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How mathematical symbols and natural language are integrated in textbooks

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In mathematical text and talk, natural language is a constant companion to mathematical symbols. The purpose of this study is to identify different types of relations between natural language and symbolic language in mathematics textbooks. Here we focus on the level of integration. We have identified examples of high integration (e.g., when symbols are part of a sentence), medium integration (e.g., when the shifts between natural and symbolic language occurs when switching to a new line), and low integration (e.g., when symbols and written words are connected by the layout).

Introduction and aims

In the mathematics classroom, the natural language is used by teachers, textbooks, and students to talk about mathematical symbols, what they stand for and how they can be manipulated. The present study aims to identify different types of relations between natural language and symbolic language in mathematics textbooks. The study is part of a larger project that aims to create a model describing how progression in the use of natural language relates to advancements in the use and understanding of the symbolic language. During the first step of the project, we characterise how natural language exists in relation to the mathematical symbols, for example, to what extent and in what ways symbols and words are mixed. As the textbook is often the foundation of mathematics instruction (Mullis, Martin, & Foy, 2008), it is a relevant source of data when it comes to questions about language use and progression in mathematics.

Method

To identify as many different types of relations as possible, we use an exploratory approach. The data includes a selection of different types of explanatory texts in various mathematics textbooks for school year 1-9. In the explanatory texts, we noted all symbols and symbolic expressions, including for example, numbers, arithmetic operators (e.g., "+"), and the equal sign. For each symbolic expression, we then noted if and how it was related to any natural language in the same text.

Tentative findings

Our preliminary findings show that there are many different types of relations between mathematical symbols and natural language, but here we focus on one aspect that we (so far) denote as *level of integration*.

We have found instances of *high integration* of two types: Firstly, when words are included in a formula, for example, "Circumference = $\pi \cdot$ diameter". Secondly, when symbols or symbolic expressions are included in a sentence. For example, "Om $x = 2$, så är $x + 1 = 3$" (Eng. "If $x = 2$, then $x + 1 = 3$"). The high level of integration becomes particularly apparent when considering the pronunciation in Swedish. The first "=" would in this context commonly be read out loud as "*är* lika med" (literally in English: "*is* equal to") and the second only as "lika med" to fit the sentence structure. In this case, the natural language affects the pronunciation of the symbol.

We have also found examples of *low integration*, when natural language and mathematical symbols occupy separate parts of a page but are still connected, typically by layout or text referencing. Many of the connections between natural language and symbols are in fact directed by the layout. For example, text squares, speech balloons, arrows, colour coding, and closeness are used as instruments to guide the reader when connecting mathematical symbols to other text sections.

In addition, we found what we denote as *medium* integration. An example is a sentence ending with a colon, followed by a symbolic expression (e.g., a formula or a step in a calculation) on the next line. In this case, the reader does not have to switch between symbols and natural language several times in the same sentence, as is the case for high integration.

In textbooks from the first school years, when many children are beginning readers, natural language is sparsely used in print. The few words that occur are often in the form of headings or short instructions, that is, low integration. When the amount of written natural language increases, examples with high integration of symbolic and natural language are often present. As an example, in school year two, we found a speech balloon saying: "2·5 eller 5·2, det beror på hur jag vrider rutorna" (Eng. "2·5 or 5·2, it depends on how I rotate the squares").

The project will continue to explore other relations between natural language and mathematical symbols and, further on, also investigate how the degree of integration of symbolic and natural language might affect the reader and the readers' ability to grasp the meaning of the mathematical symbols.

References

Mullis, I. V. S., Martin, M. O., & Foy, P. (2008). *TIMSS 2007 International Mathematics Report: Findings from IEA's Trends in International Mathematics and Science Study at the Fourth and Eighth Grades*. Chestnut Hill, MA, USA: TIMSS & PIRLS International Study Center, Lynch School of Education, Boston College.