna studie- eller nöjesbesök till Uppsala.

Avslutningsvis vill vi ge följande tips inför den kommande vinterkylan. Kryp ned i ett varmt bad i sällskap med ett glas rött och *Mathematical Expeditions* - och bara njut!

Litteraturtips

Mathematical Expeditions. Cronicals by the Explorers, av R. Laubenbacher & D. Pengelley, utgiven av Springer 1999.

A History of Mathematics. An Introduction, andra utgåvan, av V.J. Katz, utgiven av Addison-Wesley Educational Publishers 1998.

/ *Linda Mattsson* och *Johanna Pejlare*

En nordisk forskarskola i matematikdidaktik

Nordiska forskarutbildningsakademin, NorFA, lyste hösten 2002 ut möjligheten att söka medel för nordiska forskarskolor för en period av fem år från 2004 och med ett anslag på fem miljoner för perioden. Högskolen i Agder, Kristiansand i Norge sände på initiativ av mig och efter samråd med många av de matematikdidaktiska forskningsmiljöerna i Norden in en sådan ansökan. Det kom in totalt 57 ansökningar om forskarskolor. Av dessa valdes 10 ut till en fördjupad ansökan för att få medel och bland dem fanns forskarskolan i matematikdidaktik. Under våren 2003 utarbetades en fördjupad ansökan och i den processen hade jag kontakt med de flesta forskningsmiljöerna för diskussioner och för att höra om intresse för deltagande fanns. Det resulterade i att 37 miljöer i Norden och Baltikum sände brev där de förklarade att de avsåg att medverka i forskarskolan och vid dessa miljöer redovisas mer än 70 doktorander i matematikdidaktik. Den slutliga ansökan lämnades i juni till NorFA och beslut var utlovat till den 1 september. Emellertid blev beslutsprocessen fördröjd då NorFA hade problem med att få garantier från de nordiska länderna om medel för alla fem åren. Två och en halv månad försenat kom så äntligen det glädjande beskedet att medel beviljats till den nordiska forskarskolan i matematikdidaktik. Starten är planerad till den 1 januari 2004 och därmed blev det en starkt tidspressad förberedelsetid.

I ansökan beskrivs syfte och mål medverksamheten så här:

Huvudsyftet är att stödja och utveckla utbildningen av forskare i matematikdidaktik i de nordiska och baltiska länderna, varhelst denna utbildning ges eller planeras bli given. Målet är att skapa konstruktivt samarbete i avsikt att öka den vetenskapliga kvaliteten i forskning i matematikdidaktik. Vi kommer att sträva efter att ge alla doktorander i matematikdidaktik tillgång till verksamheten i forskarskolan och att skapa samarbete mellan en större grupp doktorander och handledare för att kunna dela erfarenheter och möjligheter att förbättra forskarutbildningen. Det yttersta målet är att skapa ett nätverk av samarbetspartners, som kan fortsätta att arbeta tillsammans efter den period av fem år då den nordiska forskarskolan får ekonomiskt stöd.

I ansökan skrev vi på följande sätt om verksamheten.

Aktiviteter i den nordiska forskarskolan

De huvudsakliga aktiviteterna i forskarskolan kommer att vara

- Gemensamma kurser skapade med hjälp av den samlade kompetens som finns bland alla forskare i de nordiska länderna
- Sommarskolor som bygger på tidigare goda erfarenheter av liknande arrangemang
- Seminarieserier inom speciella forskningsområden som ett komplement till lokala serier och verkstäder om ämnen eller frågor av stor betydelse
- Kompetensutveckling för handledare och utbyte av erfarenheter
- Partnerskap och samarbete med utvalda internationella forskare och miljöer
- Skapandet av en databas för pågående studier, avhandlingar och större utvecklingsarbeten i matematikdidaktik
- Mobilitetsstipendier och speciellt ekonomiskt stöd till doktorander i matematikdidaktik

Styrelsen består av Barbro Grevholm (ordförande), Gudmundur Birgisson (Island), Trygve Breiteig (Norge), Ole Björkqvist (Finland), Mogens Niss (Danmark) och Rudolf Strässer (Sverige). Styrelsen har utgjort den arbetande gruppen i förberedelserna med den fördjupade ansökan och har redan påbörjat sitt arbete inför starten 2004. Den 19 december möts styrelsen i Luleå för att mera i detalj planera vårens aktiviteter.

Doktorander inbjuds att söka stöd för deltagande i någon av två kurser som ges under våren 2004. Stödet avser främst att täcka reskostnader för deltagande vid annat universitet än det egna. I mån av resurser kan visst stöd ges till lågkostnadsboende.

Kurserna i vår är kursen MA605 *Research design and methods in mathematics education*, som ges vid höskolan i Agder, Norge. Kursbeskrivning finns tillgänglig på

http://www.hia.no/realfag/matinst/doktorgrad/english/MA_605.pdf

Den andra kursen är *Skrivprocessen i matematikdidaktik*, som ges inom ramen för den svenska forskarskolan i matematikdidaktik vid högskolan i Kristianstad. En kursbeskrivning kan fås genom att maila till Barbro.Grevholm@hia.no .

Kursen i Norge ges med fysisk närvaro under fyra veckor, vecka 3, 8, 14 och 18, och resten av tiden arbetar man vid sin heminstitution men i kontakt med medstudenter och handledare. Kursen i Kristianstad innehåller två fysiska samlingar, den 10-12 mars samt den 21-23 april.

Vi hoppas att några nordiska studenter ska vara intresserade av vardera kursen. Om många söker stöd kommer ett urvalsförfarande att ske.

Mobilitetsstipendier

Den nordiska forskarskolan har planerat att kunna ge fem doktorander per år mobilitetsstipendier för en månads vistelse för studier vid ett annat universitet än det egna. Ett stipendium kan vara på 25-30000 kronor och ska täcka i första hand kostnader för resor och boende. Det är dags att redan nu börja fundera på om ett sådant stipendium är intressant för dig eller någon du vill rekommendera det till. Sprid informationen till berörda. Även här måste ett urvalsförfarande tillgripas om intresset är för stort.

Forskarskolan ska inte anta egna doktorander utan avsikten är att den ska fungera som stöd och som ett nätverk för redan befintliga forskarutbildningsmiljöer. De miljöer som har intressanta erbjudanden att ställa till förfogande kan höra av sig till styrelsen. Det kan gälla att erbjuda till alla lämpliga kurser som ändå ska ges lokalt, önskan om att få ta ansvar för att arrangera en sommarskola eller ett seminarium för handledare. Alla initiativ välkomnas så länge de ryms inom de aktiviteter NorFa avser att stödja.

De fyra åren av uppladdning inför ICME10 är snart slut och vi ska genomföra det stora nordiska evenemanget under sommaren 2004. Vi hoppas att den nordiska forskarskolan ska medverka till att det samförstånd och den gemenskap som byggts upp under förberedelserna till ICME10 ska kunna leva kvar och komma framtida doktorander och handledare tillgodo. På så sätt kanske matematikdidaktiken kan växa sig stark både som kunskapsfält och som forskningsområde i Norden och Baltikum.

Hemsida

Inom kort kommer en hemsida för forskarskolan att läggas ut vid Högskolen i Agder (www.hia.no) och vi rekommenderar alla att ta del av löpande information där. Önskemål om och synpunkter på verksamheten kan riktas till mig eller någon i styrelsen. Vi hoppas att alla ska uppfatta den nordiska forskarskolan som en värdefull resurs för matematikdidaktiken.

/ Barbro Grevholm

Några intryck från PME 27 / PME-NA 25

Årets PME-konferens var den tjugosjunde i ordningen och med närmare 500 deltagare en av de största som ägt rum under konferensens historia. En bidragande orsak till konferensens storlek var att den i år samordnades med PME-NA25. Konferensen ägde rum under fem varma sommardagar i Honolulu. Den invigdes i Kawaiaha'o Church där konferensdeltagarna fick ta del av Aloha andan som råder på Hawai'i och blev mottagna med kindkyss och halsband.

PME

För de av läsarna som inte är bekanta med PME[1] så är det en del av ICMI[2] och har så varit sedan 1976 då den bildades i samband med ICME3[3]. Huvudmålen för PME är (fritt översatt ur stadgarna):

- att verkar för internationella kontakter och utbyte av vetenskaplig information inom matematikdidaktik med anslutning till psykologi.
- att verka för och stimulera tvärvetenskaplig forskning där psykologer, matematiker och lärare samverkar.
- att främja en fördjupad insikt i psykologiska aspekter vid undervisning och lärande i matematik och implikationer därav.

PME anordnar årligen en forskarkonferens som pågår under fem dagar. De tre senaste årens konferenser har varit i Japan, Nederländerna, Storbritannien och kommer nästa år att vara i Norge. Årets konferens anordnades på Hawai'i, 13-18 juli, och samordnades med den nordamerikanska underavdelningens konferens PME-NA25.

Konferensen skiljer sig från andra konferenser i det en artikel lämnas i ett skick färdigt att publiceras i proceedings. Granskarna utgörs av disputerade PME-medlemmar som fått minst två artiklar accepterade vid tidigare PME-konferenser. Varje artikel har tre granskare och en artikel som accepterats publiceras därefter i proceedings.

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[1] International Group for the Psychology of Mathematics Education
[2] International Commission on Mathematics Education
[3] Third International Congress on Mathematics Education

Deltagare

Uppåt 30 nationaliteter var representerade på konferensen där USA var det land som i antal deltagare dominerade. Sverige hade två representanter och de utgjordes av Barbro Grevholm och mig själv. Mitt eget deltagande möjliggjordes då jag, med Barbro som medförfattare, fick en artikel accepterad som jag presenterade vid ett forskningsrapportseminarium under konferensen. Artikelns titel är *Preservice teachers' conceptions about y=x+5: Do they see a function?* och som titeln antyder behandlas lärarstudenters syn på utsagan y=x+5 och då speciellt hur funktionsbegreppet kommer till uttryck. Studien omfattar två studentgrupper vilka går sin tredje respektive sjätte termin på lärarutbildningen i matematik och naturvetenskap för grundskolans åk 4-9. Datamaterialet består av enkäter, intervjuer och generaliserade begreppskartor där studenterna får möjlighet att utveckla sin syn på y=x+5. Teorigrunden är i stort baserad på begreppsbilder (Vinner, 1983) och en nätverksmetafor som bl.a. kommer till uttryck i Hiebert and Carpenter (1992) samt Ausubel (2000). Artikeln är den första i en planerad svit av artiklar om lärarstudenters begreppsuppfattning om funktioner baserad på data från de tre senaste vårterminerna vid lärarutbildningen i Kristianstad.

Konferensdagarna

Konferensen ägde rum på det centralt belägna Hawai'i Convention Center som är ett modernt konferenscenter med en kapacitet som långt översteg antalet deltagare på konferensen. Programmet var ambitiöst med programpunkter från kl. 8.00 till närmare 17.00 varje dag med undantag från sista konferensdagen som avslutades vid lunch. Det fanns även tid avsatt för olika sociala evenemang så som middag med dans och en utflykt till Polynesian Cultural Center på norra delen av Oahu.

Programpunkterna bestod övervägande av forskningsrapporter (11 parallellsessioner) med ett femtontal parallella seminarier om vardera 40 minuter. Två sessioner med korta muntliga redogörelser med tre presentationer om vardera 15 minuter och två postersessioner fanns även med i programmet. Vidare fick konferensdeltagarna möjlighet att närmare diskutera olika frågeställningar i utbudet av ett tiotal arbetssessioner och diskussionsgrupper som anordnades vid två tillfällen samt vid två forskningsfora. En plenarföreläsning gavs även varje konferensdag.

Mycket kan skrivas om programpunkternas innehåll som alla var av hög kvalité. Personligen hade jag nog mest utbyte av ett antal forskningsrapporter (och de diskussioner som följde i anslutning till dem) där jag fann flera forskare som hade många beröringspunkter med mina egna forskningsintressen. Här kan t.ex. nämnas Mike Thomas artikel *The role of representation in teacher understanding of function* som berör lärares ämneskunskaper och förmåga att tolka olika representationsformer av funktionsbegreppet. Thomas knyter bl.a. an till Williams (1998) studie som med hjälp av begreppskartor studerar disputerade matematikers begreppsuppfattning och för en diskussion om dess konsekvenser för matematiklärare. En annan, av många forskningsrapporter jag fann intressanta, presenterades av Rossana Falcade vars artikel *Function and graph in DGS environgment* (Mariotti et al, 2003) behandlade kommando i Cabri som ”semiotic mediator” för funktionsbegreppet. Ett tema som för övrigt även dök upp under en workshop om ”Symbolic cognition in advanced mathematics” ledd av Stephen Hegedus. Under konferensen kom jag i kontakt med många intressanta människor, idéer och referenser. Det senare dök upp vid flera oväntade tillfällen som t.ex. Koyama (1992) under en mycket komprimerade kort muntlig framställning av honom själv.

Jag upplevde konferensen som mycket givande och när den var tillända hade jag många intryck och idéer att bearbeta under den 24 timmar långa hemresan till Sverige.

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/ Örjan Hansson

ICTMA - An international community

The abbreviation ICTMA stands for International Conference on the Teaching of mathematical Modelling and Applications and is a 20-year-old enterprise. The most recent conference was held in Milwaukee, USA in July 2003, which was the eleventh conference in a row. More information about that conference and about ICTMA can be found at http://www.infj.ulst.ac.uk/ictma/. So far there has been eleven international conferences in:

1 – England	7 – Northern Ireland
2 – England	8 – Australia
3 – Germany	9 – Portugal
4 – Denmark	10 – China
5 – Netherlands	11 – USA
6 - USA	

The reason for the first ICTMA conference seems to have been born in the fact that students who graduate from universities with an exam in mathematics often are good in solving mathematical problems served directly to them, but not so good in doing mathematical modelling of "real world" problems, not so good in communication and not in working in teams (Houston, 2003). Evidently, there are voices who claims that this is still true today:

> The SIAM (Society for Industrial and Applied Mathematics) report on Mathematics in Industry (SIAM, 1995) contained data from a survey of PhD graduates working in industry. The report indicated that modelling, communication and teamwork skills together with a willingness to be flexible are important traits in employees. However the PhD graduates themselves indicated that they felt inadequately prepared to tackle diverse problems, to use communication effectively and at a variety of levels, or to work in teams. (Challis, Gretton, Houston, & Neill, 2002).

There were and are of curse many other influences behind a theme such as ICTMA that still exists after some 20 years and some of the strong forces behind this community have resulted in products such as the eleven ICTMA books in which selected conference papers are published after the conference. The first two ICTMA conferences was held in England and David Burghes, by Ken Houston (Houston 2003) named "the father of ICTMA" managed to start publishing a journal called *The journal of Mathematical Modelling for Teachers* in 1978. This journal is since 1981 published under the name *Teaching*

Mathematics and its Applications, a journal of the UK-based Institute of Mathematics and its Applications.

The journal is available online at http://www3.oup.co.uk/teamat/.

The international flavour of this community was evident from the beginning with delegates from 23 different countries already at ICTMA 1 in Exeter 1983. It is worth mentioning that the addendum "*and Applications*" first occurred at the ICTMA 3 in Kassel, Germany. From that conference and onward, the phrase "applications and modelling" occurs frequently in literature and underlines the subtle but significant difference between a study of mathematical models and the process of mathematical modelling. In his plenary lecture at ICTMA 3, Mogens Niss (Roskilde University) said that an *application* of mathematics was the *result* of the process of applying mathematics to any area of extra-mathematical reality – a mathematical model – whereas mathematical modelling is the *process* (Niss, 1989).

The ICTMA organization of today is really an international community with many different nationalities and perspectives present at both conferences and in the excellent series of eleventh ICTMA books. Nevertheless, as far as I know, there have not been many Swedes present at the ICTMA conferences and only two Swedes have so far been published in the ICTMA series: Lingefjärd and Holmquist (1999), Holmquist and Lingefjärd (2003), and Lingefjärd and Holmquist (2003). Hopefully this will change in the future, since mathematical modeling is visible both in the Swedish curriculum for compulsory school as well as for the gymnasium at present time:

> The importance of mathematical models has increased in the age of information society. Everything that happens inside a computer is the result of a mathematical model, as one example. It is important that this area is acknowledged in mathematics education. (Skolverket, 1997, p. 19)
>
> The school in its teaching of mathematics should aim to ensure that pupils:
> - develop their ability to design, fine-tune and use mathematical models, as well as critically assess the conditions, opportunities and limitations of different models, (English version of the Swedish curriculum for the gymnasium, 2000, p. 61.)
>
> - develop their knowledge of how mathematics is used in information technology, as well as how information technology can be used for solving problems in order to observe mathematical relationships, and to investigate mathematical models. (English version of the Swedish curriculum for the gymnasium, 2000, p. 61.)

The ICTMA community has not only grown stronger but also more mature along the years, with a constitutional democracy that elects President as well as members to the Executive committee. Applications and modelling has also been on the agenda of the ICME conferences since ICME 6 in Hungary in 1988 (Blum, Niss, and Huntley, 1989). It was therefore especially joyful that ICTMA was accepted by ICMI as an Affiliated Study group earlier this year (other such study groups are HPM, PME, IOWME, and so forth).

Last, but not least, an ICME Study Volume is underway, with the Study Conference planned for in February 2004 in Dortmund, Germany. Werner Blum is the chair of the International Programme Committee and the richness and variation of the conference contributions so far looks very promising. Unfortunately, Sweden has not many contributions to the ICME Study although there is still some time for anyone out there that is willing to contribute. More information about the ICME Study conference in Dortmund can be found at http://www.mathematik.uni-dortmund.de/didaktik/_aktuelles/ICMI.htm.

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/ Thomas Lingefjärd

Senaste nytt från
Forum for matematikkens didaktik

Forum, vår danska systerförening, firade sitt tioårsjubileum med en konferens den 27 september 2003 och SMDF skickade hälsningar och lyckönskningar till Forums ordförande Lisser Rye Ejersbo.

Vid ICME10 I Köpenhamn planerar *Forum* att tillsammans med SMDF i den nordiska presentationen ge en föreläsning om de båda föreningarna och visa en utställning om dess verksamheter. Vår danska systerförening är dubbelt så gammal som SMDF men båda organisationerna har liknande mål och verksamheter. Båda organiserar seminarier och konferenser som riktar sig till lärare, doktorander och forskare intresserade av matematikens didaktik. Båda publicerar medlemsblad av lite olika form och utförande och båda ger ut skrifter som dokumenterar matematikdidaktisk forskning. Tyvärr har vi änu inte riktigt hittat formerna för hur vi ska samordna aktiviteterna. Just när SMDF arrangerar MADIF4 den 21-22 januari 2004 ska *Fourm* ha en workshop med Koeno Gravemaijer och Paul Drijvers från Freudenthalinstitutet i Holland. Det ska handla om att utveckla en realistisk matematikutbildning, alltså det som brukar kallas den holländska skolan, *Realistic Mathematics Education*. Workshopen sker i Learning Lab Denmark, dvs på Danmarks Pedagogiska Universitet den 20 januari och på Dansk Institutt for gymnasiepedagogikk den 21 januari 2004. Intresserade kan vända sig till Lisser Rye Ejersbo, e-postadress lisser@dpu.dk eller Claus Michelsen, e-postadress Claus.Michelsen@dig.sdu.dk .

En skrift som utkommit alldeles nyligen har författare som är verksamma i *Forum*. Bokens titel är *Kan det virkelig passe?* och är redigerad av Ole Skovsmose och Morten Blomhøj (ISBN: 87-25-00050-0).

Bland de medverkande finner vi bland annat Helle Alrø, Kristine Jess, Anna Jørgensen, Lena Lindenskov, Paola Valero och Tine Wedege. Så här presenterar redaktörerna boken:

> *Kan det virkelig passe? En bok om matematiklærging.*
> *Matematiklæring er mange ting. Det foregår i skolen; det kan ske i køkkenet; det efterspørges af Dansk Industri; og det er måske nødvændigt før fortsat demokratisk udvikling. Det er nyttigt for et almindeligt liv i vores nutidige samfund. Det kan også være svært. Det har med kommunikation og socilaisering at gøre, og det er stærke følelsesmæssige aspekter knyttet till det.*

Boken består av 16 kapitel och handlar om sådana och andra aspekter på hur vi lär matematik. Författarna närmar sig ämnet på ett undersökande och diskuterande sätt och boken vill öppna för en pedagogisk debatt om hur vi lär matematik bland pedagoger, didaktiker, lärarutbildare, lärare och föräldrar. Kapitlen i boken är skrivna med en koppling till undervisningssituationer och exempel på hur matematik lärs. Det betyder att boken i hög grad kan tjäna som inspiration för matematiklärare som vill utveckla sin undervisning. Boken har sin grund i arbeten som har utförts vid *Center for forskning i matematiklæring* (se webbsidan http://mmf.ruc.dk/~bds/123.htm) och innehåller även ett kapitel om centrets verksamhet.

/ Barbro Grevholm

E-postdresser till medverkande i *Medlemsblad* nr 7

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Anslagstavlan

Aktuella konferenser

ICME10 Köpenhamn 4-11 juli 2004
http://www.icme-10.dk

HPM 2004, Uppsala 12-17 juli 2004
http://www-conference.slu.se/hpm/index.html

PME 28, Bergen14-18 juli 2004
http://www.pme28.org/

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