How epistemological characteristics influence the design of a course in projective geometry

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In this short presentation, we report parts of an ongoing project building on the Anthropological Theory of the Didactic (ATD) to investigate how the epistemological characteristics of the topic to be taught influences the design of university mathematics courses. We focus our attention on a course in Affine and Projective Geometry. Analysis is ongoing, but we present some initial observations and discuss them in relation to other courses previously analysed.

There is a growing amount of research investigating factors influencing university mathematics teachers' (UMTs') didactical choices when planning and preparing courses or individual lectures. These factors can be individual, depending upon the UMT's own educational background, research practice, and beliefs about mathematics and students (e.g. Hernandes-Gomes & González-Martín, 2016); pedagogical, such as the choice of textbooks and other educational resources (Mesa & Griffiths, 2012); or institutional, for instance the dominant epistemology of the institution (Barquero, Bosch & Gascon, 2013). In a recent paper (Viirman & Jacobsson, in press) we considered another influential factor: the inherent structure of the mathematical domain covered by the course. Building on the construct of *praxeology* from ATD (e.g. Bosch, 2015) we studied two different university mathematics courses: General Topology (GT) and Differential Geometry (DG), characterizing the first as logos- and the second as praxis-driven.

Here we continue this line of inquiry, investigating a third course, Affine and Projective Geometry (APG), with the aim of analyzing whether and how the epistemological characteristics of the topic of this course has influenced its design. The APG course is designed and taught by the second author, and has certain institutional characteristics in common with the GT and DG courses, making it a suitable choice for further study. All three courses have the same prerequisites, mainly multivariable calculus and linear algebra, and are thus aimed more or less at the same cohort of students. Furthermore, all three courses serve as introductions to specific mathematical topics – algebraic geometry, in the case of APG.

The APG course has a distinct narrative structure, in that it is built around the statement and proof of one particular theorem, namely Bezout's Theorem on the intersections of plane projective curves. This theorem is proved about halfway into the course, and the remainder of the course is then devoted to applications of the theorem and some further related topics. In this way, the introduction of new mathematical notions and concepts is motivated by their use in removing obstacles for formulating and proving Bezout's Theorem. Still, these concepts, central to algebraic geometry, are at least as important as the proof *per se*. Comparing the APG course to the GT and DG courses, there are distinct similarities but also differences. Like DG, APG builds on students' previous familiarity with concepts and techniques from multivariable calculus. However, where DG grew from the wish to use these techniques on more complicated, abstract surfaces, APG rather introduces new ways (complex and projective) of viewing familiar objects, namely curves given by polynomial equations. Thus, APG does not appear to be praxis-driven in the way that DG is. On the other hand, where the logos-driven GT course grew out of the definition of a topological space, APG is built around a particular, albeit highly theoretical, task: wanting to formulate and prove a particular theorem. New definitions are introduced to this end. In fact, the "proper" algebraic definition of intersection number does not appear until the very end of the course. Hence, although the course clearly aims at developing theory, it cannot properly be described as logos-driven. Another important aspect of the APG course that we are still in the process of analyzing is the way it serves as a bridge between algebraic and geometric reasoning, showing by example how powerful algebraic techniques can be for solving geometric problems, while at the same time showing the importance of geometric arguments for understanding the algebraic manipulations.

As is clear from the above, analysis is still ongoing, and at the conference we aim to present a more detailed praxeological analysis of the APG course, and to give some implications for course design and planning in university mathematics.

References

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