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Computational Thinking in the Mathematics Classroom

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CMT Research Project Fact Sheet

- **Title:** ***Computational thinking and mathematical thinking: digital literacy in mathematics curricula***
- **Consortium: Utrecht University, Radboud University, SLO, and five schools**
- **Duration: feb 2019 – jan 2022**
- **Granted by "NRO langlopend Onderwijsonderzoek"**
- **Context: 11th grade pure and applied math curriculum**

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NRO NATIONAAL REGIEORGAAN ONDERWIJSONDERZOEK

Informal questions

- **How to deal with CT in math education (e.g., in the current curriculum reform in NL)?**

- **How to exploit the manifest links between mathematical and computational thinking?**

Research question

How can a teaching-learning strategy, focusing on the use of digital tools, support 16-17 years old pre-university students in developing computational thinking skills related to mathematical thinking in pure and applied mathematics courses?

The question is

… not only about the intersection of mathematical and computational thinking

… but also about the aspects of computational thinking that can be addressed in mathematics education, as instances of problem solving through the use of calculational tools

Research sub-questions

1. What are core common aspects of computational thinking and mathematical thinking?
2. How can these common aspects be promoted in learning activities in which 16-17 years old pre-university education students use digital tools in pure and applied mathematics courses?
3. Do such learning activities lead to learning gains with respect to computational and mathematical thinking?

2 x 5 lessons, e.g., using R for stats, using GGB CAS in calculus, using DME

Research set-up

A theory-informed design study including four phases:

- **Inventory phase (literature + Delphi + expert interview study)**
- **First design cycle (grade 11)**
- **Second design cycle phase**
- **Concluding phase**

As first cycle, but revised and focus on learning outcomes

Synthesis, dissemination

Targeted Results

1. A literature-based, but practice-oriented identification of key elements of computational thinking that relate to mathematical thinking.
2. A set of empirically validated learning activities for upper secondary pre-university education students in applied (Wiskunde A) and pure (Wiskunde B) mathematics courses.
3. A set of assessment instruments to assess learning outcomes in the field of computational and mathematical thinking.
4. A guide for mathematics teachers who design or carry out learning activities targeting computational thinking and mathematical thinking using digital tools.

Outline

1. What is Computational Thinking?
2. How do Computational and Mathematical Thinking relate?
3. How to address Computational Thinking in the Mathematics classroom?
4. Conclusion

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Computational thinking (CT) …

… is "in the air", both at national and at international levels,

… but is also a "container concept".

So what do we mean?

A first definition

Wing (2006):

Computational thinking involves solving problems, designing systems, and understanding human behavior, by drawing on the concepts fundamental to computer science.

[...] Thinking like a computer scientist means more than being able to program a computer. It requires thinking at multiple levels of abstraction.

The T in CT: more possible interpretations

Computational…

- **Thinking**
- **Transformation**
- **Transposition**
- **Transparency**
- **….?**

Some more definitions

Lu & Fletcher (2009):

CT involves the thought processes used to understand and phrase problems in such a way that they can be solved in terms of computations

Not necessarily carried out by a machine

5 Computational Thinker

Students develop and employ strategies for understanding and solving problems in ways that leverage the power of technological methods to develop and test solutions.

CLOSE —

5a Students formulate problem definitions suited for technology-assisted methods such as data analysis, abstract models and algorithmic thinking in exploring and finding solutions.

5b Students collect data or identify relevant data sets, use digital tools to analyze them, and represent data in various ways to facilitate problem-solving and decision-making.

5c Students break problems into component parts, extract key information, and develop descriptive models to understand complex systems or facilitate problem-solving.

5d Students understand how automation works and use algorithmic thinking to develop a sequence of steps to create and test automated solutions.

International Society for Technology in Education, https://www.iste.org/standards/for-students

Aspects of computational thinking
(CSTA & ISTE, 2011)

- formulating problems in a way that enables us to use a computer and other tools to help solve them,
- logically organizing and analysing data,
- representing data through abstractions such as models and simulations,
- automating solutions through algorithmic thinking (a series of ordered steps),
- identifying, analysing, and implementing possible solutions with the goal of achieving the most efficient and effective combination of steps and resources, and
- generalizing and transferring this problem-solving process to a wide variety of problems.

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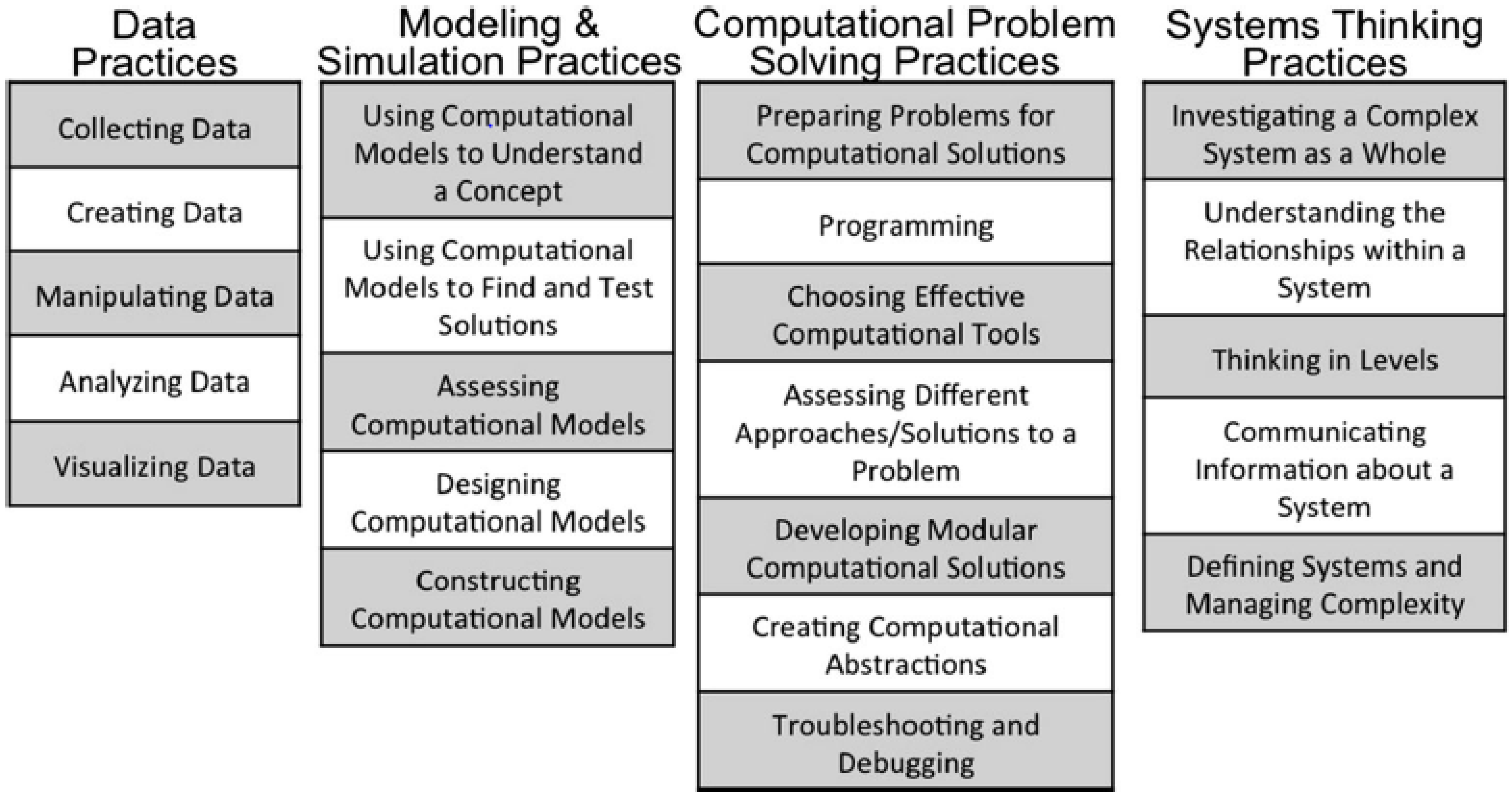
CT aspects according to Weintrop et al. 2016:



A taxonomy of computational thinking in mathematics and science

Aspects of computational thinking

Angeli et al. (2016):

- Abstraction
- Generalization
- Decomposition
- Algorithms
- Debugging

Grover & Pea (2013); Selby & Woollard (2015):

- Problem-solving
- Abstraction
- Generalization
- Decomposition
- Algorithmic thinking
- Modularization

Is problem solving in CT …

… Problem solving* through the use of calculation?

… Problem solving through the use of calculational tools**?

***Problem solving is as old as mathematics, even if mathematics has not the exclusive right to claim it**

**** Tools can be material, digital, mental, plugged and unplugged**

In short,...

... CT is a rich but somewhat vague notion

... which clearly shares common ground with notions of mathematical thinking

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Mathematical Thinking (MT): why?

- The goal of math ed is ”to make young people think” (Polya)
- The need to go beyond routine procedural work
- The need to be able to deal with (results from) sophisticated embedded mathematical tools

-> MT as a core goal in the Dutch 15-18 curriculum reform in 2015, and in the ongoing one for 4-18.

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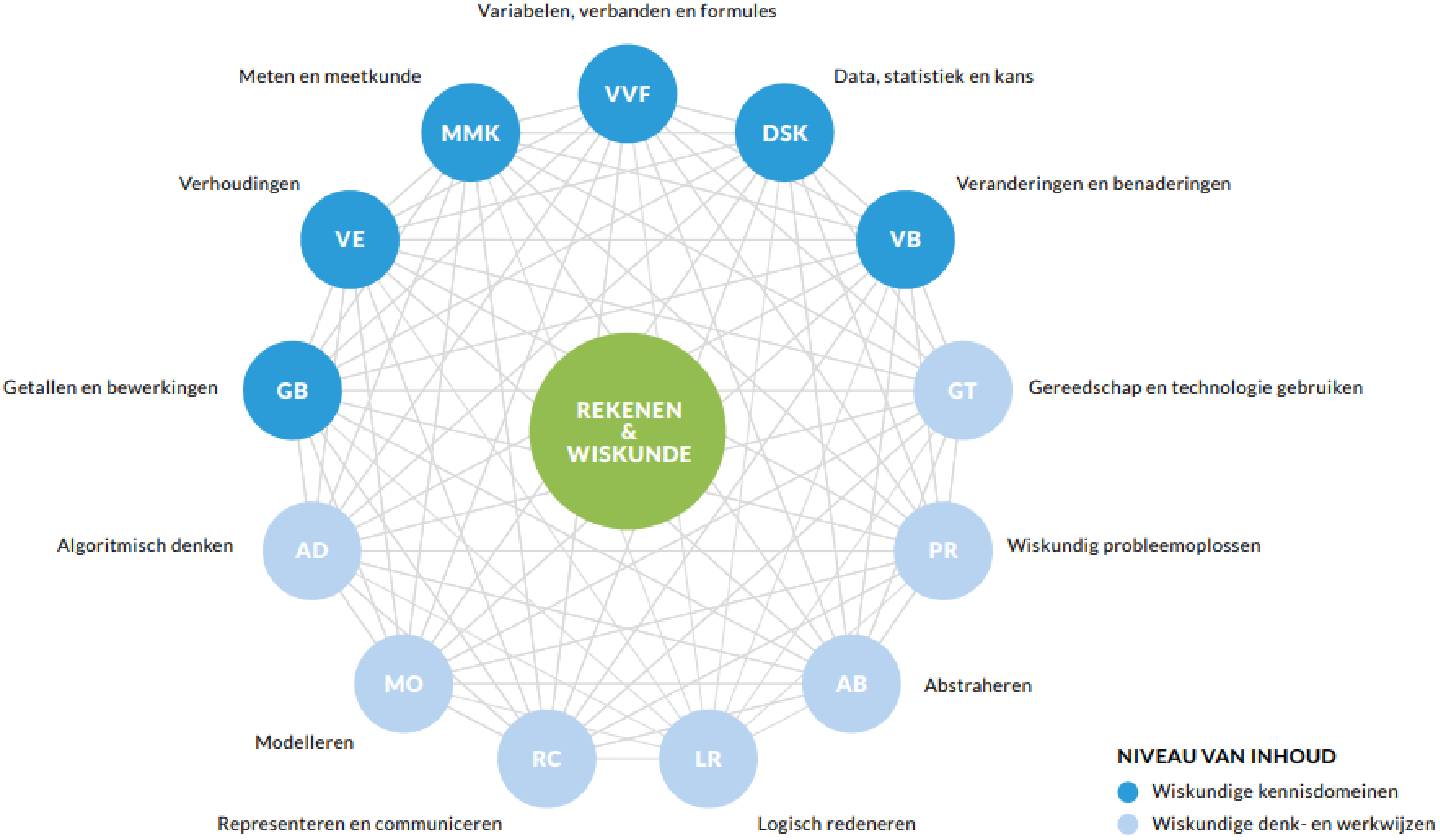

https://curriculum.nu/

MT according to the NL cTWO (2015) curriculum reform

two

Eindrapport van de vernieuwingscommissie wiskunde cTWO

www.ctwo.nl

4

Standpunt 4

Kernconcepten in het wiskundeonderwijs van havo en vwo zijn getal, formule, functie, verandering, ruimte en toeval. Centrale denkactiviteiten zijn modelleren en algebraïseren, ordenen en structureren, analytisch denken en probleemoplossen, formules manipuleren, abstraheren, en logisch redeneren en bewijzen. Deze kernconcepten, denkactiviteiten en de bijbehorende vaardigheden moeten als lange leerlijnen door het gehele programma van havo-vwo lopen.

- **Modelling and algebraizing**
- **To orden and structure**
- **Analytical thinking and problem solving**
- **Manipulate formulas**
- **Abstract**
- **Logical reasoning and proving**

My MT triangle

http://www.fisme.science.uu.nl/publicaties/literatuur/Oratie_Paul_Drijvers_facsimile_20150521.pdf

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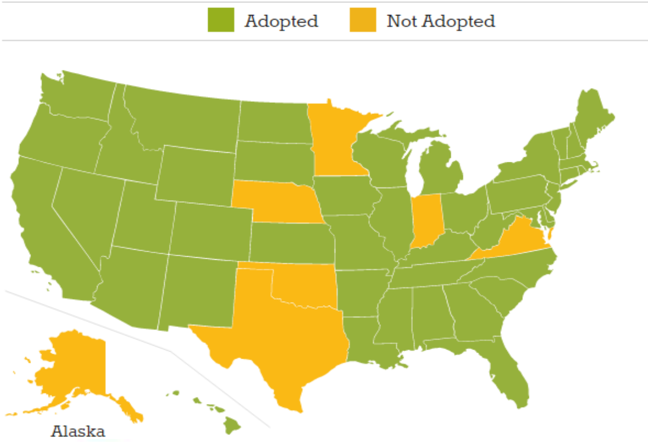
Focus on MT not just a Dutch Disease, but an international trend

US Common core standards for mathematical practice:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

http://www.corestandards.org/Math/Practice



COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

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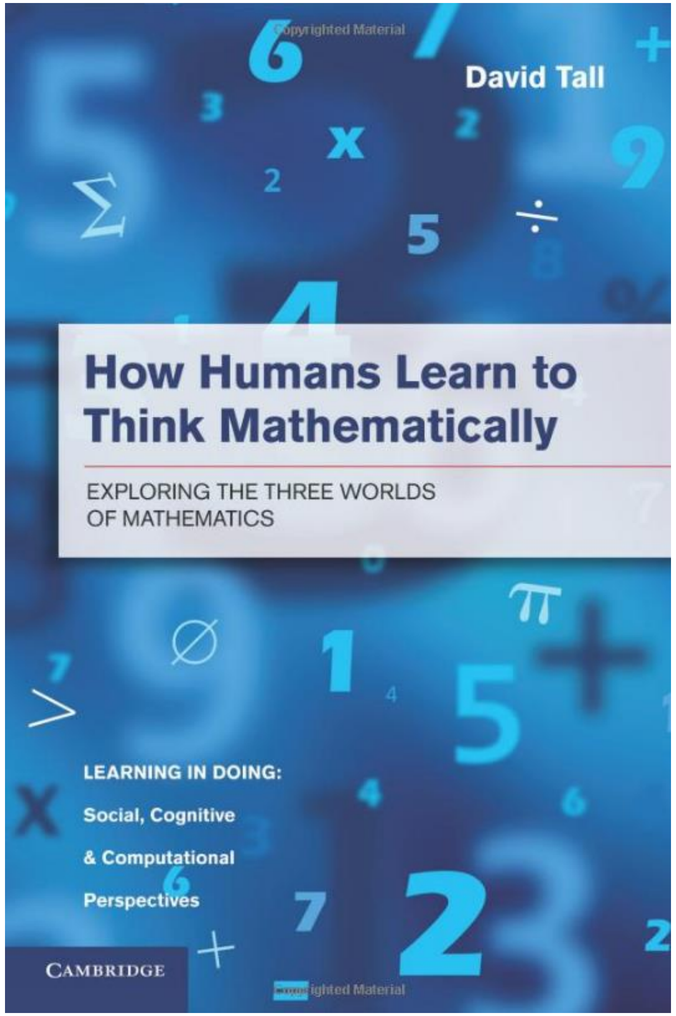
Zooming in on abstraction

"The greatest barrier to doing mathematics" (Devlin, 2000, p.11)

"Abstracting is an activity by which we become aware of similarities … among our experiences." (Skemp, 1986, p. 21)

"Abstraction is the isolation of specific attributes of a concept so that they can be considered separately from the other attributes" (Tall, 1988, p. 2).



Central in abstraction: Object formation

rekenen
met variabelen

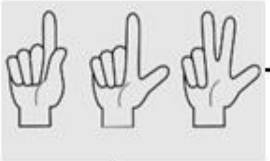
$$x \to x^2$$

$$a^2 + b^2 = c^2$$

vergelijken,
generaliseren

$$a$$

tellen

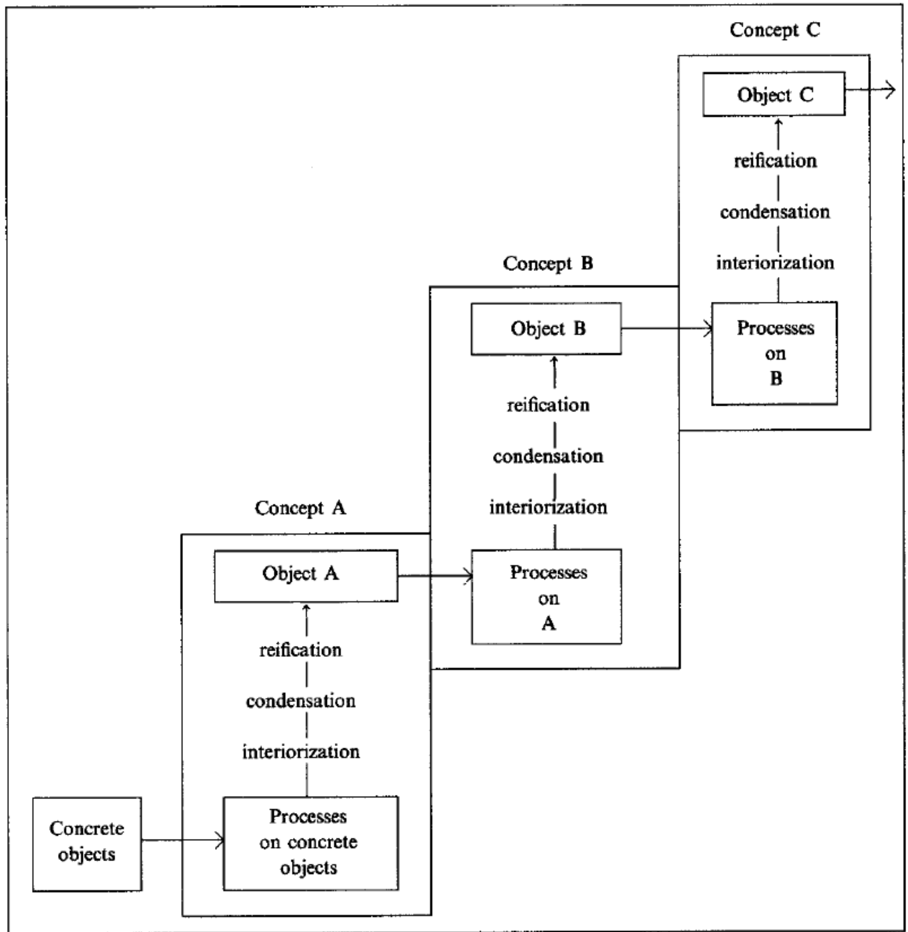


$$32$$

Sfard 1991:"On the dual nature..."

... of mathematical conceptions"

- **Math concepten have a process and an object face, an operational and a structural character**
- **In learning, the first usually precedes the latter**
- **Reification involves including the latter.**



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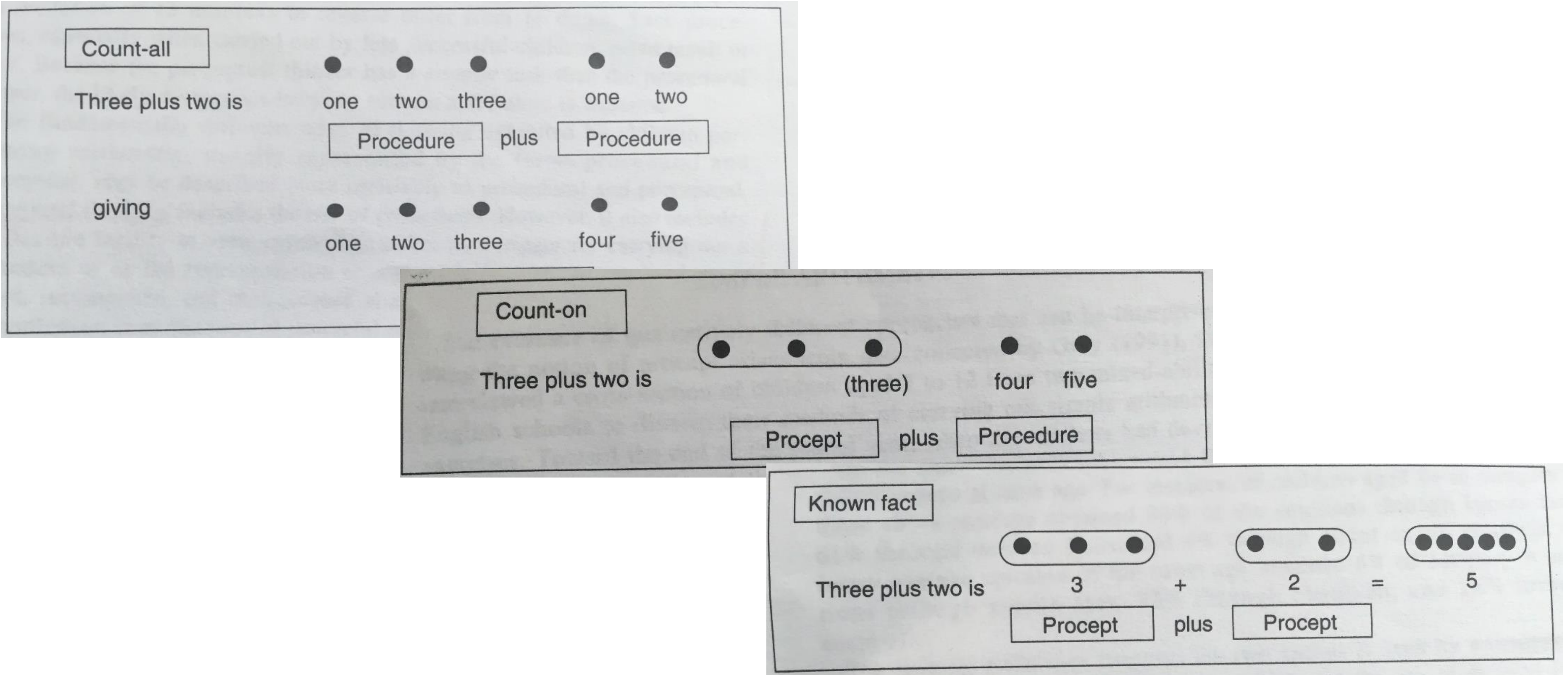
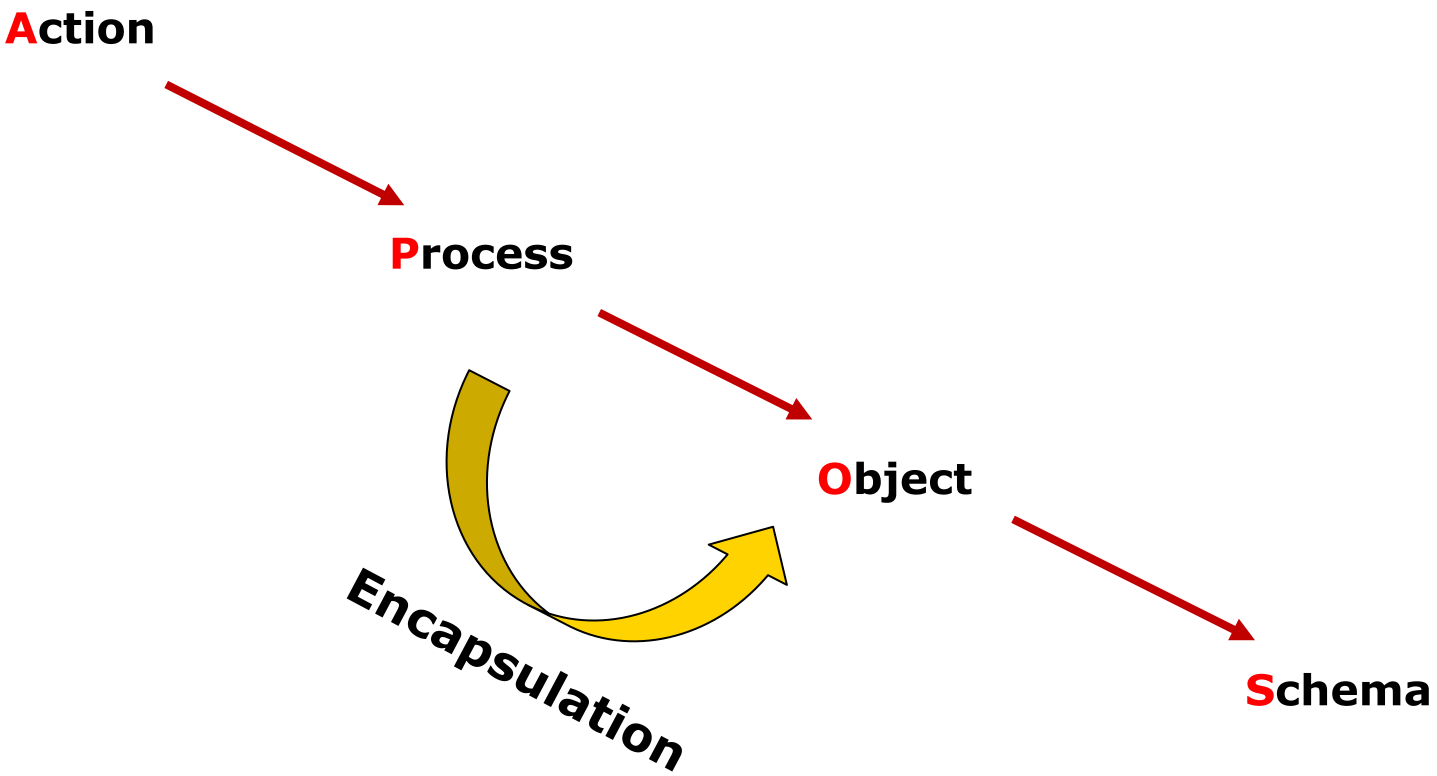
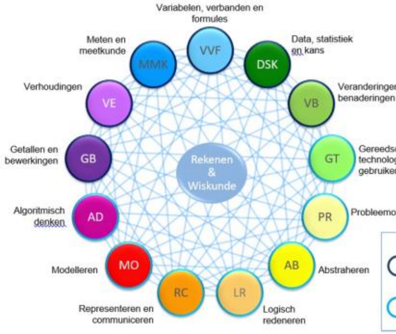
Gray & Tall 1994: procept

Procept = process + object (+ symbol)



Cotrill et al. 1996: APOS theory





Mathematical thinking and computational thinking: shared foundations

MT (cTWO, 2013):

- Modelling and algebraizing
- To orden and structure
- Analytical thinking and problem solving
- Manipulate formulas
- Abstract
- Logical reasoning and proving

Grover & Pea (2013); Selby & Woollard (2015):

- Problem-solving
- Abstraction
- Generalization
- Decomposition
- Algorithmic thinking
- Modularization

Mathematical thinking and computational thinking: shared foundations

Our aims

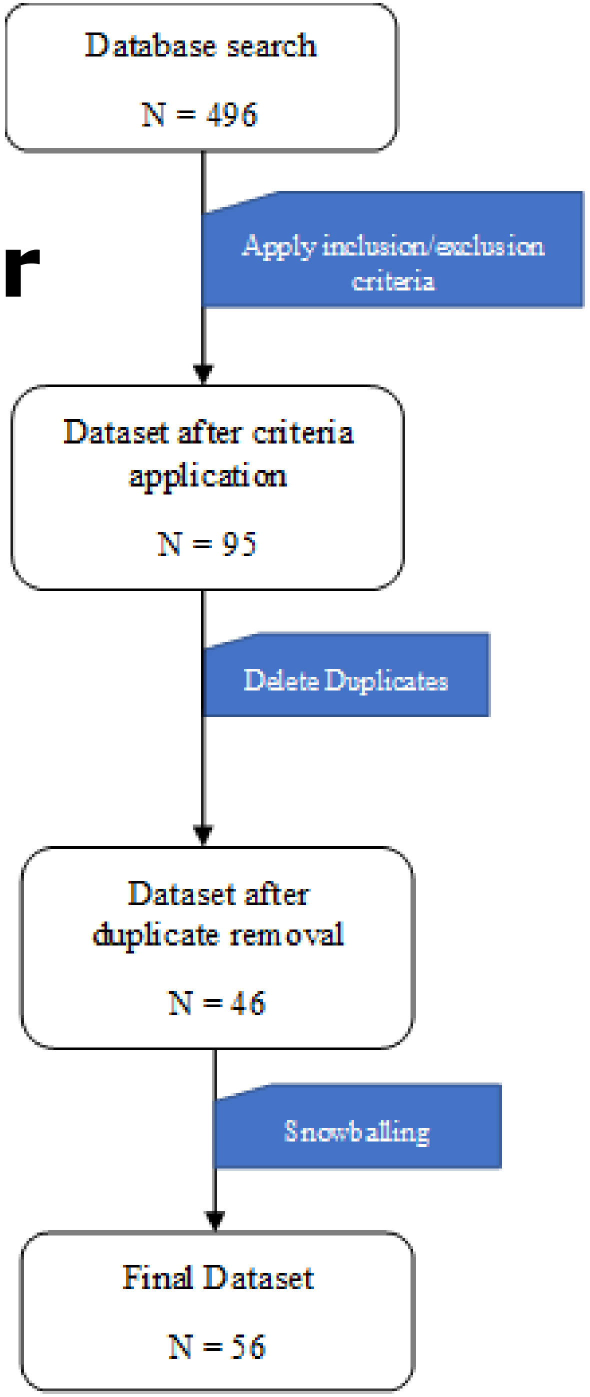
a. To exploit these common grounds
b. To design intra-curricular mathematics lessons in which CT aspects and the use of digital tools are addressed

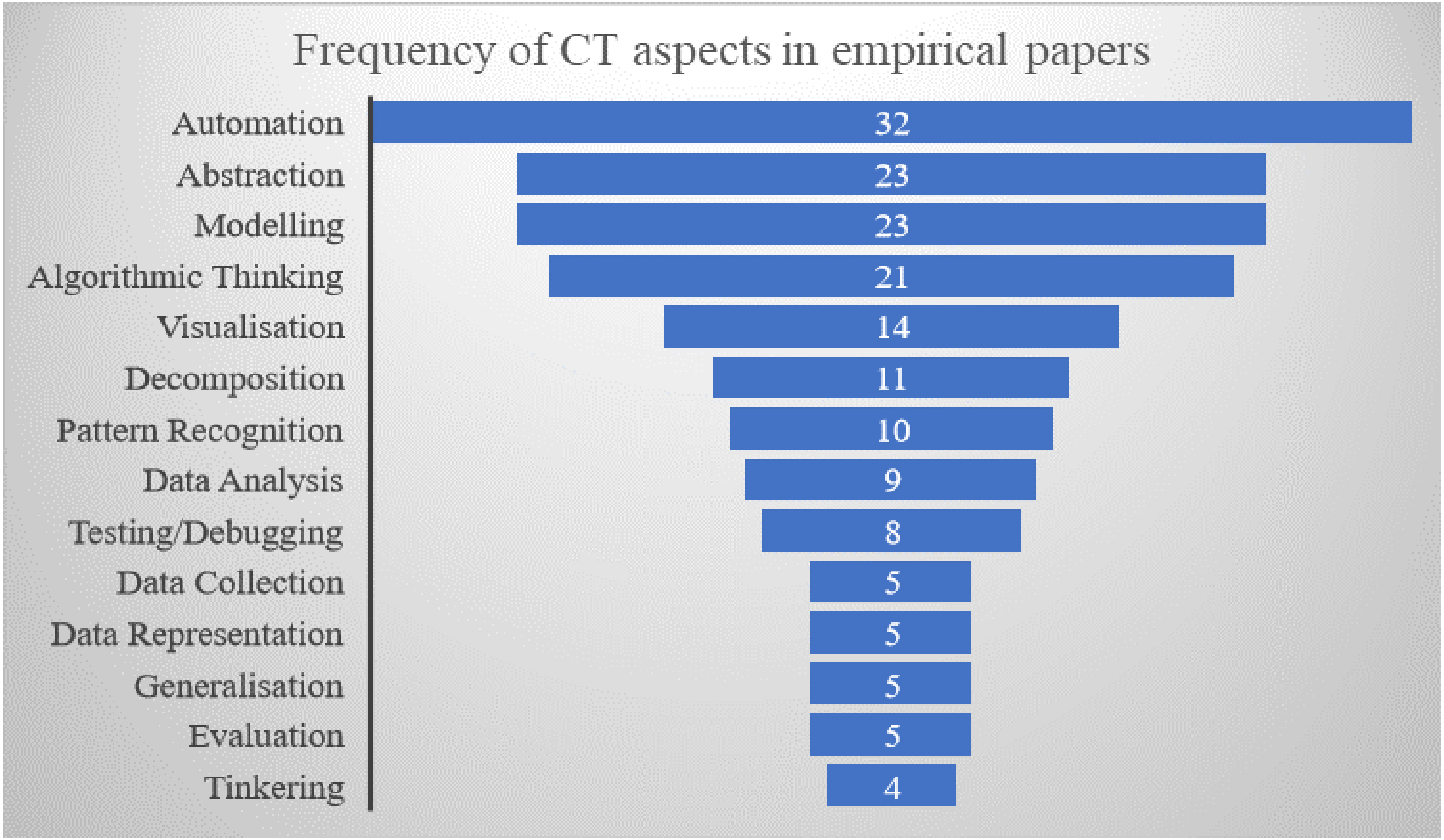
Ad a. -> A Delphi Study

(Kallia, Van Borkulo, Drijvers, Tolboom, & Barendsen, in preparation)

Literature study as starting point for Delphi study

- 6 databases (ACM, IEEE, Web of Sciences, ERIC, Scopus, and PsycINFO)
- Query: "computational thinking" AND mathematic*
- 479 papers, 56 included in review
- Both empirical and theoretical papers





Frequency of CT aspects in empirical papers
Automation
Abstraction
Modelling
Algorithmic Thinking
Visualisation
Decomposition
Pattern Recognition
Data Analysis
Testing/Debugging
Data Collection
Data Representation
Generalisation
Evaluation
Tinkering
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23
23
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11
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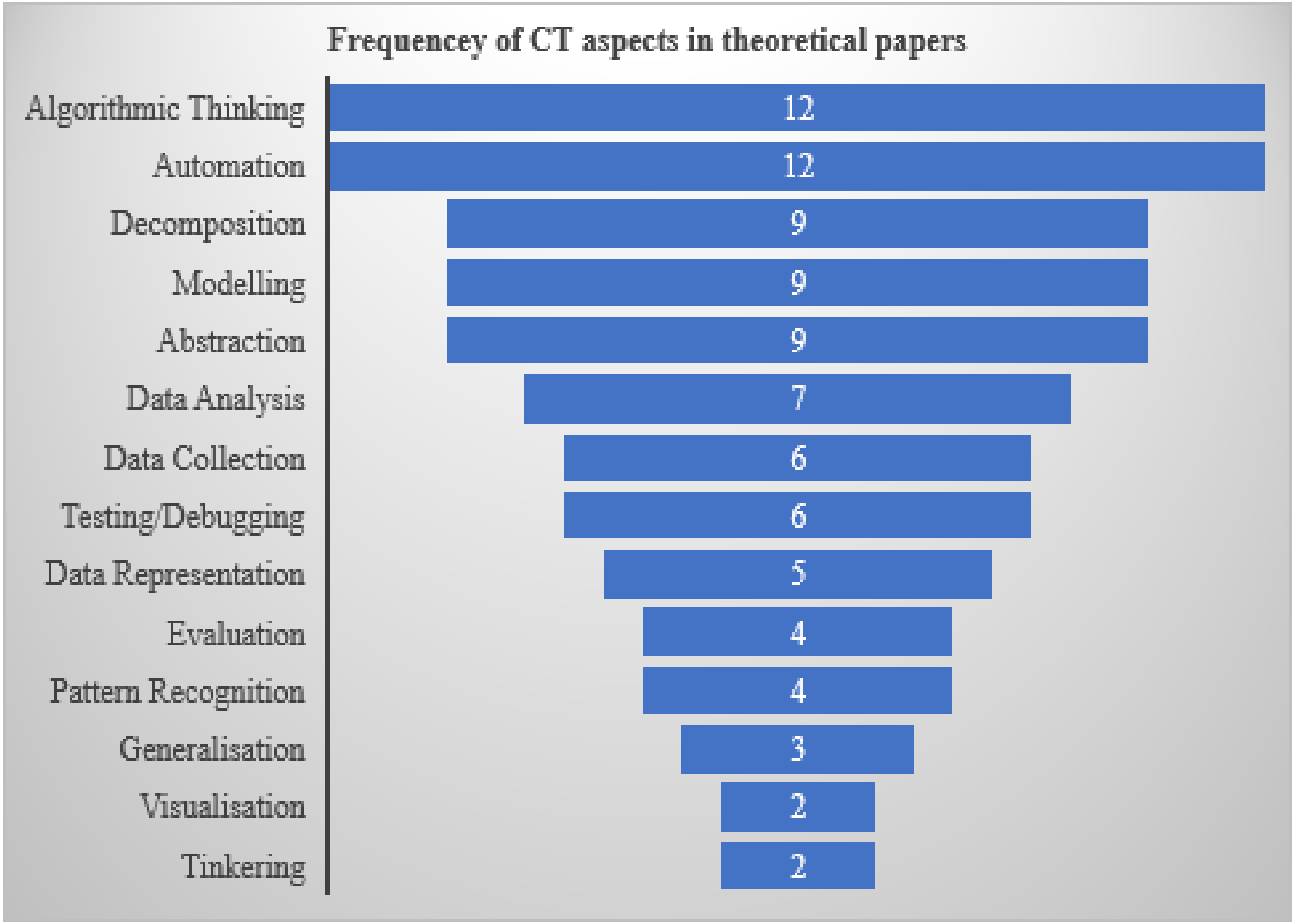
Frequencey of CT aspects in theoretical papers
Algorithmic Thinking
12
Automation
12
Decomposition
9
Modelling
9
Abstraction
9
Data Analysis
7
Data Collection
6
Testing/Debugging
6
Data Representation
5
Evaluation
4
Pattern Recognition
4
Generalisation
3
Visualisation
2
Tinkering
2

Delphi Study

9 teachers, 16 researchers

Questions:

1. **What characterizes computational thinking in mathematics education?**
2. **What aspects of CT can be addressed in mathematics instruction?**
3. **What aspects do computational thinking and mathematical thinking have in common?**
4. **Digital tools or unplugged?**
5. **Is programming essential?**

1. What characterizes computational thinking in mathematics education?

Agreement upon:

- abstraction
- decomposition
- pattern recognition
- algorithmic thinking
- modelling
- logical thinking

2. What aspects of CT can be addressed in mathematics instruction?

Agreement upon:

- abstraction
- decomposition
- pattern recognition
- algorithmic thinking
- modelling
- data analysis and representation
- automation
- generalization
- evaluation

3. What aspects do computational thinking and mathematical thinking have in common?

Agreement upon:

- abstraction
- decomposition
- pattern recognition
- algorithmic thinking
- modelling
- logical thinking
- Generalization, evaluation
- problem analysis
- analytical thinking

4. Digital tools or unplugged?

A mixture of both digital tools and unplugged activities is ideal in mathematics education:

- **Digital tools and unplugged activities are useful for developing computational thinking skills in mathematics education**
- **Unplugged activities are useful for conceptual understanding**

5. Is programming essential?

Programming and the way of thinking it involves are useful in mathematics education (e.g., as a skill for today's society, debugging, visualization).

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Design principles derived from literature and Delphi study

We set out to…

- **Design activities that invite both mathematical and computational thinking**
- **Focus on problem solving, data representation and algorithmic thinking**
- **Address topics core to the math curricula**
- **Make students use digital tools that are relevant for mathematics**
- **Integrate plugged and unplugged activities**

Designing Student Activities

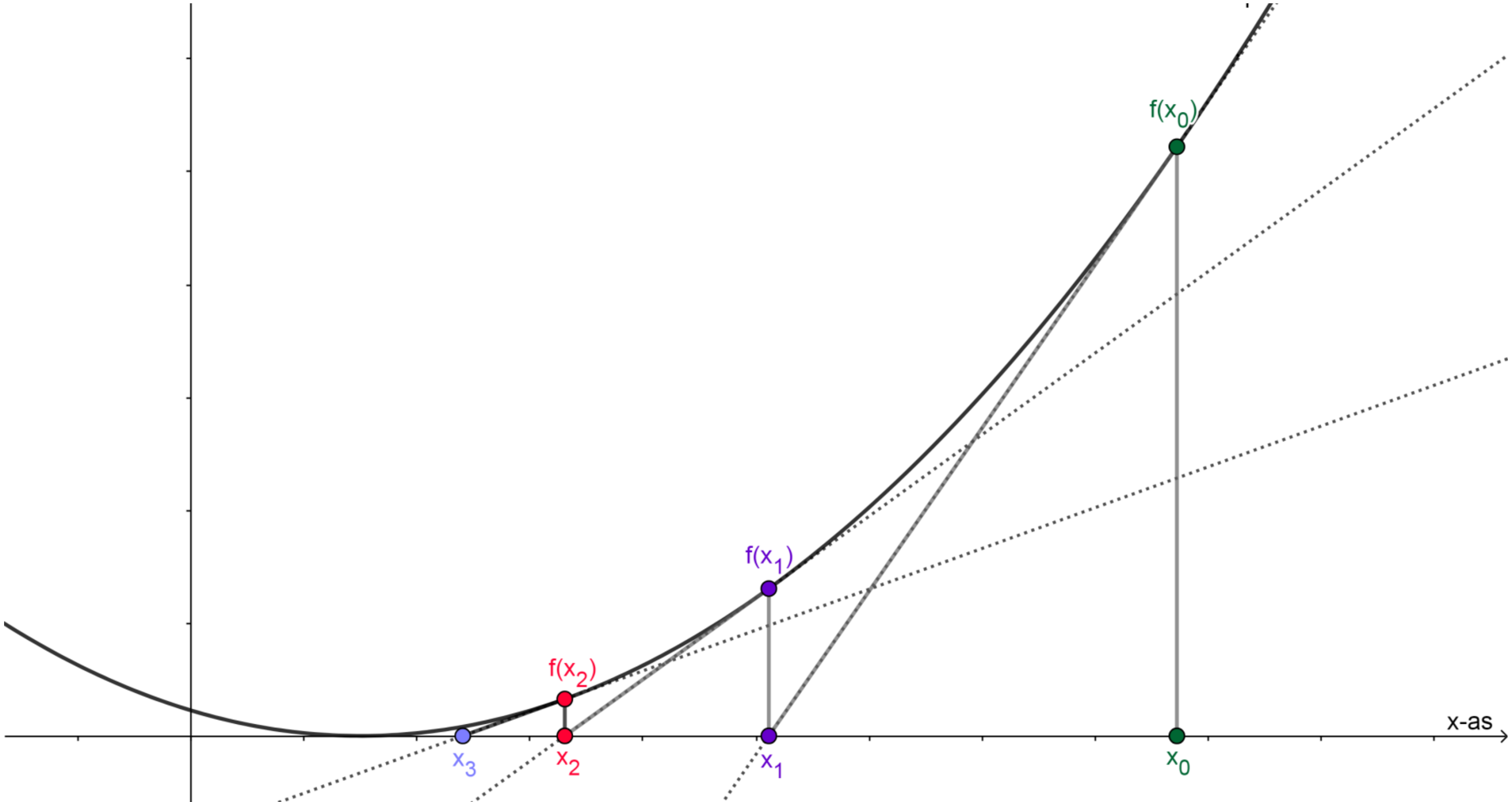
	Math A	Math B
Subject	Statistics	Algebra/calculus
CT/MT aspects	Data representation Data structures	Algorithmic thinking Problem solving
Digital technology	Excel	GeoGebra

Math A Design Ideas

Titanic: “Women and children first?”

- Explore dataset, make suitable for analysis
- Specify question into sub-questions (decomposition)
- Choose mathematical tools
- Create graphs/tables (representation)
- Evaluate results

Math B Pilot study: Newton-Raphson



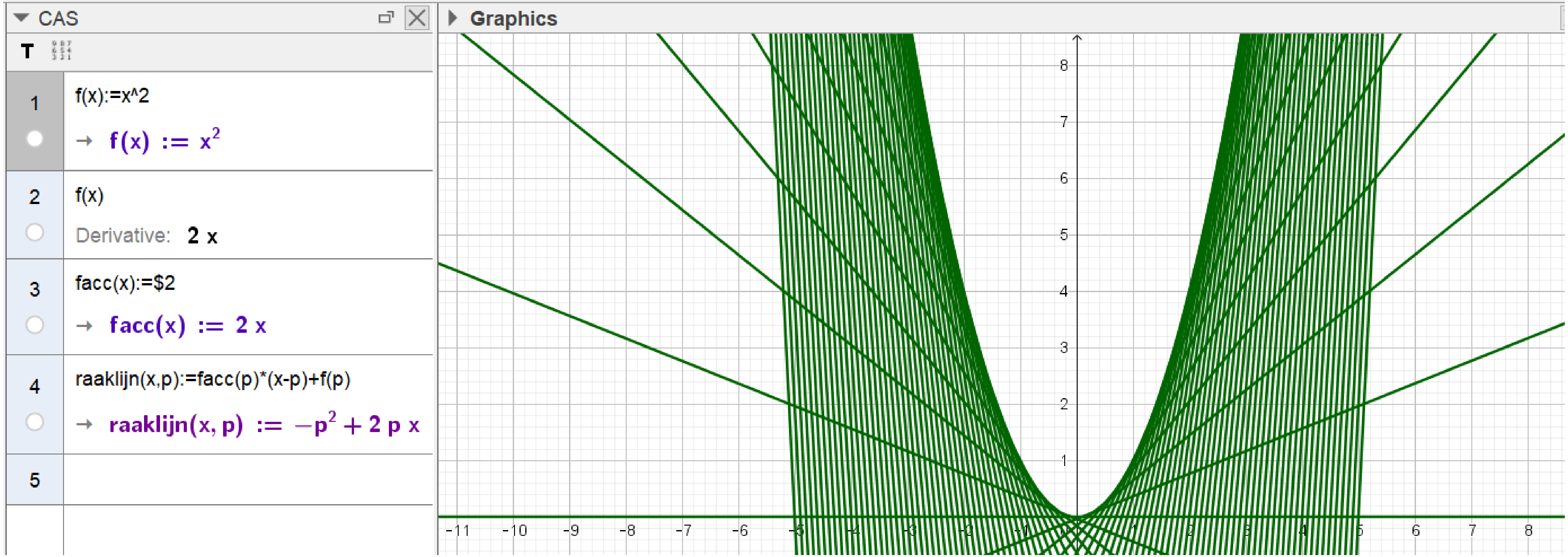
Master thesis Ivana den Hartogh

	1	2	3	4	5	6	7
1	Use newton-raphson to find roots of f(x) = x^3 - 6 x^2 + 11x -6.1						
2							
3	iter	x_r	f(x_r)	f'(x_r)	delta_x	x_r+1	rel_error
4	1	15	2183,9	506	-4,31601	10,68399	
5	2	10,68399	646,0908	225,2352	-2,86852	7,815476	-26,85%
6	3	7,815476	190,7625	100,4593	-1,8989	5,916572	-24,30%
7	4	5,916572	56,06181	45,0186	-1,2453	4,671268	-21,05%
8	5	4,671268	16,29003	20,40702	-0,79826	3,873012	-17,09%
9	6	3,873012	4,597842	9,524524	-0,48274	3,390275	-12,46%
10	7	3,390275	1,196938	4,798594	-0,24944	3,14084	-7,36%
11	8	3,14084	0,243981	2,904546	-0,084	3,05684	-2,67%
12	9	3,05684	0,023556	2,350733	-0,01002	3,046819	-0,33%
13	10	3,046819	0,000317	2,287492	-0,00014	3,046681	0,00%
14							

This requires “encapsulation” of calculation processes in formulas (objects!) that Excel can deal with.

Math B Design “Functional Algorithms”

Theme: lines, tangent lines, sheaves of tangent lines

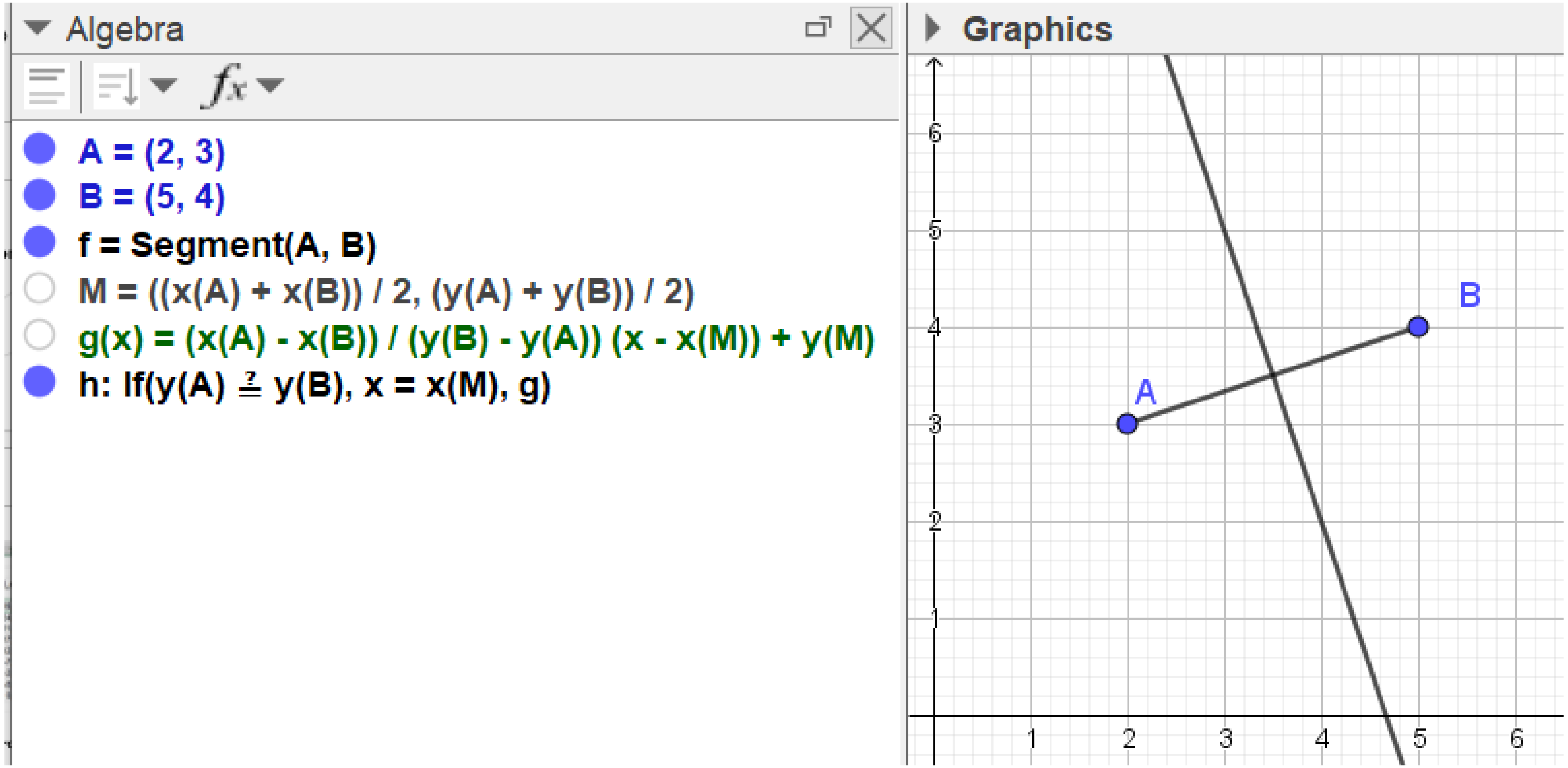


MT/CT aspects and GGB techniques

MT/CT glimpses	GGB technique
Conditional reasoning	If-then-else command
Generalizing	Using parameters
True or not true reasoning	Using Boolean variables
Object formation	Functional definitions
Repetition	Sequence command
Iteration	IterateList command
Object formation	Merging steps into macros

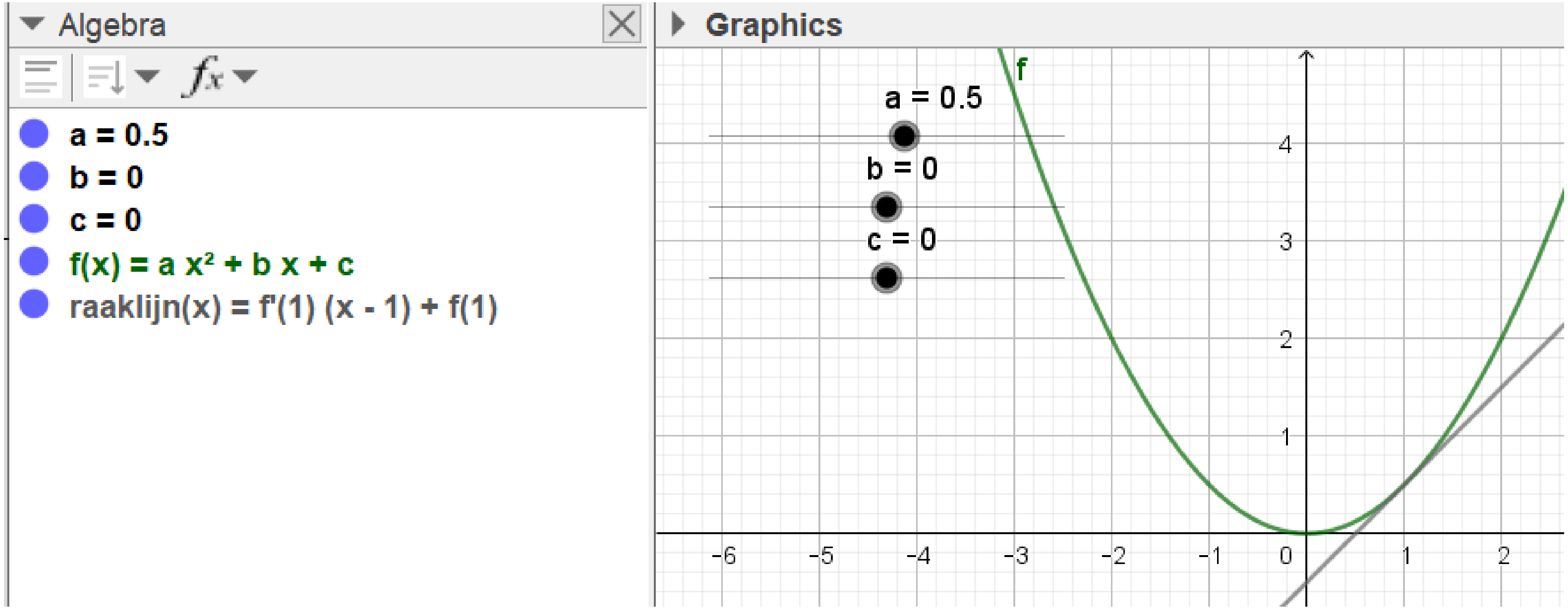
Examples / snap shots (1)

Reconstruct a Perpendicular Bisector Procedure



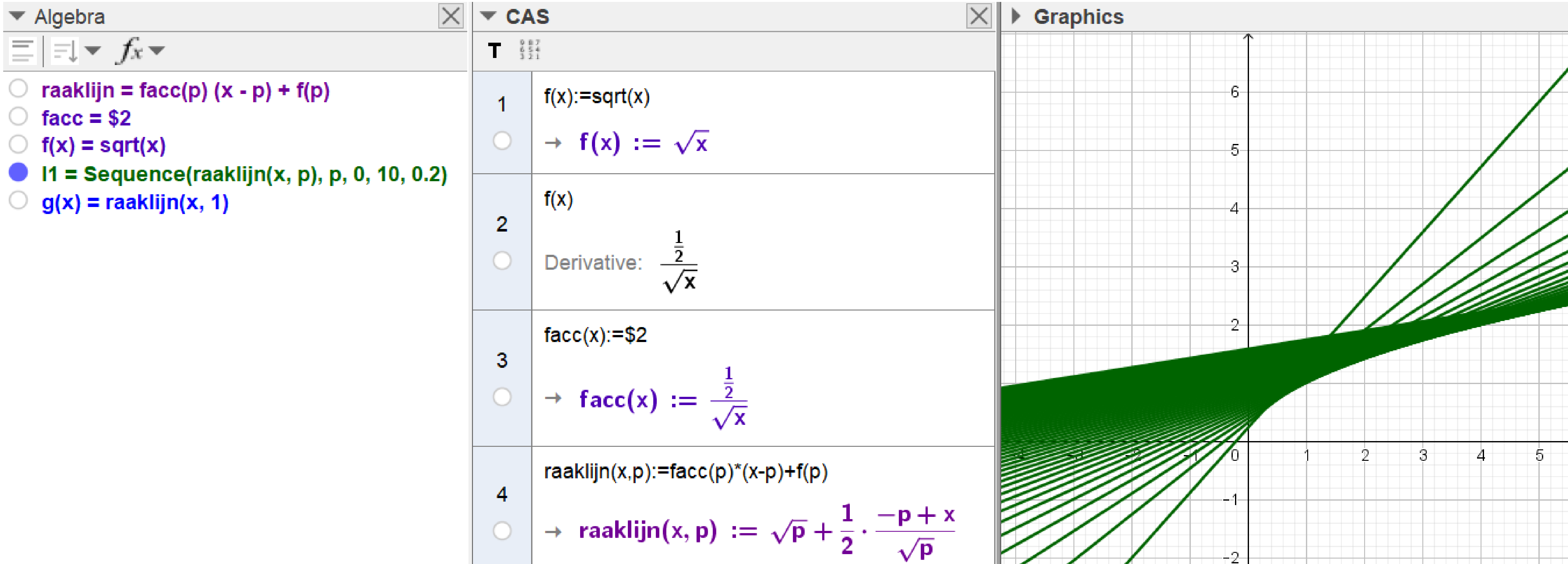
Examples / snap shots (2)

Reconstructing a Trangent Procedure for a general case



Examples / snap shots (3)

Sheaves of tangent lines for a general case



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Conclusions & Dilemmas

Conclusions:

- **Results from literature study and Delphi study suggest that MT and CT have much in common**
- **Design and pilot activities suggest that there are obvious means to address CT aspects in the mathematics classroom**

Dilemmas:

1. **Thinking <-> Tool use**
2. **Algorithms unplugged <-> Coding**

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Thank you for your attention!

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References

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