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INNEHÅLL

Redaktör för Medlemsblad nr 10 har varit *Christer Bergsten*

Medlemsblad nr 10

Under rubriken *Några rader från...* berättar inledningsvis föreningens ordförande om vad som är på gång i SMDF och svensk matematikdidaktik i övrigt.

Medlemmarna i SMDF inbjuds att till medlemsbladet skicka in kortare artiklar eller berättelser, som kan vara av intresse för föreningens medlemmar att ta del av. Detta nummer speglar den stora aktivitet som ägde rum under 2004 inom vårt område.

Från världskongressen ICME10 i Köpenhamn i juli återges Barbro Grevholms presentation vid IOWME:s session, dvs 'ICMI Affiliated Study Group' för kvinnor och matematik[1]. Thomas Lingefjärd berättar vidare om hur en ämnesgrupp ("Topic Study Group") vid ICME10 fungerade. Semiotiken är ett område som fått allt större betydelse för matematikdidaktiken och Christer Bergsten hade en presentation i den särskilda diskussionsgrupp med denna inriktning som fanns vid PME, en årlig konferens som detta år arrangerades som en satellitkonferens till ICME10 i slutet av juli 2004 i Bergen. PME är en annan av ICMI:s 'Affiliated Study Groups'.

Ytterligare några av doktoranderna i den nationella forskarskolan i matematikdidaktisk forskning, Örjan Hansson, Johan Prytz, Kerstin Pettersson och Magnus Österholm har lagt fram sina licentiatavhandlingar. I detta medlemsblad ges en kort presentation av dessa uppsatser. Vidare annonserar Arne Engström innehållet i en nyutkommen konferensrapport om ett angeläget område inom matematikdidaktiken, specialundervisning i matematik. Avslutningsvis rapporterar Barbro Grevholm från vår danska systerförening Forum.

/ Christer Bergsten

[1] Se hemsidan på http://www.mathunion.org/ICMI/affiliated_study_groups.html

Några rader från…

ICME10 är över och vi kan nu njuta av frukterna av allt arbete som gjordes i samband med kongressen. För SMDFs del är vi särskilt glada att vi kunde presentera Madif4 boken, *Mathematics and language*, lagom till kongressen och att boken från symposiet om lärarutbildning i Malmö också blev klar just då. Den boken har titeln *Educating for the future* och kan beställas från NCM i Göteborg. SMDF deltog aktivt i det arbetet med symposiet och boken genom medverkan båda av ordförande, vice ordförande och flera av medlemmarna.

Vi vill nu passa på att rikta ett tack till alla i SMDF för goda insatser under ICME-10. Utställningen om SMDF var bra placerad och tilldrog sig stor uppmärksamhet. Vi hade en egen hantlangare under hela kongressen (min man Lennart hjälpte till både med att utforma utställningen, trycka A3-blad, transportera, sätta upp den och bemanna den och att bära dit alla våra trycksaker). Flera medlemmar var på plats och hjälpte till att svara på frågor. Vi lyckades dela ut ett antal MADIF4-böcker till medlemmar och därmed sparade vi en del portokostnader och arbete med utskick. Det blev också klart för mig att vi bör marknadsföra våra gula böcker i Madif-serien på ett bra sätt. Intresset från internationellt håll för böckerna var påfallande stort. Jag sålde några böcker på plats och har girerat över dessa pengar till vår kassa. Vi bör sända exemplar av böckerna till ZDM för att få dem granskade där och så att de kommer med i olika databaser. Kanske skulle vi även sända dem för review i någon eller några tidskrifter? De borde i varje fall till NÄMNAREN och NOMAD.Vi har även annonserat dem på hemsidan och det innebär kanske att vi får en och annan beställning av dem. Kassören sköter denna försäljning om det är någon som skulle vilja beställa böcker för att ge bort. Det är ingen stor belastning för det handlar säkert endast om några få böcker per år (jag vet av erfarenhet att marknaden för detta slag av böcker är begränsad).

Alla våra Newsletter (Medlemsbladet på engelska) tog slut och de mottogs med stort intresse av dem som kom förbi och såg utställningen. Tack alla ni som bidrog till dem med innehåll.

Lisser Rye Ejersbo och jag hade en gemensam föreläsning i den Nordiska presentationen om SMDF och Forum. Den blev uppskattad även om en liten grupp lyssnade (det var massor av parallella presentationer). Nu ska dessa före-

läsningar dokumenteras i en bok som redigeras av en grupp inom den nordiska kontaktkommittén NCC.

I materialet som matematikdelegationen lämnat till departementet har Bengt Johansson refererat till vårt *Newsletter* och SMDF nämns i arbetet. Vi är även remissinstans för delegationens förslag.

Frågan om nordiskt samarbete rörande föreningar för matematikdidaktisk forskning har väckts i samband med ICME-10. Vi har i SMDF redan medlemmar både från Danmark och Norge.

Jag har föreslagit att vi kunde bilda en slags paraplyorganisation som kan förena de olika nationella föreningarna för matematikdidaktisk forskning. En sådan organisation skulle kunna ta till vara föreningarnas ekonomiska behov på ett nordiskt plan samt fungera som en samordnare. De nordiska forskningskonferenserna som äger rum vart tredje eller fjärde år borde kunna skötas av en sådan organisation. Som det är nu hänger dessa konferenser i luften och det är mera tur om den som vid en konferens lovar att ta hand om nästa verkligen gör det. Norma01 ägde rum i Kristianstad och Norma05 kommer att bli i Trondheim den 2-6 september 2005. Där hoppas vi få se många från SMDF.

De olika nationella föreningarna som nu finns i Finland, Danmark och Sverige kunde utöka sitt samarbete och samverka om arrangemang och på så sätt få bättre utnyttjande och bättre ekonomiska förutsättningar. Diskussionen om forskningsprojekt och andra pågående verksamheter kunde bli mera spridd och bidra till ökad kunskap om forskningsresultat och annat.

Tidskriften Nomad skulle naturligt kunna knytas till denna paraplyorganisation. Det finns en förening bakom Nomad, men den har ingen annan verksamhet än att skaffa pengar till Nomad. Kanske kan den föreningen fyllas med mer liv av det slag jag beskrivit här?

Sedan sist har ännu fler av forskarskolans doktorander lagt fram och försvarat sina licentiatavhandlingar. De flesta går nu in i slutfasen i sin forskarutbildning och därmed är det aktuellt att fördjupa diskussionen om kvalitetsfrågor. Forskarskolan hade ett tvådagarsmöte i våras om kvalitet i artiklar, som var värdefullt. Den nordiska forskarskolan ordnade ett tredagars seminarium i Vasa i september, där handledare träffades och arbetade med kvalitetsfrågor. En workshop ägnades åt att formulera kriterier för kvalitet i artiklar. Men även kvalitet i program för forskarutbildning och för avhandlingsarbete diskuterades. Detta kommer att fortsätta genom att nordiska forskarskolan i matematik-

didaktik ordnar ett liknande seminarium i slutet av april 2005 och ett endagsmöte dagen före Norma05, den 1 september för handledare av doktorander. I Danmark håller den nationella forskarskolan för naturvetenskapens didaktik ett ”vejlederopgraderingskurs” för handledare och doktorander den 10-12 januari 2005 i Korsör. Här kommer man bland annat att se på hur hållbara forskningsfrågor kan arbetas fram och på vad som kännetecknar kvalitet i en avhandling. Vi kan alltså konstatera att det finns krafter igång för att synliggöra frågan om kvalitet i den matematikdidaktiska forskningen. Vad jag efterlyser är dock en öppen debatt om kvalitetsfrågor. I vetenskapsteori framhålls ofta hur väsentligt det är med en öppen debatt för att ett forskningsområde ska kunna betraktas som god vetenskap. Den debatten måste kunna föras även utanför de seminarierum där avhandlingar försvaras?

Frågan om kvalitet i forskning är naturligtvis inte oberoende av frågan om god handledning. Vad menar vi med god handledning? Så här formulerade vi det i en presentationen vid den första nordiska forskarhandledningskonferensen i Umeå 2003, som nu kommer ut på engelska:

> *Our definition of effective supervision*
>
> It is worth discussing what is meant by effective supervision in our model. We want to define it as follows:
>
> 1. The studies can be finished within the five years (with 20 % teaching) and the quality of the Ph.D. theses is high.
> 2. The new doctor wants to and is able to continue as a researcher, e.g. she/he
>
> - is able to formulate new research problems,
> - is able to get financial support for the research,
> - is able to work in new research groups and continue to publish,
> - has her/his own network of international contacts,
> - has knowledge of her/his own about important organisations in her domain, conferences and journals and has been active also in this connection in various ways.

Vi ställer frågan hur det är möjligt att få handledningen dithän. Det vore värdefullt att få reaktioner på detta försök att definiera god handledning och vi hoppas på en debatt. I artikeln har vi utvecklat mera ingående hur vi karaktäriserar den goda handledningen (Grevholm, Persson & Wall, 2004).

Detta har vår danska systerförening tagit fasta på och använder det som en startpunkt i sin tidskrift för en debatt om handledning och kvalitet (Wedege,

2004). De efterlyser debatt i sitt nyhetsbrev. Det gör vi också här i SMDFs medlemsblad.

.... från ordförande

/ Barbro Grevholm

Referenser

Grevholm, B., Persson, L-E. & Wall, P. (2004). A dynamic model for education of doctoral students and guidance of supervisors in research groups. *Educational Studies in Mathematics* (forthcoming).

Wedege, Tine (2004). Kvalitet i ph.d.vejledningen? *Forum for Matematikkens didaktik*, 8 nr 3, 1-2

Vad händer i SMDF under år 2005?

Under oktober hade styrelsen för SMDF ett innehållsrikt styrelsemöte per email som pågick i en veckas tid. Här planlades bland annat en del av verksamheten för 2005.

Året börjar med årsmötet som äger rum på Lärarhögskolan i Stockholm fredagen den 28 januari kl 15-ca 17. Efter de stadgeenliga förhandlingarna kommer vi att debattera frågan om vilka möjligheter det finns för matematikdidaktisk forskning efter matematikdelegationens förslag. Alla inlägg, erfarenheter och synpunkter från våra medlemmar är välkomna.

Den 18 mars ordnar SMDF i samarbete med Svenska matematikersamfundet och SMaL en workshop om matematik och språk, som kan ses som en fortsättning av det arbete som startade vid Madif 4. Programmet för dagen och inbjudna att deltaga har sänts ut i december till alla medlemmar och finns även tillgängligt på hemsidan www.mai.liu.se/~chber/workshop.

Under den fjärde Nordiska matematikdidaktiska forskningskonferensen, *Norma05* som äger rum i Trondheim den 2-6 september kommer SMDF att deltaga i ett möte om frågan om vi ska bilda en slags nordisk paraplyorganisation för nationella föreningar för forskning i matematikdidaktik. Detta har närmre beskrivits i ledaren. Inför denna konferens vill vi återigen ge ut ett nummer av medlemsbladet på engelska och det kommer att förberedas under våren 2005.

Vid konferensen för lärarutbildare i matematik, LUMA, kommer SMDF som sedvanligt att ordna ett möte för medlemmarna. Förslag till program vid detta möte tar vi gärna emot från våra medlemmar. Aktuellt är naturligtvis alltid frågan hur lärarutbildningen ska kunna forskningsanknytas. Vad händer inom de olika utbildningarna i denna fråga?

En programkommitté för Madif5, som ska äga rum före biennalen i januari 2006, är under tillsättning och börjar arbeta i början av 2005. Idéer och förslag till programmet tar vi gärna emot.

/ Barbro Grevholm, ordförande i SMDF

Increasing women's participation in mathematics: The role of networking[1]

Abstract

Women's performance in mathematics is good, but their participation is not satisfactory in Sweden. Change over time has been slow (Grevholm, 1996). In an effort to speed up the rate of change in the area of gender and mathematics a network "Women and mathematics was created in 1990. The network builds on international and Swedish research results in mathematics in its efforts to influence important parts of society, teachers and students. An overview of such research will be given as a foundation for the description and analysis of the work of the network. A philosophy of critical mathematics education serves as theoretical framework and the network is seen as an intervention project. Criteria for evaluating intervention projects will be used in the discussion of the effects of the network. The claim is that Women and mathematics is one possible efficient way to implement research results in order to create actions in mathematics teaching.

Background and questions

The ratio of women in academia has increased considerably during about a hundred years since they were formally allowed to enter colleges and universities. In Sweden the women constitute around 60 % of the entrants each year and the situation in the US is similar. But there are still areas where women are not taking part in the activities to an equal degree. For a long time mathematics has been one of those areas and little change in that situation has been noted (Grevholm, 1995ab, 1996a, Skolverket, 2003), contrary to the situation in for example medicine and law studies. It has become a concern of society as the access of qualified persons going into work in science and technology is claimed by politicians and industrial decision makers to be vital for progress. In most developed countries there have been actions and activities for about twenty years in order to raise the number of students, and especially women, going into mathematics and science (Grevholm, 1993b, Solar, 1998). Through research on gender and mathematics a growing body of knowledge is available but this fact does not seem to influence the situation much. In practice changes in the area of gender and mathematics are slow and it seems that the results from research to a

[1] Presentation vid ICME10 i Köpenhamn 2004, vid IOWME:s session. Se vidare http://www.stanford.edu/~joboaler/iowme/index.html

minor degree reach teachers in schools and have impact on their teaching. Can women's participation in mathematics be improved? Can research influence practice to support increased participation of women in mathematics? What can be done?

Both researchers and teachers need multiple frameworks to help in understanding and interpreting reality but we also need to act, to build new agendas, as Leder et al (1996, p 978) writes, for change and development in the area of gender and mathematics.

The aim of the paper

I will describe, analyse and discuss the network *Women and mathematics* in Sweden, which can be seen as a long term intervention programme to bring about change in the area of mathematics and gender. The networking activities will be related to and explored against a background of research findings internationally and in Sweden. One purpose is to argue for long-term networking as one possible efficient way to bring about change. Another purpose is to interpret the activities of the network with perspectives from research on gender and mathematics.

Theoretical research issues on gender and mathematics

Gender and mathematics has been an international research focus for about thirty years. Some examples of relevance for this paper from international findings and from Swedish research will be discussed.

International findings

The International Commission on Mathematical Instruction, ICMI, initiated an international research conference on Gender and mathematics in 1993. One of the plenary speakers, Elizabeth Fennema (1995 p 26, 1996) in her paper summarises research findings in the area of gender and mathematics in this way:

1. *Gender differences in mathematics may be decreasing.*
2. *Gender differences in mathematics still exist in:*
   learning of complex mathematics
   personal beliefs in mathematics
   career choice that involves mathematics

3. *Gender differences in mathematics vary:*
   by socio-economic status and ethnicity
   by school
   by teacher
4. *Teachers tend to structure their lessons to favour male learning.*
5. *Interventions can achieve equity in mathematics.*

Fennema gives an overview of her own research and that of others in her paper. Reflecting upon a review of extant work on sex differences in mathematics, which she wrote in 1974 and in which she concluded that there was evidence to support the idea that there were differences between girls' and boys' learning of mathematics, she writes:

> *…it was really the writing of that 1974 article that turned me into an active feminist, compelling me to recognize the bias that existed toward females, which was exemplified by the recognition and acceptance by the mathematics education community at large of gender differences in mathematics as legitimate*. (Fennema, 1995, p 22)

She discusses intervention studies as well as the different research perspectives used during three decades: traditional social science, cognitive science and feminist perspectives. Her conclusion is that: "We have come a long way. We have a long way to go to accomplish equity in mathematics education" (ibid, p 35).

She expresses her conception of feminist views as follows: "Feminist scholars argue very convincingly that most of our beliefs, perceptions, and scholarship, including most of our scientific methodologies and findings, are dominated by male perspectives or interpreted through masculine eyes. […] because females have been omitted, the view of the world, as interpreted through masculine perspectives, is incomplete at best and often wrong" (ibid p 32).

As well as the review by Fennema (1974) already mentioned, two major reviews of research (Leder, 1992, Leder, Forgasz & Solar, 1996) indicate the issues and concerns that have been in focus during the last three decades in the area of gender and mathematics. The chapter by Leder (1992) in the International Handbook on Mathematics Education is called "Mathematics and gender: Changing perspectives." She notes that more than 10 % of the articles in *Journal for Research in Mathematics Education* during 1978 to 1990 are about sex differences. Leder discusses the question of terminology (sex or gender) and finds it appropriate to select a terminology that emphasises cultural pressures and socialisation processes. Particular issues in the review are participation

rates, performance and the differential course work hypothesis. The theoretical models include influence from the social environment, from significant others, from culture and the context in which the learning takes place as well as affective and cognitive variables. None of the models uses biological variables and the reason given for that is that no evidence has been found for mechanism based on biological influence. The environmental variables are school variables, teacher variables, the peer group, the wider society and parents. The learner related variables are cognitive, such as intelligence, spatial ability, internal beliefs, confidence and related variables, fear of success, attributions and persistence. Leder notes that "Even though gender is often a significant determinant of aspirations, expectations, and behaviour, there are many other variables, including race and class for example, which have an important and interactive impact" (p 617).

In Leder, Forgasz and Solar (1996) a summary is given of research into gender issues and in particular into the effectiveness of related intervention programs (p 945 ff). They examine and discuss models of gender equity and the progression from empirical research to feminist perspectives. They list four models that they claim to address equity issues: assimilationist, deficit, pluralistic and social justice. With the inclusion of each of these perspectives, research and practice are becoming more complex.

Among contributions from feminist theories, Rogers and Kaiser (1995) discuss the stages of curriculum development called Womanless mathematics, Women in mathematics, Women as a problem in mathematics, Women as central to mathematics and Mathematics reconstructed. Leder, Forgasz and Solar (1996) underline the apparent overlap in the four models, the five stages and the three generations of feminism spelt out by Noddings (1990): 1) women seek equality with men, 2) women embrace their own special qualities and reject uncritical assimilation into the male world and 3) women critique what they sought and accomplished in the first two phases and seek solutions that arise out of a careful synthesis of old and new questions.

After a rich overview of research from the nineteen-nineties on gender issues, Leder, Forgasz, and Solar turn to a discussion of intervention programs. They give a general overview and discuss historical and political influences on intervention programs. A discussion of different classifications of programs and elements of success is followed by characteristics of exemplary programs. The following criteria for assessing programmes (taken from Malcolm, 1984) are presented:

- achievement of primary goals as measured by staff, participants or external evaluation;
- length of time of the program's operation;
- ease in attracting outside support;
- ratio of applicants to participants (program popularity);
- reputation of program with scientists from relevant fields;
- program imitation or external expansion;
- cost effectiveness;
- the strength of the academic content: and
- competence and orientation of teachers for programs with academic orientation.

The conclusions by Leder, Forgasz, and Solar (1996) end with this sentence:

> *Scholars concerned with girls' and women's learning of mathematics now have a solid basis of research, achieved in less than 30 years, on which to build new agendas for the attainment of gender equity in mathematics education (p 978).*

They also clearly point out that they see all the presented different approaches to empirical research or development work as complementary. Whether classical approaches or feminist critique, they believe that the activities should continue.

Solar (1998) and Wilson (1998) give overviews of intervention programmes in the United States. As examples of interventions they mentions conferences, educational activities, activities within schools, community activities, institutional strategies, summer activities, nation wide campaigns, governmental actions, and exhibits. According to Solar, intervention programmes are claiming and acting for change. According to Malcolm (1984) they emerged with the civil rights movement. In spite of all the intervention programmes that have been carried out there is still a need for action in order to change the conditions in the area of mathematics and gender. Such needs became visible in PME27.

The discussion group "Research on gender and mathematics from multiple perspectives" aimed to initiate a dialogue that moves away from current methods and frameworks to new perspectives and new methodologies for considering gender and mathematics (Becker & Rivera, 2003). Ferdinand Rivera took his start from a chart in progress by Patti Lather (1991). Four paradigms of post-positivist inquiry were described:

	Predict	**Understand**	**Emancipate**	**Deconstruct**
Method of inquiry paradigmatic	Positivist	Interpretive Naturalistic Constructivist Phenomeno-logical Hermeneutic Symb. inter-action Microethnog.	Critical Neo-marxist Frereian Race-specific Praxis-oriented Feminist Participatory	Postmodernist Poststructural Post- diaspora Queer Theory
Focus	Study of hum. behavior Study of how structures in-fluence behav. or action	Study of how people under-stand or make sense of their realities … construct meanings	Study of marginalis. emancip. oppression related to race, class ethn & gender …ways to link practice with theory	Study of how multiple voices could lead to diplacement of narratives of of progress f. all Theory Western ethnocentric rationalism Perspective and ess. assumpt.
Feminist appropriation	Studies involve gender diff. in math perf. concerns ability, achievem. & attitude	Womens's ways of knowing in math	Possibility of a feminist math fem. epistem.	Construction of identities, differences
Exemplars	Fennema, Leder	Becker, Erchick	Damarin	Walkerdine Walshaw

In the discussion at PME27, impatience about the slow development concerning gender equity in mathematics was expressed. What are the new paradigms researchers are searching for and what change might they bring to the development? I will return to this question later.

Swedish findings

The Swedish government today consists of 50 % women and Parliament has more that 40 % women. Equity is supported in the law and Sweden has an ombudsman for equity questions. But the picture is not as simple as you might believe because Sweden has the most gender-segregated labour market in Europe (SCB, 1994, 1995, 2003) and young people still make traditional career choices after having graduated from upper secondary school (SCB, 1995, 2003).

Sweden is regarded as a country, where equity between men and women has improved quickly. But it is not true for advanced mathematics, which still has an extreme majority of men. The educational system is supposed to offer equal opportunities of education to all (Utbildningsdepartementet, 1994ab). Sweden has compulsory nine years of schooling, where all pupils study the same mathematics. Almost all pupils continue from compulsory school to the voluntary three year upper secondary school. In upper secondary school mathematics is compulsory for the first course Mathematics A, with 100 study hours. After that mathematics is optional (courses B-E and special courses). The pupils chose among 16 different programmes in upper secondary school with different amount of mathematics. Differing patterns for the participation of boys and girls are found here (SCB, 1995, 2003).

Looking at participation in mathematics we can see that less than one fifth of a year-group chooses the natural science programme, which opens opportunities to go to the university in fields of mathematics and science. About 40 % is girls in this group.

The upper secondary school as a whole could be classified as two separate schools, a girls' school and as a boys' school, in the sense that most of the programmes have a decided majority of either girls of boys (SCB, 1995, 2003, Skolverket, 2003). Only two programmes are balanced in gender participation.

As a consequence of the situation in upper secondary school only about one third of the students in mathematics at university level are women. Earlier at the doctoral level in mathematics only few of the students were women. Historically fewer than 30 women in Sweden received a research degree in pure mathematics before 1994 (Ph D or equivalent) (Grevholm, 1994b). The number of female Ph Ds has increased slowly after that time and in 2001 about 1 out of four doctoral students were females (in mathematics and related subjects) (SCB & Högskoleverket, 2000-2002a). Partly this is a consequence of the broadening of Ph D programmes and of new subjects being included (mathematics education).

The poor record of female participation in mathematics in Sweden is further reflected in the fact that during the early nineties less than 5 % of mathematics senior lecturers in the universities were women. The only female professor of mathematics before 1997 was Sonja Kovalewsky, who died in 1891. In other subjects in the early nineties, on average there were about 8 % female professors and 20 % female senior lecturers (Wittenmark, 1993). For 1999 the numbers for females in mathematics were 3 % professors, 16 % senior lecturers, and 20 % teachers without a Ph D (SCB & Högskoleverket, 2000-2002b).

As seen from above participation in mathematics continues to be problematic but that is not the case with the performance of females in mathematics in Sweden. Generally mathematics in compulsory school is the most common subject for pupils to fail in. The girls as a group are successful in their studies. They leave school with better marks than boys in mathematics (and in most other subjects too). More details about the performance of girls and women have been given in earlier reports and I will just refer to them here (Ljung, 1990, Grevholm & Nilsson, 1994, Kimball, 1994, Grevholm, 1998, PISA 2000).

The reports on women's success at tertiary level in mathematics performance continue to come. A recent study at Umeå University (Arbetsgruppen för anpassade studiegångar, 2002) shows that women have better study results in the first mathematics course than men. Bylund & Boo (2003) claim that earlier investigations at the same university show that women and men have equivalent pre-knowledge in mathematics when they enter university mathematics, with a more homogeneous group of women. Although women and men have comparable results in a diagnostic test the women succeed better with the mathematics studies. A comparison over time from the same university shows that the results of men have decreased strongly from 1999 to 2001 but women's results have only changed marginally according to Bylund & Boo (2003).

After this overview of the participation and performance of women at different educational levels I turn to some specific issues and gender perspectives.

Swedish textbooks in mathematics were shown to present a world that consists of roughly 60 % men (Areskoug & Grevholm, 1987; Rönnbäck, 1992). Pictures in the books show men more often and choices of contexts in the problems are mainly male. The group of textbook authors consists of almost only men. Most teachers are unaware of these facts. In an investigation of teacher's beliefs it is shown that more than one third of the teachers think that textbooks are gender-neutral (Grevholm, 1994c, 1996d).

Terminology and theory in this study

By intervention project (in this paper) I mean systematic and organised work for actions and activities made with the purpose and intention to change a situation of which one is critical. Leder et al (1996) interprets interventions as programmes that "aim to foster in the sex and race composition of specific fields of study and work in which women and minorities are still underrepresented" (p 967).

The network explores and criticizes conditions from a gender-perspective and acts for change. The theoretical foundation for the networking project is Skovsmose's philosophy of critical mathematics education (1994). To be critical means to draw attention to a critical situation, to identify it, to grasp it, to understand it and to react to it (ibid p 16). A critical theory is characterised by a critic of ideology directed towards certain belief systems and attempts to do so in a theoretically based and organised way (ibid p 17). The goal of critical activity can be described as emancipation, meaning a freedom from stereotypes of thought (ibid p 19). The critical activities of the network Women and mathematics aim at the emancipation of both men and women, and the removal of stereotypical ideas and constructs. In Lather's chart (1994) a critical method of inquiry is placed in the emancipatory paradigm and related to feminist methods.

Methodological issues

In this paper I use written, published or generally shared documents from the activities in the network Women and mathematics as my data source. The documents are proceedings from conferences, newsletters, articles in journals, reports, conference presentations and notes from discussion groups and meetings. As I have myself been part of the network since its start I am aware of the risk for subjectivity. To avoid unwanted consequences of that I try to be as open as possible and offer the reader the opportunity to judge on the basis of the data presented. It is important that a project as Women and mathematics can be reported and discussed from a scientific point of view. Leder et al writes: "Yet intervention programs and strategies are rarely reported in research journals, even though links with research are often apparent" (1996, p 966). Solar (1998, p 196) in her paper on intervention projects writes "The lack of data from many countries prevents a more detailed analysis". A critically remark is that this lack of data must also be a consequence of a lack of preparedness to read and take part of reports in other languages than English as data are obviously available for example in Swedish. Development work based on research such as the network has to be made visible in the discussion on the need to improve

educational research and make it more useful and influential (Burkhardt & Schoenfeld, 2003).

Theory into practice in the area of gender and mathematics

Burkhardt and Schoenfeld expose six models of linkage between research and practice. They claim that translating research into practice is a decidedly non-trivial task (ibid, p 4). How can research on gender and mathematics influence practice? In Sweden there is a need to increase female participation in mathematics both at the upper secondary level and at the university level. Two points in a girl's life seem to be crucial in mathematics, the points where the young person has to make a choice. The first is the choice of study programme in upper secondary school. The second is the passage to graduate studies. Interventions in the educational system could be a way to change the traditional pattern for these choices. The system evidently changes very slowly when it is not placed under pressure from outside. I will argue that networking can be one efficient way for change and a way to let research inform and influence practice.

Now let me come back to the question from PME27. To me it is obvious that we have a gap between theory and practice here. Do we need a new research paradigm? Will more research papers speed up the pace of changes in practice? Do we not need to implement consequences of what research has shown during 30 years? I claim that implementation can not only be done through writing more research papers with new theoretical frameworks. As said before, we need multiple frameworks to help us understand and interpret reality but we also need to act, to build new agendas as Leder et al (1996, p 978) writes, for change and development in the area of gender and mathematics. It seems to me that sometimes in the discussion, there is not a clear distinction between what the researcher can do through writing a research paper and what has to be done in practice, drawing the consequences of what has been found in research. Society has expectations for educational research to be useful and influential. Researchers have a responsibility to assist in making the possible conclusions of research clear and not just believe that the publishing of academic papers will make a difference in practice (Burkhardt & Schoenfeld, 2003).

Intervention programmes are one way of taking this responsibility and as can be seen from the overview of Leder et al (1996) many researchers have tried such programmes. Below I will describe and analyse the network “Women and mathematics” in Sweden, which can be seen as a long term intervention programme. The debate in PME27 convinced me again of the importance of such

programmes. Research is needed but action must also be taken based on the research results. The learning community created in the network is one way of bridging the gap between theory and practice (Jaworski, 2002). Teachers, student teachers, students, and researchers learn from each other in the community in their efforts to critique gender bias and create changes.

The Swedish network Women and mathematics

Creation and ways of working

The international Organisation of Women and Mathematics Education, IOWME, is a study group affiliated to ICMI. It started in 1976 at an ICME conference in Karlsruhe (Shelley, 1995). Shelley writes

> *"… and out of that meeting IOWME was born. IOWME has affected the format of each ICME since, helped to bring the question of women and mathematics into the arena, and now has branches in more than forty[2] countries" (p 255).*

The idea to start a Women and mathematics network in Sweden was born after the IOWME meetings during the ICME6 conference in Hungary in 1988. At that time IOWME had no branch in Sweden. The practical process has been described elsewhere (Grevholm, 1995b, 1997). Thus the IOWME activities and the research presented there (Burton, 1990) led to the

constitution of the Women and mathematics network in April 1990 (Grevholm, 1990, 1992a). From the beginning it was decided to use an informal structure and spend as little energy as possible on organisational matters. All activities have been organised as separate projects with different groups of initiators and workers in different geographic places in each case.

Aims set in 1990

The aims of the network Women and mathematics in Sweden as stated in 1990 are to

- create contacts between those who are interested in women's/girls' conditions in studies or research of mathematics
- spread information on projects and research about women/girls and mathematics
- suggest speakers (preferably female) in subjects concerning women and mathematics

[2] The number of countries in 2002 is 43 (IOWME Newsletter 2002).

- be a national suborganization of the international network IOWME (International Organisation of Women and Mathematics), (Grevholm, 1991).

The Swedish network of women wants to increase the number of females in mathematics by engaging them in various kinds of projects. A theoretical model of how this is done is shown and discussed below. From a theoretical point of view the network as such can be seen as an intervention project. According to Mura (1995) it can also be classified as a feminist or segregation project.

After ten years of activity in the network some additional aims were formulated (Grevholm, 2001, p 61-62).

Some of the additional aims set in 1999

- We want 50 % girls in all mathematics courses at upper secondary school.
- We want 50 % women in mathematics course at university level.
- We want 50 % women among the doctoral students in mathematics
- More researcher education programmes in mathematics education must be developed
- We want 40 % women among the senior lecturers at university
- We want five female professors of mathematics
- All textbooks at all levels will be inclusive for both girls and boys
- All teachers will in development work and competence development get experience from gender perspectives in mathematics education

These goals will be evaluated in 2009 and new goals set again. The fourth point was almost prophetic because in 2001 eight new such programmes were set up (Leder, Brandell & Grevholm, 2004).

The conferences and books

The five conferences given (every third year) since 1990 have attracted many participants, both men and women. For brevity I will refer to the conferences and their documentation as M90, L93, G96, U99 and K02. They have been the most important way to introduce the international research base through personal influence and writings for the activities in the network. From the group of international researchers in gender and mathematics (many mentioned above in the theoretical overview) the following have visited the conferences: Burton M90, Hoyles, L93, Keitel, Owens, G96, Fennema, U99 and Leder, Horne K02. The lectures and writings from these researchers have given Swedish teachers new perspectives on gender and mathematics. It is an important way to dissemi-

nate research results and inspire teachers to act from new knowledge. For a number of women the conferences have offered an arena for debut in public as a speaker. They have experienced the support from more experienced women in the network as a safe environment. The documentation of the conferences in books has grown in quality and more and more papers are research based (Grevholm, 1992a, b, 1996b; Brandell et al, 1994; Lindberg & Grevholm, 1998, Grevholm, Vretblad & Sigstam 2001, Grevholm & Lindberg, in press). Reports with good quality from teachers work give evidence of knowledge of the research issues discussed in the theoretical part above.

Multiple frameworks are used in the research papers presented in the conferences as seen from the examples of research in the proceedings:

Content	**Author**	**Theoretical perspective**	**Type**	**Conf**
Science ed	Staberg	Feminist	Qualitative	M90 G96 U99
Math	Sjöstrand	Analysis		M90
Math ed	Kristjánsdóttir	Feminist	Quantitative	L93
Math ed	Linnanmäki		Quantitative	L93
Math	Stocke	Measure theor		L93
Math ed	Wernersson	Feminist	Quantitative	L93 G96
Math ed	Ahlberg	Cognitive	Mixed	G96
Comp ed	Erson	Feminist	Qualitative	G96
Math ed	Grevholm	Feminist	Quantitative	G96
Math ed	Grönmo	Cognitive	Quantitative	G96
Math ed	Keitel	Res. overview		G96
Math ed	Owens	Res. overview		G96
Math ed	Wilson	Res. overview		G96
Math ed	Fennema	Cognitive	Qualitative	U99
Math	Fainsilber	Algebra		U99
Scien ed	Sjöberg	Feminist	Mixed	U99
Math ed	Nevanlinna	Phenomenological	Qualitative	U99
Math ed	Wedege	Sociocultural	Qualitative	U99
Math & sci ed	Wistedt	Res. evaluat	Mixed	U99
Math ed	Leder	Feminist	Quantitative	K02
Math ed	Horne	Feminist	Mixed	K02
Scien ed	Lindahl	Constructivism	Mixed	K02
Math ed	Leder Brandell	Feminist	Qualitative	K02

The proceedings also show a wide variety of development works by teachers and teacher educators building on research on participation, performance, attitudes, beliefs, single-sex education, textbooks, collaborative work, assessment forms, recruitment, ICT, alternative work forms, career choice, and so on. For more details I refer to the proceedings.

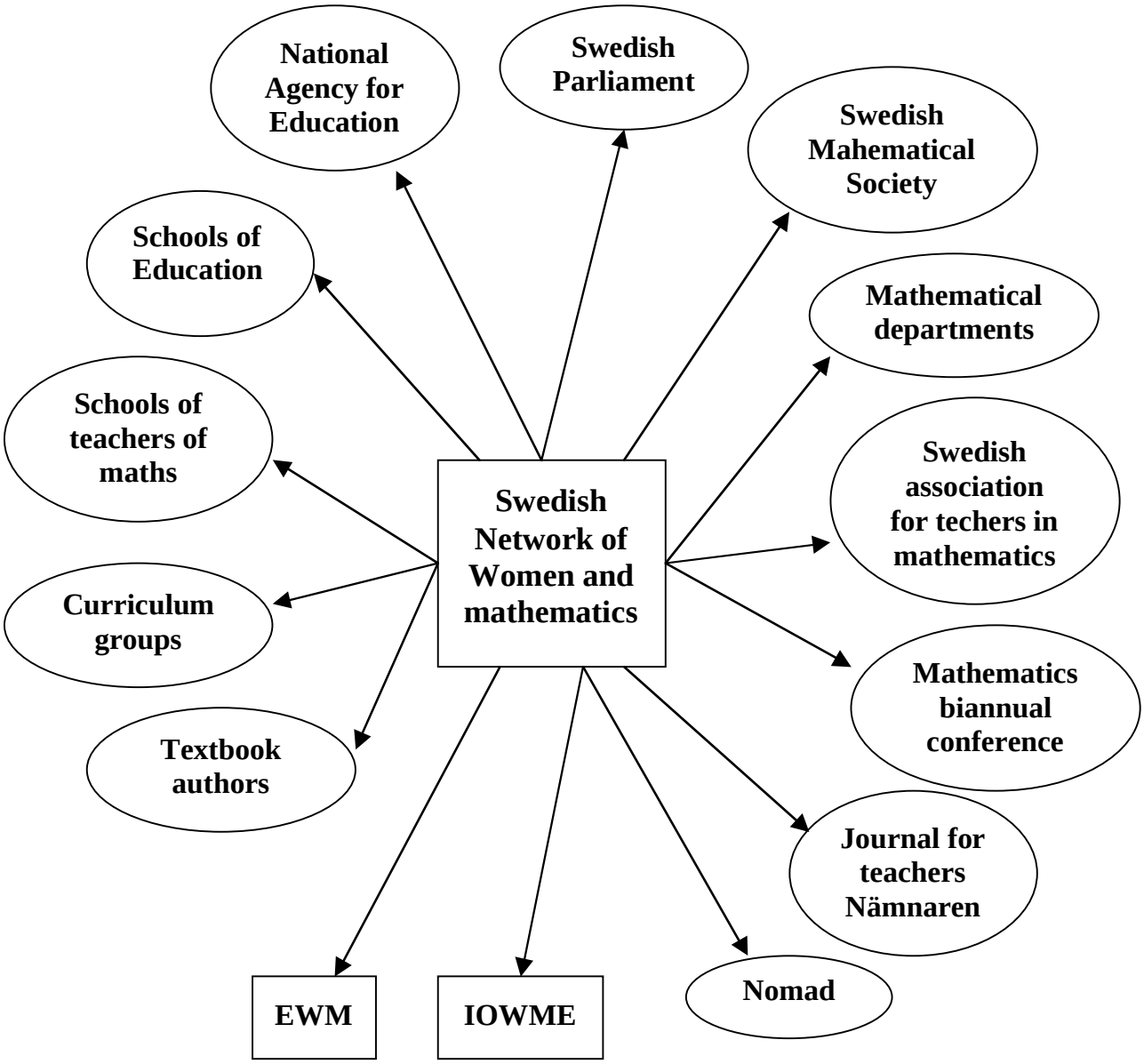
A theoretical model of how the network is working

The contacts and ways of the network influencing the surrounding society through women active in the network are shown in the model below. An arrow indicates that women in the network have direct opportunities to influence that part of the surrounding society.

The spider web activities and women's participation in important organisations have made it possible to influence the development in many different ways. Through illustrating with examples what the arrows mean I will show how theoretical perspectives and research permeates the activities of the network.

The link to the *Swedish Parliament* stands for example for the parliament member, chairing the JÄST-group (Parliament's equity group), who took part in the Luleå conference in 1993 lecturing about the work of the group (Lewander & Jordansson, 2000). One consequence of that was funding for work with equity issues over two years and including all university staff at the University Colleges of Malmö and Kristianstad (Grevholm & Lindahl, 1998).

One close connection to the *Swedish Mathematical Society* has been through one of its former chairs, who lectured at several of the conferences and was responsible for the summer school for doctoral students in 1996 (Klisinska & Persson, 1997; Persson, 2001, in press). The only woman at the board of the Society for many years is an active member of the network. Her own development work with upper secondary students has been reported through the network (Backlund, 2001). Women from the network have been invited as speakers at several occasions in the Education Days of the Mathematical Society, thereby influencing mathematicians.



Many of the most active women in the network have positions at *mathematical departments* all over Sweden. Their work with gender questions are supported by the network and they can report back to give the network insights about the development in different departments. Their contributions to the conferences have been many: 6 in M90, 9 in L93, 6 in G96, 15 in U99 and 8 in K02. Research on gender and mathematics reaches the departments through them and they can act as critical friends in all the activities at the departments from a gender perspective. Thus the network is an arena for meetings between mathematicians and mathematics educators.

The *Mathematics Biannual conferences* from the start used to have an overweight of male speakers and organisers. After the appearance of the Women and mathematics network it has become evident that there are many competent women who could contribute. A fruitful cooperation has developed and working groups on Gender and mathematics have been included since 1992 in the biannual conferences (see for example Emanuelsson et al, 1992, 1994; Olsson et al, 1996). Members in the network participated in the national conferences Matematikbiennalen through programmes about Women and Mathematics (see for example Grevholm 1992c, 1994d, 1996d).

The mathematics teacher journal *"Nämnaren"* has reported frequently on gender and mathematics. (Fennema, 1994; Boaler, 1997). Members of the network also contribute continuously to issues Nämnaren (Grevholm, 1991, 1993a, 1994a; Rosén, 1990, 1993; Lindberg, 1994). Through this medium many teachers get access to both research reports and more popular scientific reports on gender and mathematics. Documents on gender and mathematics from abroad have been translated and published in Nämnaren (see for example Grevholm, 1994a).

The mathematics education research journal *Nomad* has via members of the network got suggestions for papers on gender and mathematics, which made the issue visible for a Nordic readership (Leder & Forgasz, 1995; Grevholm, 1997).

The *National Agency for Education* has been an important partner for the Women and mathematics network for funding reasons and as a way for members of the network to enter curriculum groups. Members of the network have been involved in the discussions about new curriculum and commentary material in Sweden (Brandell et al, 1994). The conferences of the network have been supported both financially and by representatives from the Agency as participants in the activities of the network (Lindberg & Grevholm, 1998 p 6; Backlund, 1999, Mattsson, 1995). This collaboration has ensured that the Agency has knowledge about research reports on gender and mathematics and a critical judgement on what goes on in Swedish schools in the area of gender and mathematics. The National Agency for Education has supported the production of written material concerning equity in mathematics education and supported members' participation in international conferences. The ICMI Study 93 on Gender and Mathematics Education received generous support from the Agency. The existence of the network was one reason to choose Sweden as the host country of the conference. The local organising committee was formed of members from the network. This study resulted in two scientific books (Grevholm &

Hanna, 1995; Hanna, 1996). The theme gender and mathematics was included as a working group focus in the regular programme of ICME 8, and thereby the work was carried on from the ICMI study conference in 1993 (Grevholm & Evans, 1998). At ICME 8 in Seville 1996 there were two main lectures on issues concerning gender and mathematics by Gilah Leder and Gila Hanna in addition to the traditional IOWME programme.

Many mathematics teacher educators working in *schools of education* are active in the network and participate in conferences and other kinds of work. They have been able to put the issue of gender and mathematics on the agenda for student teachers. Evidence of that is the number of written final exam essays on the theme gender and mathematics, often supervised by members of the network (e. g. Engström, 1994; Karlsson, 1993; Flinck & Åberg, 1993; Edman, 1995; Thörnqvist, 1995; Lindh et al, 2001; Larsson, 2002). The conference proceedings of the network are used as textbooks in teacher education.

The majority of participants in the work of the Women and mathematics network are *teachers in schools* and at tertiary level. Many of them have been inspired by lectures in the network to start actions in their own work. On later stages such work has been reported back to the network in the conferences. An ongoing dialogue back and forth from teachers in schools to researchers in the network and back again takes place. Members in schools and schools of education help to spread information about research results on gender and mathematics and also bring good examples of intervention or segregation projects that are going on in education. The total impact of different teachers' work in the area of gender and mathematics is hard to estimate but has had a great importance.

Curriculum groups in Sweden have normally consisted of only men. In the groups for upper secondary school the network had two female members, who could ensure that gender aspects were included in the curriculum texts. (Grevholm, 1999; Backlund, 2001)

At least ten women in the network belong to the group of *textbook authors*. Results from investigations of textbooks have reported by women in the network. Thus the female authors are aware of the possible lack of equity in textbooks and are able to influence their male co-authors. Their influence contributes to a raised awareness of gender issues and textbooks.

Other activities in the network

Summer schools for female doctoral students in mathematics have been organised twice (Klisinska & Persson, 1997). The first time it was initiated by the network and carried out by an experienced group of academic teachers. The second time participants in the first summer school took the initiative and the more experienced teachers were just resource persons in the planning phase. Such passing on of responsibility is evidence of the viability of the activities. The group of students was also broadened to mathematics and related subject, such as mathematics education.

A *national newsletter* was produced and sent out twice a year (Grevholm, 1996b) between 1990 and 1998. Later email communication substituted regular postings. The international IOWME Newsletter is distributed, which means that news from all over the world about gender and mathematics reaches many women in Sweden.

In 1993 the idea was born to *produce a video with interviews* of female mathematicians to use as an inspiration for young persons in their choice of studies. Three quarters of a million SEK was raised from funds and donators and the video project started in 1996. Four years later the video existed in reality and *Formulas and imagination* was sent twice in Swedish television. The video is now available in all municipalities through libraries together with a study handbook (Dahl & Grevholm, 2002).

Other activities of regular character are discussion groups at the biannual conferences for mathematics teachers, presentations of gender research for teacher groups, invited lectures for student teachers and in schools and Nordic contacts and influence through cooperation in Nordic working groups. In 1991 a conference took place in Denmark and in 1992 a conference in Norway was arranged on Women and Mathematics and Mathematics Didactics (Tingleff,1991, NAVF, 1993) A second conference in Norway took place in 1999 (Hag, Holden & van Marion, 2000). As the three groups of women involved have interests in common they form a strong opportunity for women to communicate and have served as impulse to keep in touch and use each other as resources in different ways.

Members of the network initiate local activities on equity issues. For example equality groups at universities have been inspired by the network reports. Teachers in schools have started development projects thereby replicating research reported in the conferences of the network. Documentation from such projects and further examples of how the network is functioning can be studied

in the published reports on the network (Grevholm, 1992a, b, 1996b; Brandell et al, 1994; Lindberg & Grevholm, 1998, Grevholm, Vretblad & Sigstam 2001, Grevholm & Lindberg, in press).

What have been achieved through the network?

To summarize we can mention that the network Women and mathematics has

- placed the issue of gender and mathematics on the agenda
- contributed to making women visible in mathematics
- worked on raising awareness of research results on gender issues
- created lasting documentation on gender and mathematics
- proved that women are there and are willing to contribute in mathematics
- inspired to investigations and essays by students and teachers on gender issues.

Evaluation of the effects of the network

I will use the criteria for assessing programmes of intervention by Malcolm (1984) for the discussion. I take each of his criteria presented above and comment on them on the basis of evidence from the presentation.

1 Achievement of primary goals as measured by staff, participants or external evaluation;

The primary goals from 1990 have been achieved: contacts created, research and information disseminated, speakers made visible, and acting as national sub-organisation of IOWME. The network set complementing new goals in 1999 in a twelve point programme (Grevholm, 2001).

2 Length of time of the programme's operation;

The length of the programme is now 14 years and the network will continue until it has made itself unnecessary (the twelfth point in the new aims). This ability to survive without a formal organisation and permanent financial support is evidence for the demand of a network, and a forum a meeting place.

3 Ease in attracting outside support;

Outside support has been given generously by different departments for conferences, by the National Agency for Education, and by funds and donators to the video project. Permanent support has not been given, but applications had to be made each time.

4 Ratio of applicants to participants (programme popularity);

The activities have attracted many participants, sometimes more than what was planned to be the case. New women ask to be included in the network continuously. That new groups of women are prepared to take responsibility for organising conferences every third year shows that the networking idea is viable. There is a need for contacts between women and a need for opportunities to communicate.

5 Reputation of programme with scientists from relevant fields;

Invited academics from Sweden and abroad in mathematics and mathematics education have been eager to participate and present as evidenced by the proceedings. The programme seems to serve as a reminder in many situations that there is a group of persons who will evaluate and criticize from a gender perspective. Certain respect in academic environments for this fact can be noted. When the network awarded its price of honour in 2002 it was received with great enthusiasm by a professor of mathematics.

6 Programme imitation or external expansion;

The first conference in Malmö served as an impulse to women in the other Nordic countries to start working in similar ways. For example women from all the Nordic countries have presented in the Swedish conferences. The programme has grown outside the borders of Sweden.

7 Cost effectiveness;

It is very cost effective as women involved work idealistically. The only paid work has been that of the producer of the video. The work teachers do is integrated as part of their normal teaching work.

8 The strength of the academic content.

The academic content, that is the content of the conferences, has been raised step by step when the participants have become used to what is expected. From mainly reports from development projects in 1990 there is a tendency toward a larger part of research papers through the years. Following this development students' essays have a stronger academic base.

9 *The competence and orientation of teachers for programmes with academic orientation.*

The women, who have been central in driving the organisation, have been more and more deeply involved in research and teacher development work. Many

teachers have taken competence development in mathematics education with a growing interest in making their own inquiry in the classroom.

Thus according to Malcolm's criteria the intervention programme is successful.

Discussion and conclusions

Do we really need a national network for Women and Mathematics in Sweden? Would not the development towards better gender balance in mathematics come by itself? What are the advantages of a network? A formal organisation would offer possibilities to meet and create contact, but would take energy for the formal organisational parts. A network offers good opportunities to collaborate in a flexible way and be in touch without forcing the activities into special formal arrangements. A network empowers women by making them visible to each other and outsiders, creating personal contacts, making it possible to explore each others work and results, opening communication, giving creative impulses and gathering women's force by using the strength united for influence in different situations.

Why prefer a segregated network for women and not a network for all mathematicians and teachers of mathematics? Do women need other ways to handle the problems, other methods to reach each other than men? Are women's ways of knowing other than that of men? Maybe Belenky et al are right (1986) when they claim it to be so. At the moment many women seem to value the segregation. The network is open to both women and men and men have taken part from the beginning. The name of the network "Women and mathematics" has been discussed and other suggestions as Gender and mathematics have been put forward. As long as there is a need for a separate arena for women in mathematics the network will survive. The aim is to achieve such a situation of gender balance that the network will have no reason to exist any more.

By creating contacts women have been able to use each other in work as speakers, as references, and sources of information and inspiration. Women's work and results are often invisible, but through the network members get to know about other women's work.

Women work united for better equality, higher consciousness about the gender and mathematics perspective, better curriculum, influence on textbooks and commentary material.

There is a value in women's contributions to mathematics and their perspectives being involved, not least from a democratic perspective. Such a view is part of a critical perspective. There is practically no literature in Sweden about women and mathematics except what was produced through the network. Gender and mathematics has been put on the agenda. Through the network women have been asked to participate and contribute to other conferences, working groups and have exchanged ideas of work. The network also created contact between researchers abroad and Swedish women and opened ways for research to practice.

Future plans

There is some evidence that might make us hopeful for the future of women in mathematics. First of all the Nordic Summer school in 2003 for female doctoral students is a valuable initiative, that was taken by a new generation of women in mathematics. The conferences still attract interest. A group of women in Northern Sweden has volunteered to organise the next one in 2005. The Swedish Parliament has stressed the fact that all academic teaching should be done in such a way that female students are included as well as male. Funding has been arranged for special positions for women. We have had some female mathematics guest professors during the years after 1996. A permanent position for a female professor of mathematics at Uppsala University was created in 1997. In one of the universities the mathematics department has 50 % female doctoral students (Grevholm, Persson & Wall, 2003). Two investigations of academic mathematics in Sweden have among other things focussed on the problem with the lack of women in the subject (UK, 1995; SNSCR, 1995). The government has funded five important new university programmes in mathematics and science, with the aim of attracting more women than traditional programmes (Wistedt, 1996, 1999). We can see that the problems are officially recognised. The network will follow the development carefully and continue to offer criticism when problems arise and support through its work and actions. It is important that research and intervention programmes or other actions go hand in hand and they complement each other in the work towards changing the conditions for women in mathematics. “That information gathered from research can be translated into action through intervention programs is made explicit in the recommendations for teachers by Leder and Forgasz […]” (Leder et al, 1996, p 970).

Reports on intervention programmes are often invisible and Leder et al (1996) indicates this problem: “Traditional empirical research monitoring females’

participation and performance in mathematics and related career activities should continue, as should documenting the effects of intervention programs" (p 978).

This paper tries to make one intervention programme visible and argues that such a programme is one possible efficient way to implement research into practice, express criticism and create action and activity based on research. Evidence has been presented to support this claim. The evidence consists of the collaborative work of many women over a long period of time. What the paper cannot convey is the joy and satisfaction this work has created in the group.

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/ Barbro Grevholm

Att medverka i en ämnesgrupp vid ICME-10

När detta skrivs vävs Sverige in i det disiga novembermörkret och sommarens ICME-10 i Köpenhamns utkant tycks avlägset. Själv har jag en hel del arbete kvar i samband med arbetet i den ”Topic Study Group” som jag medverkade i och samma typ av anknytning efter ICME-10 har säkerligen också många av er som läser dessa rader. Denna situation förefaller bli allt vanligare och kan nästan ses som det ”normala” nu för tiden med internationella konferenser. Arbete pågår såväl innan som efteråt och själva konferenstillfället blir alltmer huvudsakligen en möjlighet till personliga träffar för många som samarbetar även mellan konferenserna.

Själv var jag knuten till TSG 20: *Mathematical applications and modelling in the teaching and learning of mathematics***,** en ämnesgrupp som också är nära ansluten till ICMI-Study 14: Applications and Modelling in Mathematics Education samt till **ICTMA** (International Community of Teachers of Mathematical Modelling and Applications), en så kallad Affiliated Study Group. Se SMDFs medlemsblad nr 8 för mer information om dessa två initiativ.

Dessa tre grupperingar bestod under ICME-10 till stor del av samma människor, om än i olika roller, vilket innebar att arbetet i TSG 20 blev tämligen varierat och eventuellt kanske framstod som förvirrande för en icke insatt åskådare. TSG 20 leddes av en så kallad Team Chair, nämligen Ken Houston, från University of Ulster, Northern Irland. Dessutom fanns det tre så kallade Team Members: Toshikazu Ikeda, Yokohama National University, Japan, Nikos Klaoudatos, University of Athens, Greece och João Filipe Matos, University of Lisbon, Portugal.

Eftersom jag själv var med i alla tre grupperingarna innebar det för min egen del att jag var tämligen upptagen med enbart TSG 20 och inte hade tid att följa så mycket annat under ICME-10. Det var med nöd och näppe som jag hann närvara när Survey Team 5, som jag hade arbetet tillsammans med under ett år inför ICME-10, presenterade sin rapport via gruppens ordförande Frederick Leung. Arbetet i TSG 20 blev produktivt, men möjligheterna att närvara vid spontant valda konferensbidrag förefaller för min egen del att minska konferens för konferens.

Arbetet i TSG 20 går för närvarande ut på att hitta en lämplig förläggare eller utgivare för några av de bidrag som blev presenterade under konferensen. Dessa

finns fortfarande att ta del av och hämta hem på adress http://www.icme-10.dk/ för den som är intresserad. Att hantera ett drygt 20-tal artiklar inom ramen för de tre möten som varje Topic Group blev tilldelade under ICME-10 var inte lätt. Därför presenterades artiklarna gruppvis av någon som läst 3-4 artiklar och ombetts ge utlåtande om varje artikel. Mitt och Mikael Holmquists bidrag vid namn *To assess students' attitudes, skills and competencies in mathematical modelling* blev på så sätt presenterat av Chris Haines vid City University i London. En mycket intressant upplevelse, att få sin artikel kritiserad i direktsändning, så att säga. Förfarandet påminde till viss del om arbetssättet under de CERME konferenser som jag varit involverad i. Chris Haines är för övrigt huvudansvarig för nästa ICTMA konferens, ICTMA12, som går av stapeln i London 10 - 14 July 2005.

För fyra år sedan deltog jag i TSG9: Mathematical Modeling and Links between Mathematics and Other Subjects under ICME-9 i Tokyo/Makuhari och efterarbetet till den konferensen ledde så småningom fram till att ett antal konferensbidrag inom TSG9 publicerades via tidskriften *Teaching mathematics and its Applications* (http://teamat.oupjournals.org). Tiden får utvisa vad som händer med konferensbidragen till TSG20 2004.

/ Thomas Lingefjärd

Beyond the representation given[1] – The parabola and historical metamorphoses of meanings[2]

Abstract

Tracking the history of a mathematical object reveals a double influence on its development - from inside and outside the pure mathematical realm. From a semiotic point of view, a chain of metamorphoses of meanings is created, which becomes critical as the mathematical object by the social process of didactic transposition is turned into a presented object of teaching and learning. As an example, by representing the mathematical object "parabola" by means of its historical progression within classical geometry, analytic geometry, and dynamic geometry software, it is seen, by using analytic tools from semiotics, anthropological theory of didactics, and embodied cognition, how and why students' concept images and problem solving techniques may become disconnected and instrumental.

Introduction

In today's school mathematics concepts and methods are often treated in isolation, with only surface level theoretical embedding within a mathematical sub-domain. In addition, this treatment is most often, at least in Swedish secondary school, more or less dominated by algebraic tools. However, many students have significant problems to handle and appreciate this particular semiotic tool, both regarding the meaning of symbols used and the ways to handle them. As a consequence, an image of mathematics as disconnected and difficult to understand may emerge. Applications used in teaching to enhance motivation then easily get the character of decorations rather than parts of an integrated knowledge structure. One example that well illustrates this didactic phenomenon is the parabola. Problems and techniques related to this ´classical´ graph are ubiquitous in school mathematics:

[1] The title alludes to Bruner (1973) who argues that "when one goes beyond the information given, one does so by virtue of being able to place the present given in a more generic coding system and that one essentially reads off from the coding system additional information either on the basis of learned contingent probabilities or learned principles of relating material" (p. 224).

[2] Paper presented at DG7 at PME28 in Bergen 2004 – Draft version

- the concept of square root, often used as a basis for the extension of the number concept from rational to real numbers,
- the solution of second degree equations, often used as a basis for the extension of the number concept from real to complex numbers,
- a prototype for polynomials and the connection between zeros and factoring of polynomials,
- second degree polynomials often serve as a prototype for the study of derivatives and optimisation problems,
- a prototypical example for mathematical modelling, e.g. of reflections (parabolic antennas) and projectile motion.

It is obvious that a student's conceptualisation of a parabola is coloured by the way it is described, defined, treated, used, and so forth. The term concept image has been coined to capture the (non-static) outcome of this process (Tall & Vinner, 1981). A key role is here taken by the representations used, and how the object in focus (in this case the parabola) is being objectified by the mediation of these representations (Radford, 2003). However, also at the cultural level of the mathematical community the representations used influence the views and the development of the object, which in turn have an influence on the development of mathematics itself. In today's school mathematics these different ways of looking at a mathematical object, such as the parabola, all live in a mixed world of mathematical ideas and tools, not always treated by a systematic approach, as seen from the students' perspective.

In this paper, a short account of the historical progression of the mathematical object "parabola", within classical geometry, analytic geometry, and dynamic geometry software will be given. From a semiotic point of view, a chain of metamorphoses of meanings will be described, and how this becomes critical as the mathematical object by the social process of didactic transposition is turned into a presented object of teaching and learning. The purpose of the paper is to open up for questions, by linking different approaches, rather than to try to answer them.

Historical metamorphoses

Tracking the history of a mathematical object reveals a double influence on its development - from inside and outside the pure mathematical realm. In the works of Euclid, Archimedes and Apollonius the parabola is a geometric object, defined rhetorically and analysed by the tools of constructive and deductive geometry. In the famous treatise by Archimedes of the quadrature of the parabola, he speaks of the parabola as "a section of a right angled cone" (Archimedes, 1952, p. 527)

and refers to ”the elementary propositions in conics which are of service in the proof” (p. 527), three of which are referenced to the work (now lost) by Euclid and Aristaeus (p. 528). By this definition a parabola is the section produced by a plane cutting a right angled cone at a right angle to an element of the cone. Historically, the conic sections were invented, it is believed, by the attempts of Menaechmus (approx. 350 B.C.) to solve the problem of duplicating the cube (Eves, 1983, p. 80). One of the ”elementary propositions” listed by Archimedes (1952, p. 528) is of special interest here[3]:

> If from a point on a parabola a straight line be drawn which is either itself the axis or parallel to the axis, as *PV*, and if from two other points *Q*, *Q´* on the parabola straight lines be drawn parallel to the tangent at *P* meeting *PV* in *V*, *V´* respectively, then $PV : PV' = QV^2 : Q'V'^2$.

The property identified here is what relates the parabola, in modern language and within another mathematical domain, to the quadratic function (see e.g. Charbonneau, 1997, p. 20, however discussing Apollonius). The basic idea in the classic constructions involved in relating the parabola to a square, in line with the proposition above, builds on the Pythagoren method of applications of rectlinear areas[4]. The construction in figure 1 is based on Euclid I.43 (see Thompson, 1991, p. 494).

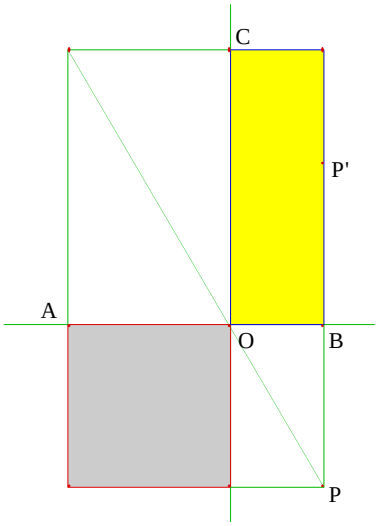

Figure 1. Area application

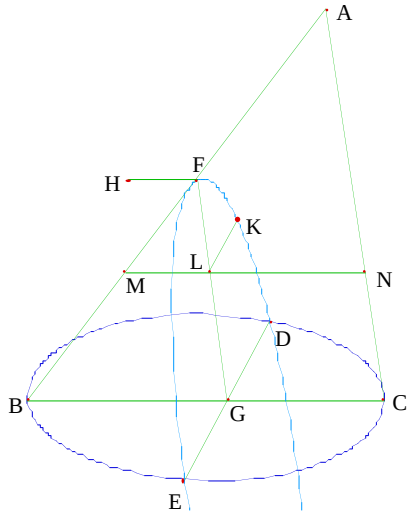

Figure 2. Definition of parabola (Apollonius, 1952)

[3] The power notation on *QV* is used by the editor to refer to the square with each of its sides equal to *QV*.
[4] The names that Apollonius introduced for the conic sections (ellipse, parabola, hyperbola) have their origin in such applications; see e.g. Eves, 1983, p. 128).

In figure 1, OC is a given segment (parameter), apply the square on the (variable) segment OA to OC by the method in Euclid I.43. This will produce the point P, choose P' so that BP' and BP are equal. As the point A is moving horizontally the points P and P' will move along a parabola. Using algebraic notation, with OB = x, OA = y, and OC = $2p$, we have $y^2 = 2px$. In the more general setting of Apollonius, the parabola is defined, along with a proposition (*Conics* I.11) similar to the one given above, by the following wordings after the proof (see figure 2):

> And let such a section be called a parabola, and let *HF* be called the straight line to which the straight lines drawn ordinatewise to the diameter *FG* are applied in square [...], and let it also be called the upright side [...]. (Apollonius, 1952, p. 616)[5]

Apollonius defines the diameter of a section as the line passing through the midpoints of parallel chords, and the axis of the section as the diameter which is perpendicular to the parallel chords. The focal property is not discussed for the parabola but may be inferred as limit cases from such discussions about the hyperbola and the ellipse (see e.g. the note in Apollonius, 1952, p. 788). The terminology used, based on drawings where the size of segments are supposed to vary, may be seen as predecessors of both the coordinate system of analytic geometry and the concepts of variable and parameter as used in the study of functions (see e.g. Charbonneau, 1997). The semiotic register found in the works of Euclid, Archimedes and Apollonius consists of natural language (i.e. symbols) and diagrams (drawings with letters indicating points, i.e. icons and indices).

After the event of analytic geometry, most notably by Descartes and Fermat, where the more modern algebraic notation in Descartes' book *La géomètrie* from 1637 was the most influential, a line (curve) could be described by the relation between coordinates (referring to distances along given directions) and studied by algebraic treatment. By representing a point on a parabola by an algebraic equation of the coordinates at the point, it was, by *a historical metamorphosis*, transformed from a geometrical object into an algebraic object: from being described by a rhetoric sequence (referring to a configuration) or a

[5] Apollonius defined a conic surface and a cone as follows: "1. If from a point a straight line is joined to the circumference of a circle which is not in the same plane with the point, and the line is produced in both directions, and if, with the point remaining fixed, the straight line being rotated about the circumference of the circle returns to the same place from which it began, then the generated surface composed of the two surfaces lying vertically opposite one another, each of which increases indefinitely as the generating line is produced indefinitely, I call a conic surface, and I call the fixed point the vertex, and the straight line drawn from the vertex to the center of the circle the axis. 2. And the figure contained by the circle and by the conic surface between the vertex and the circumference of the circle I call a cone, and the point which is also the vertex of the surface I call the vertex of the cone, and the straight line drawn from the vertex to the center of the circle the axis, and the circle the basis of the cone." (Apollonius, 1952, p. 604)

diagram (drawing), it showed itself as an algebraic expression such as $y = x^2$. By defining the geometrical properties of a curve, it was possible by inserting a coordinate system to find an algebraic representation[6], provided the algebraic manipulations could be coped with. As an example, from the defining property for a parabola as the loci for points with the same distance to a fixed point (focus) as to a fixed line (directrix)[7], a simple calculation gives the equation $x^2 = 4ay$ if the focus point is in $(0,a)$ and the directrix is $y = -a$, using an ON system. The fact that Fermat reverted this process by finding the geometric counterparts starting with given algebraic equations (Eves, 1983, p. 265), shows how the power of representations through semiotic invention and chaining may produce new and unexpected knowledge. Curves never before thought of, such as the 'parabolas of Fermat' (given by the equations $y^n = ax^m$), or in modern time fractals, could be described and pictured. Descartes performs in the second book of his *Géomètrie* a classification of all possible curves produced by a quadratic equation in two variables, and opens the way for higher order equations. His analytic method is clearly expressed in these lines:

> I could give here several other ways of tracing and conceiving a series of curved lines, each curve more complex than any preceding one, but I think the best way to group together all such curves and then classify them in order, is by recognizing the fact that all points of those curves which we call "geometric", that is, those which admit of precise and exact measurement, must bear a definite relation to all points on a straight line, and that this relation must be expressed by means of a single equation. If this equation contains no term of higher degree than the rectangle of two unknown quantities, or the square of one, the curve belongs to the first and simplest class, which contains only the circle, the parabola, the hyperbola, and the ellipse; but when the equation contains one or more terms of the third or fourth degree in one or both of the unknown quantities (for it requires two unknown quantities to express the relation between two points) the curve belongs to the second class; and if the equation contains a term of the fifth or sixth degree in either or both of the unknown quantities the curve belongs to the third class, and so on indefinitely. (Descartes, 1954, p. 48)[8]

Thus, curves corresponding to the equation $y^n = px$, $n > 2$, are considered parabolas of higher order (Eves, 1983, p. 264). When the properties of geometric

[6] In the embodied cognition literature this phenomenon is termed a "conceptual blend" (Lakoff & Nuñez, 2000, p. 48).

[7] In the *Conics* of Apollonius we find the corresponding properties of the hyperbola and the ellipse in propositions III.51 and III.52, respectively.

[8] It can be observed here how Descartes still sticks to the tradition when interpreting second order terms like *xy* or *xx* geometrically ("rectangle", "square"), but calls higher order terms more freely as "third or fourth degree", and writes them with exponential notation.

objects be studied by work in the algebraic domain, the latter may serve as a means to better understand, or gain new knowledge, of the former (Bolea et al., 1999). The symbolic language of algebra can thus be seen as a *didactic tool* to learn about the other domain, in this case geometry, fundamentally different (Bergsten, 2003).

As an example of Descartes' algebraic work, in performing the classification of the second degree equation, the following short passage gives a flavour of both his skill and enthusiasm for his new method[9]:

> Again, for the sake of brevity, put $-\frac{2mn}{z}+\frac{bcfgl}{ez^3-cgz^2}$ equal to *o*, and $\frac{n^2}{z^2}-\frac{bcfg}{ez^3-cgz^2}$ equal to $\frac{p}{m}$; for these quantities being given, we can represent them in any way we please. Then we have
>
> $y = m-\frac{n}{z}x+\sqrt{m^2+ox+\frac{p}{m}x^2}$. (Descartes, 1954, p. 63)[10]

As soon as the sometimes heavy geometric analysis had been complemented with smooth algebraic calculus, the development of mathematics exploded. Not only the ancient conics, now called second degree curves, but also corresponding 3D second degree surfaces[11] could be systematically studied (as for example by Euler, who introduced coordinate transformations to find the canonical form representation; see Kline, 1972, p. 545-547), also in the general setting of *quadric forms*, which by matrix notation and eigenvalue theory were given a unified and systematic treatment during the 19th century (see e.g. Kline, 1972, pp. 799-812). The formal notation of analytic geometry was easy to expand into any dimension, as shown by the first printed publication focused on higher-dimensional point geometry by Cayley in 1843 and other early independent work by Grassmann and Schäfli (Eves, 1983, p. 415).

With the computers of present time, performing high speed complex matrix and numerical calculations, the user can study the geometrical properties of the parabola (and other curves) as points on a screen, by doing simple hand movements (using the "drag mode" of a dynamic geometry software). The geometric (iconic) object is again in focus and this time directly available, manipulative like the

[9] This enthusiasm also echoes in the very last words of the treatise: "I hope that posterity will judge me kindly, not only as to the things which I have explained, but also to those which I have intentionally omitted so as to leave to others the pleasure of discovery." (Descartes, 1954, p. 240)

[10] Descartes used the notation *xx* for x^2 and a special sign for equality.

[11] Already Fermat had in 1643 anticipated coordinate descriptions of these 3D surfaces (Kline, 1972, p. 321).

algebraic equations used to describe it and put it on the screen, i.e. the algebraic register being used but hidden. By a historical metamorphosis the parabola has reappeared as a *dynamic object* on a computer screen. Mathematics has given itself, provided the hardware technology, one more *didactic tool*, possibly to better understand itself.

The parabola and school mathematics

A theoretical framework

The anthropological theory of didactics, based on the work of Yves Chevallard, identifies two inseparable aspects of mathematical activity, the *practical block* and the *knowledge block*. The former consists of *types of problems* that are studied and *techniques* used to solve them, while the latter is formed by the corresponding discursive environment, i.e. issues of *technology* (the word is here used in the sense of the discourse/ingredients related to the techniques) and *theory* (deeper justification). Types of problems, techniques, technologies and theories form *mathematical organisations* or *praxeologies*, the term indicating that the two blocks are inseparable. (Barbé et al, in press).

As an example, the problem of finding the midpoint of a given circle may be solved by the technique of constructing midpoint perpendiculars to chords in the circle, thus using technological ingredients such as chords to circles, a solution process justified by theorems in Euclidean geometry. Mathematical organisations can be considered at different levels, where a *punctual* mathematical organisation of a special type of problems and techniques used to solve it can be embedded in a *local* mathematical organisation with the technology available for using those techniques. Some local mathematical organisations may build on the same theoretical discourse to form a *regional* mathematical organisation. (*ibid*.)

By the process of didactical transposition, different levels of determination form didactic constraints of classroom activity: from society to school, pedagogy, discipline, area, sector, theme to question. The activity of the teacher is more or less restricted to the last two levels, with the mathematical scope of the three preceding levels. (*ibid*.)

Given this theoretical framework, students' first acquaintance and work with the parabola is limited by the local mathematical organisations previously explored by the curriculum. The available level of justification varies with the kinds of problems that are studied.

Examples of didactic transpositions

For the upper secondary science student in Sweden in the 1960s, a parabola was the geometric loci of points equidistant to a given point (*focus*) and a straight line (*directrix*), a property which was immediately elaborated by the algebraic register of analytic geometry to the formula $y^2 = 4ax$, upon which a systematic treatment of a number of geometric and algebraic properties was based (e.g. Sjöstedt & Thörnqvist, 1963, pp. 62-76)[12]. The study of the parabola was embedded in a local mathematical organisation of analytic geometry, including techniques of Euclidean geometry and elementary algebraic equation solving. Techniques from the study of functions, such as the derivate, were not used.

Later, in the 1990s, for the ”same” student the parabola was presented as an algebraically defined curve of second degree, $y = x^2$, the shape of which was plotted, based on a table of values (e.g. Björk et al., 1990, pp. 264-274). There was only little or no discussion about geometric properties of the parabola (such as the equidistance or reflection properties), other than the obvious symmetry, and the vertex point. Tangents were treated by means of the derivative. The study of the parabola was embedded in a local mathematical organisation of functions. Techniques from Euclidean geometry were used only casually, as well as analytic geometry other than the basic idea of coordinate representation.

For university students in Sweden the acquaintance with (3D) second degree surfaces, and the corresponding plane curves, is accomplished by the study of quadratic forms, embedded in the domain of linear algebra. However, the application of integrals on rotational volumes for the same mathematical objects, in the calculus course, is done in isolation within the local mathematical organisation of calculus.

Will the students of the new millennium meet the parabola as a dynamic object on a computer screen, the properties of which are explored by experimentation, within the iconic transformations made possible by the specific software, and validated by geometric or algebraic tools where “possible”? If so, in which mathematical domain (local mathematical organisation) will this study be embedded?

Objectification of knowledge

In many educational situations, the student is presented objects of learning previously not known, as might be the case with the parabola in a mathematics class. For the student to become aware of this object the teacher might use

[12] The approach of Apollonius was also included in this textbook, for optional deepening studies.

"*semiotic means of objectification* – e.g. objects, artifacts, linguistic devices and signs that are intentionally used by the individuals in social processes of meaning production"(Radford, 2002, p. 14). The drawing of a parabola may seem the optimal such means but is by itself void of meaning besides its appearance. It is necessary to go beyond the representation given to find meaning. The same remark holds, of course, for the algebraic representation $y = x^2$ or $y^2 = 4ax$. In the case of the icon, a defining property such as the equidistance property (focus-directrix) gives not only the shape of the icon (the curve) but also a basis for further explorations and analysis of the object. Using another defining non-algebraic property, such as the application of area in figure 1 above, gives the same shape but offers another basis for further explorations. The objectification process for the learner diverges between the two options, even more so by providing an algebraic formula as the main means of objectification, leading into a different mathematical organisation (see below) by the different techniques available.

As a way to enhance the objectification process, what may be called *linking tasks* can be used, provided the techniques and technology of the different mathematical organisations are available. An obvious example of a linking task is to show that the curve defined algebraically by the equation $y = x^2$ has the equidistance property, and vice versa. As a more demanding linking task, investigate if the two (non-algebraic) constructions shown in figures 3 and 4 produce the same curve. Figure 3 is based on the equidistance property and figure 4 is a simplified version for the method of area application.

Another linking task is to demonstrate the reflective property of a parabola, as defined by figure 3, and as defined algebraically by the equation $y = x^2$. In the latter case, the technology of the mathematical organisation of the study of functions may include the derivative. Linking tasks may also be designed to use the history of mathematics in teaching by the "voices and echoes game" (Boero et al, 1997).

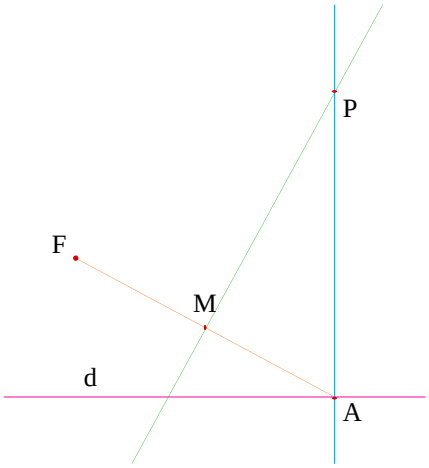

Figure 3. Equidistance construction[13]

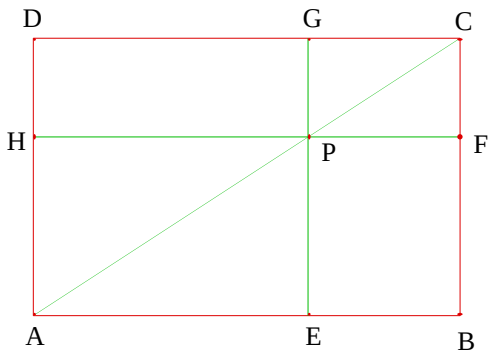

Figure 4. Area application.[14]

Discussion

The historical development of the parabola may be seen as an illustration to the idea of algebraization of a mathematical organisation (or work): "a *mathematical work is algebraized* if it can be considered as an *algebraic model* of another mathematical work, the *system to be modelled*" (Bolea et al., 1999, p. 142). This kind of modelling should respond to all the techniques and technological questioning in the organisation being modelled as a whole (ibid.). This seems exactly what Descartes aimed at with his analytic method (cf. quotation on pages 4-5 above, and footnote 8). His method proved self-generating for the development of mathematics.

By the "parabolic" cases in 2 and 3 dimensions, i.e. $y = x^2$ and $z = x^2 + y^2$, the obvious (in some sense) "parabolic" extension to *n* dimensions would be $y = x_1^2 + x_2^2 + x_3^2 + ... + x_{n-1}^2$, where $\bar{x} = (x_1, x_2, x_3, ..., x_{n-1}) \in R^{n-1}$. From the concrete action on an object – cutting a cone – the iconic "section" of a parabola by a process of metamorphosis returned in the shape of a symbolic representation in the semiotic register of algebra. By the development of this register, hitherto unknown objects were hidden beyond transformation, variation and generalisation by elaborations on this representation. With the new eyes new things were

[13] To the segment between the (fixed) focus point F and a (variable) point A on the (fixed) directrix d, draw the perpendicular at the midpoint M, intersecting at the point P the perpendicular to d at A. The point P is on a parabola by the equidistance definition. Note that the distance from F to d is the parameter.

[14] AB is a given segment (parameter), BC equal to AD constructed equal to AE, where E is a (variable) point on AB (or AB produced). EG is parallel to BC, P the intersection of AC and EG. As the rectangle ABFH is equal to the square AEGD, the point P is on a parabola from the definition by area application.

found, and old things were looked at in new ways. By the human quest for understanding, when images from experience are missing (as in the case of multi-dimensional "geometry" in R^n, $n > 3$), the phenomenon of *metaphorical mapping* (Lakoff & Nunes, 2000, p. 43) comes into play, witnessed by the choice of wordings and diagrams most often produced when communicating about these things. Examples include the expression "unit ball" when referring to the set $\left\{ \bar{x} \in R^n : x_1^2 + x_2^2 + x_3^2 + ... + x_n^2 < 1 \right\}$, or the use of the word "distance" between functions as elements in a Hilbert space.

As a remark, the wordings by Apollonius used to define a conic surface (see footnote 4) display an image schematic character (the *path schema*; see Lakoff & Nunes, pp. 37-39 and 141) in being thought-of generalised bodily based images.

In a learning setting the situation is different, as the objectification processes evoked by work on the different representations may result in fundamentally different outcomes. The options to relate to other representations or other semiotic registers are constrained by the overall didactic situation, as a consequence of the didactical transposition that has taken place. By those limitations, students' work will remain at the punctual level, resulting in concept images often too vague for successful problem solving in situations spanning over more than the restricted punctual or local mathematical organisation that has been covered in class.

Then what is a parabola, beyond the representation given? Is there a "truth" hidden at the horizon, in line with the Peircean dream? The semiotic perspective shows how meaning is transferred through the representational forms to what the interpreter within a given situation and background constructs from them. And the historical development of mathematics shows how the cultural meaning changes its face when new semiotic registers are developed and used on objects that used to be studied by other registers. Through a semiotic chain the intersection of a cone with a plane (i.e. an icon) has been linked to a quadric form (the notation of which is a symbol), represented by a symmetric matrix (symbol), and to electronic dots on a computer screen (together constituting an icon). However, new developments do not always replace old meanings with new but rather grow as a new layer of signification outside the old. A process of a similar kind, at the individual level, has been described by Presmeg (2002) by a Peircean nested triadic model of semiotic chaining. The model may be applied also to a historical-cultural development of meaning, as the one of the parabola sketched above.

An additional conception of Peirce may overarch and in the realm of reasoning and problem solving integrate the iconically based argumentation of Euclidean geometry and symbolically based in analytic geometry, i.e. *diagrammatic reasoning*:

> … all deductive reasoning, even simple syllogism, involves an element of observation; namely, deduction consists in constructing an icon or diagram the relations of whose parts shall present a complete analogy with those parts of the object of reasoning, of experimenting upon this image in the imagination, and of observing the result so as to discover unnoticed and hidden relations among the parts (Peirce, quoted in Dörfler, 2004, p. 7)

Also the manipulation of algebraic formulas is a work with icons, the patterns of formulæ: "These are patterns, which we have the right to imitate in our procedure, and are the icons par excellence of algebra." (*ibid.*; cf. the concept of *mathematical form* in Bergsten, 1999). This kind of diagrammatic reasoning is of such generality that it may function, for the individual, as a linking force between local mathematical organisations, since it is not primarily focused on the referential function (or meaning) of the forms.

To conclude, it is a question of curriculum development to design mathematical schooling to facilitate students' development of concept images and problem solving techniques not limiting students' thinking to the punctual level. Only by linking the different representations of the objects of learning (e.g. the parabola) to the local mathematical organisations into which they can be embedded, the flexibility needed for solving more non-routine problems by the availability of adequate technology can be developed.

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/ Christer Bergsten

Fyra nya licentiatavhandlingar i forskarskolan

I SMDF:s medlemsblad nr 9 beskrevs fem licentiatavhandlingar presenterade inom RJ:s nationella forskarskola i matematikdidaktik. Sedan dess har ytterligare fyra av doktoranderna lagt fram sina uppsatser för licentiatexamen. Här kommer vi att kort ge en beskrivning av innehållet i dessa, vilket tillsammans med de fem tidigare ger en framväxande bild av hur den matematikdidaktiska forskningen i Sverige sprider sig över ett allt bredare forskningsfält.

Örjan Hanssons licentiatavhandling har titeln *Presevice teachers' view on the concept of function, A study including the utilization of concept maps.* Opponent vid presentationen av licentiatavhandlingen på ett seminarium den16 december vid Luleå tekniska universitet var Morten Blomhöj från Roskilde universitetscenter i Danmark. Avhandlingen består av tre artiklar och en sammanbindande kappa. Den första artikeln *Preservice teachers' conceptions about* $y = x + 5$*: Do they see a function?* är författad tillsammans med handledaren Barbro Grevholm och publicerad i PME-proceedings från 2003. Utgående från en longitudinell studie som genomförts av Grevholm (med ett 40-tal lärarstuderande) har Hansson upprepat en liten del av studien i en grupp av 19 lärarstuderande. En aspekt rör hur lärarstuderande ser på påståendet $y = x + 5$ före och efter kurser i algebra respektive funktionslära. Få studenter kopplar det till begreppet funktion och även begreppet linjär ekvation lyser med sin frånvaro. Ett vanligt synsätt är att påståendet är en procedur för att beräkna *y* då *x* har tilldelats ett värde. Den andra artikeln behandlar ett oortodoxt sätt att använda begreppskartor med utgångspunkt från ett matematiskt påstående istället för ett begrepp. Kartorna ses som ett diagnostiskt verktyg och Hansson diskuterar hur de lärarstuderande svarar på detta verktyg. Den tredje artikeln är en fallstudie över tre studerande på olika prestationsnivå och analyserar hur de ser på $y = x + 5$, $y = \pi x^2$ och $xy = 2$. De tre påståendena anses utgöra centrala delar av stoff som ska behandlas i grundskolan när det gäller funktioner och det är därför av värde att se hur de lärarstuderande uttrycker sig om dessa tre påståenden. Resultaten visar att lärarstuderande inte alltid har ett väl utvecklat funktionsbegrepp. Det finns en risk att det leder till att deras framtida undervisning blir alltför inskränkt och inte erbjuder deras elever tillräckligt varierade inlärningssituationer. Författaren gör även en omfattande genomgång av tidigare forskning om funktionsbegreppet och om användning av begreppskartor i studier inom matematikens och naturvetenskapens didaktik.

Licentiatuppsatsen *Samspel mellan intuitiva idéer och formella bevis* av Kerstin Pettersson är en fallstudie av fyra universitetsstuderandes arbete i grupp med en uppgift där det gällde att ge ett induktionsbevis för ett påstående i anslutning till en kurs i matematisk analys, med syftet att belysa samspelet mellan intuitiva idéer och formella bevis. Genom att använda sig av ’intentionell analys’ vill författaren ge möjlighet att synliggöra tolkningsprocessen av data (transkriberad videoinspelning), deltagarnas olika kompetenser och det matematiska innehållet i deras aktiviteter. Samspelet mellan intuitiva idéer och formella bevis och dess funktion illustreras via olika exempel, som visar hur studenternas använder rika begreppsbilder och relevanta intuitiva idéer för att lösa den förelagda uppgiften. Kraven på att ge ett formellt bevis både stimulerar och hämmar problemlösningsprocessen, som leder till att de anser sig gett ett riktigt bevis men att det inte passar in i den bild de har av ur ett sådant bevis bör se ut. Som fallstudie påvisar arbetet både existensen av ett samspel mellan intuitiva idéer och formella bevis och hur detta kan se ut. Opponent vid licentiatseminariet i Göteborg den 23 november 2004 var Johan Lithner, Umeå universitet.

Johan Prytz licentiatavhandling *A study of the angle of contact with a special focus on John Wallis’ conception of quantities and angles* har en matematikhistorisk inriktning. Genom att studera några exempel från matematikens historia kring begreppet kontaktvinkel, dvs den vinkel som bildas när en kurva tangerar en linje, ges en bild av den historiska utvecklingen av en matematisk begreppsuppfattning. Redan i Euklides *Elementa* visas hur en sådan vinkel inte kan mätas på samma sätt som ”vanliga” plana vinklar med en kvantitet som uppfyller euklidiska krav på kvantitativa mått. Huvudsyftet i uppsatsen har varit att försöka beskriva John Wallis (1617-1703) försök att inkludera ett mått på en sådan kontaktvinkel som en storhet inom en generell teori om kvantiteter. Wallis diskussioner jämförs med äldre matematikers arbeten kring samma fråga, Peletier, Clavius och Hobbes, framförallt för att belysa skillnader i begreppsuppfattningen. Problemet är intressant att studera även mot bakgrund av att Wallis arbetade i brytningstiden i samband med differentialkalkylens framväxt. I uppsatsen placeras Wallis och hans nyskapande bidrag, som han även försökte legitimisera genom att relatera till de antika klassikerna, in i den historiska kontexten och den samtida diskussionen kring matematikens och vetenskapens metoder. Uppsatsen belyser bland annat skillnader mellan kvalitativa och kvantitativa aspekter av matematiska begrepp. Opponent vid licentiatseminariet den 30 september 2004 i Uppsala var Jacqueline Stedall, Queen's College, Oxford University.

Magnus Österholm har för sitt licentiatarbete *Läsa matematiska texter: Förståelse och lärande i läsprocessen* haft ett huvudsyfte att testa och utveckla speciellt för matematiska texter en befintlig mer generell teori för förståelse av texter och förståelse i allmänhet (Kintsch, 1998). Förutom en teoretisk analys av matematiska texter genomfördes en empirisk studie där elever i gymnasiets avgångsklass (naturvetenskapsprogram) och studerande som läst inledande universitetskurser i matematik läst och svarat på frågor om en av två matematiska texter och en historisk text. De matematiska texterna hade samma innehåll men olika form, där den ena saknade matematiska symboler men den andra innehöll sådana symboler. Resultaten visar att det verkar finnas många likheter mellan läsning av ”vanlig” text och matematisk text utan symboler. Artikulering och avkodningsförmåga av symboluttryck anges som tänkbara förklaringar. Hur det matematiska innehållet presenteras i en text verkar ha större betydelse för läsprocessen än det matematiska innehållet i sig. Opponent vid licentiatseminariet i Linköping den 16 december 2004 var Rudolf Strässer, Luleå tekniska universitet.

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/ Christer Bergsten och Barbro Grevholm

A new report:
Democracy and participation – A challenge for special needs education in mathematics

From the Preface

This volume of Reports from the Department of Education, Örebro University contains the proceedings of the 2nd Nordic Research Conference on Special Needs Education in Mathematics.

Low achievement in mathematics is a social construct. It is not a fact, but a human interpretation of relations between the student and his/her environment. Special needs education in mathematics must be looked upon from a relativist view according to Magne (2003).

Research on special needs education in mathematics is lagged behind comparing research on for instance reading or writing disabilities. We also lack experience on successful developmental works in the field. Therefore the Nordic conferences held in recent years are of great importance for the further development of the field.

The 1st Nordic Research Conference on Special Needs Education in Mathematics was held at Agder University College in Kristiansand, Norway in 2001, organised by Forum for matematikkvansker on the theme Mathematics for all in a school for all. About 50 participants from the Nordic countries came together to bring up issues of special needs education in mathematics. The conference was a great success.

Inclusion and inclusive schooling is a current trend in most European countries. But what constitutes inclusion and how should it be developed in practice and policy at national, local and school levels? There is a range of different potential interpretations. By referring the term more recently to inclusive school for all its scope has been broadening to include new areas of concern as social justice and social inclusion (Campbell 2002). The rethinking of special needs education that has occurred during the latest decade has brought up new challenges for researchers and practitioners.

Democracy and Participation – A Challenge for Special Needs Education in Mathematics was consequently the theme on the 2nd Nordic Research Conference on Special Needs Education in Mathematics October 2003, organized by the Department of Education at Örebro University. More than 70 researchers, teachers and administrators from the Nordic countries came together with the intention to meet these challenges.

Four invited lectures were given by scholars from Germany, United Kingdom and Sweden. About 20 papers were presented covering different topics, from research projects to developmental works. These proceedings include the invited lectures and most of the papers presented at the conference.

The different contributions are written in English or in a Scandinavian language (Danish, Norwegian or Swedish). In the latter case there is an English abstract.

One of the more important results of the conference was the establishment of a Nordic Network for Research on Special Needs Education in Mathematics (see appendix). National networks are also established or planned in the Nordic countries. A common site for the Nordic Network on the Internet is also available at following URL: http://www.matematikkvansker.net where different activities of the network are presented.

The conference was planned during two days in May 2003 by the Programme Committee, Arne Engström chair, Örebro University, Ann Ahlberg, Sweden, Edda Óskarsdottír Iceland, Anna Kristjánsdottír, Norway, Michael Wahl Andersen Denmark, Karin Linnanmäki Finland, and two representative of the organizer for the first conference Olav Lunde and Jarl Formo, Forum for Matematikkvansker, Agder University College, Norway.

Many thanks to all those colleagues and friends in contributing to such a success of the conference.

The third conference will be held at Aalborg University in Denmark in 2005. I hope we will meet there for new challenges.

/ Arne Engström

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The report:

Engström, A. (Ed.) (2004). *Democracy and participation – A challenge for special needs education in mathematics.* Reports from the Department of Education, 7. Örebro: Örebro university.

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Nytt från *Forum for matematikkens didaktik*

Vår danska systerförening firade sitt 10-årsjubileum i fjol och vi rapporterade från det i nummer 8, 2003. I vårt engelskspråkiga nummer 9 som kom ut vid ICME10-kongressen hade vi ingen rapport från *Forum* så därför blir denna en rapport från hela 2004. Föreningen har i sin årgång 8 av tidskriften *Forum* kommit ut med tre nummer och vi sammanfattar det viktigaste från dessa.

Vi ska dock börja med nr 6 från årgång 7 eftersom den kom ut i januari i år. Två av de tunga inslagen där är presentationerna av två färska avhandlingar i matematikdidaktik. Det gäller Dinna Ballings avhandling *Grafregneren i gymnasiets matematikundervisning: lærernes holdninger og erfaringer*. Den framlades vid Dansk Institut for gymansiepædagogik i januari 2004. Hela avhandlingen kan hämtas ner från nätet. Kontakta Dinna på *dinna.balling@skolekom.dk*. Den andra avhandlingen är författad av Sören Antonius vid samma institut och handlar om *Modellering til eksamen - en analyse af modellering, IT og eksamen i matematik på højere handelseksamen.*

En tredje artikel har titeln *Jamenalligevel*, är skriven av Inge Henningsen vid Afdeling for Anvendt Matematik og Statistik vid Köpenhamns universitet och belyser vilken roll danska elevers resultat från internationella undersökningar har i den pedagogiska debatten och hur det beror på hur resultaten tolkas. Författaren menar att de kunde tolkas på annat sätt än man gjort hitintills.

I nummer 1 av *Forum* för 2004 presenteras den Nordiska forskarksolan i matematikdidaktik som startade den 1 januari 2004 vid Högskolen i Agder. Claus Michelsen presenterar *Ett eksempel på en antididaktisk inversion - gymnasiets läreböger i matematik.* Artikeln är en rapport från konferensen med Koeno Gravemeijer från Freudenthalinstitutet som ägde rum den 22-23 januari (en kollision med MADIF4, tyvärr). Michelsen gör en grundlig genomgång av de presentationer och diskussioner som fördes om läroboken i matematik. Sådana texter om läroboken i skolmatematiken har lyst med sin frånvaro i svensk debatt och lärartidningar.

I årsberättelsen för 2004 tar ordföranden Lisser Rye Ejersbo upp den bibliografi över dansk matematikdidaktik som utarbetats av Tine Wedege och Sören Antonius. Den presenterades på ICME10. Under året har föreningen även fått en egen hemsida, http://matematikdidaktik.dk .

Nummer 2 av *Forum for matematikkens didaktik 2004* handlar givetvis om läget efter ICME10-kongressen i Köpenhamn. Här beskrivs både gamla och nya programinslag såsom tematiska eftermiddagen, utdelning av nyinstiftade priser med medaljer till Celia Hoyles och Guy Brousseau samt en intervju med Paul Drijvers och Koeno Gravemeijer från Freudenthalinstitutet. Framtidens utmaningar diskuteras där och de säger att det är inte möjligt att ändra undervisningen utan att stödja läraren med nya material och det är inte rimligt att vänta att de kan använda nya material på ett nytt sätt utan hjälp att klara den nya och krävande situationen. Det finns en tendens att låta eleverna arbeta mer och mer ensamma och i sitt eget tempo. Detta gör det mycket svårt att få igång de ’communities of learners’ som är så viktiga för både matematiken och andra ämnen. En påannonsering av ICME11 som ska äga rum i Mexico 2008 sker också.

I decembernumret, nr 3, ventileras frågan on kvalitet i doktorandhandledningen. Tine Wedege refererar till det seminarium för handledare som den Nordiska forskarskolan ordnade i Vasa den 11-13 september med Frank Lester och Diana Lambdin som inspirerande och drivande ledare. Hon citerar också en artikel där Barbro Grevholm, Lars-Erik Persson och Peter Wall presenterar en modell för handledning och en definition av vad effektiv handledning kan betyda. En kurs i Danmark för handledare och deras doktorander den 10-12 januari i Korsör ska behandla samma teman. Tine inbjuder också till konferens den 20-21 januari i Nyborg med temat *Matematik - vad är det for en størrelse?* Här medverkar Paul Ernest och talar om *Mathematics as semiotic systems.* I aktivitetskalendern pålyses vidare Nordic Summer School in Mathematics Education den 8-14 augusti i Jyväskylä i Finland (för doktorander) samt den fjärde Nordiska forskningskonferensen i matematikdidaktik i Trondheim den 2-6 september 2005. I spåren av ICME10 som var ett nordiskt åtagande ser vi alltså en rad nordiska arrangemang.

/ Barbro Grevholm

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Anslagstavlan

Aktuella konferenser

Workshop om matematikens formelspråk, Stockholm 18 mars 2005
http://www.mai.liu.se/~chber/workshop/

ICTMA 12, London, England 10-14 juli 2005
http://www.city.ac.uk/conted/research/ictma12/

PME 29, Melbourne, Australien 10-15 juli 2005
http://staff.edfac.unimelb.edu.au/~chick/PME29/

Norma 05, Trondheim, Norge 2-6 september 2005
http://www.alt.hist.no/~froder/norma05/

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