Presented at Madif–12, Växjö, January 15, 2020

How mathematical symbols and natural language are used in teachers' presentations

Ewa Bergqvist, Tomas Bergqvist, Ulrika Wikström Hultdin, Lotta Vingsle & Magnus Österholm
Department of Science and Mathematics Education, Umeå university
Umeå Mathematics Education Research Centre, Umeå university

In this study, we examine how the use of natural language varies, considering the symbolic language in procedural and conceptual aspects of mathematics.

Introduction

This ongoing study is part of a large project concerning the role of natural language when learning the mathematical symbolic language. The aim of the overarching project is to create a model describing how progression in the natural language relates to advancements in the use and understanding of the symbolic language.

Learning and doing mathematics includes both procedural knowledge, such as manipulating symbolic expressions and operations according to rules, and conceptual knowledge, such as knowledge of objects and concepts, including connections between different objects and concepts (Hiebert & Lefevre, 1986). Several studies describe the lack of teaching for conceptual knowledge, while teaching for procedural knowledge is dominant (e.g., Boesen et al., 2014). In addition, students seem to have problems with conceptual knowledge about symbols (Österholm, 2006). When teaching about symbols, the natural language plays an important role to introduce them in a meaningful way and to explain their meaning and use (O'Halloran, 2008). In this study, we examine how mathematical symbols and natural language are used in teachers' presentations. More specifically, we focus on how the use of natural language varies, considering the symbolic language in procedural and conceptual aspects of mathematics.

Definition of procedural and conceptual knowledge

Procedural knowledge consists of two parts (Hiebert & Lefevre, 1986). The first is knowledge about the formal language, the symbolic representation system, where familiarity with symbols and syntactic rules are central. The second part is knowledge about the algorithms and rules of operations, often described as processes with step-by-step instructions.

Conceptual knowledge consists of knowledge about relations between mathematical entities (objects, concepts, rules etc.). We differentiate between conceptual knowledge on two levels (Hiebert & Lefevre, 1986): The primary level and the reflective level. On the primary level, the relationships are on the same (or lower) level of abstraction compared to the entities, and the relationship is tied to the specific context. On the reflective level, the relationships are on a higher level of abstraction than the entities and not depending on the context. An example of these two levels can be found when students work with a number system using base five. On the primary level, students can understand that since the base is five, the primary group is five, and that you only work with five symbols, 0–4. The relationship between these two sets of information is concrete and dependent on the context (base five), thus at primary level. At the reflective level, students can understand that the same principles will be valid for any number base (Long, 2005), the relationship is abstract and not related to a specific context.

Method

We will use transcribed secondary teachers' presentations from regular classroom presentations. The criterion for selection will be that the teacher's presentation includes the use of written symbols. We will base our analysis on definitions by Hiebert and Lefevre (1986). In our operationalisation, conceptual knowledge is identified by relationship between two pieces of information and procedural knowledge is identified as having a sequential (process) nature. For example, a teacher can indicate processes (procedural knowledge) or relations (conceptual knowledge) by the choice of words and expressions. Two concrete examples: when a teacher says, "When you move a number to the other side of the equation, remember to change the sign", which would indicate process. If the teacher says, "Remember that the two sides in the equation must be in balance", it would indicate relations. Thereafter, patterns of teachers' use of natural language and symbols will be examined in relation to the given categories.

In the presentation we will give examples from our data of the two types of procedural knowledge, and the two levels of conceptual knowledge. We will also present patterns found in the analysis.

References

Boesen, J., Helenius, O., Bergqvist, E., Bergqvist, T., Lithner, J., Palm, T., & Palmberg, B. (2014). Developing mathematical competence: From the intended to the enacted curriculum. *The Journal of Mathematical Behavior, 33*, 72-87.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27): Taylor and Francis.

Long, C. (2005). Maths concepts in teaching: Procedural and conceptual understanding. *Pythagoras, 62*, 59-65.

O'Halloran, K. (2008). *Mathematical discourse: Language, symbolism and visual images*. London: Continuum.

Österholm, M. (2006). Characterizing reading comprehension of mathematical texts. *Educational Studies in Mathematics, 63*, 325-346.