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Gazing at mathematical reasoning

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Eye tracking can be used to investigate how students perceive and solve mathematical tasks. This short presentation will report on two such studies where task design, mathematical reasoning and students' gaze are in the spotlight. Results show that task design has an effect on students' reasoning, as well as on how they perceive the information given in a task.

Introduction

Mathematical reasoning is an important component of mathematical competency (Kilpatrick, Swafford, & Findell, 2001). Being able to formulate a sequence of mathematical arguments that can be used to solve and motivate the solution of tasks is maybe something that all teachers hope for in their students. Lithner (2008) suggests that students' mathematical reasoning can be roughly categorized as either imitative or creative. Imitative reasoning includes all instances when students utilize any kind of solution templates (both written and vocal), or when a solution is memorized. Creative reasoning, on the other hand, demands that the student constructs a new solution to the task, and that this solution is based on mathematical arguments.

In a series of studies, researchers have examined the differences between imitative and creative reasoning among upper secondary students (e.g., Granberg & Olsson, 2015; Jonsson, Norqvist, Liljekvist, & Lithner, 2014). These studies have investigated the reasoning types both in regard to how students actually reason when solving tasks and in the efficiency of different task types. In line with Brousseau's (1997) Theory of Didactical Situations, the results from these studies imply that creative tasks are more efficient when it comes to remember, re-use, and re-construct the practiced solution method than tasks with a solution template.

Eye-tracking and mathematical reasoning

In a recent study Norqvist, Jonsson, Lithner, Qwillbard, and Holm (2019) utilized eye-tracking to further observe how students use the information given in tasks that are designed to promote either imitative or creative reasoning. In this study, 48 students solved two types of practice tasks (i.e., imitative or creative) and were clustered according to their main focus in five different areas of interest, illustration, description, formula, example, and question. Surprisingly, there were not two (i.e., imitative and creative), but three clusters. One that focused on the given formula, one that focused on the illustration, and one that focused on the worked example. In the first cluster there were only students that worked with imitative tasks, in the second cluster only students that practiced with creative tasks were found, and in the third cluster there were a mix between students that practiced on either imitative or creative tasks. The students that practised on creative tasks and were clustered into the third category had lower cognitive prerequisites and a lower post-test score than the students in category two.

Ongoing study

To get a deeper insight into the effect that task design can have on students reasoning, we are now analysing the eye-tracking data from all individual tasks that students solved or tried to solve in the previous study (i.e., Norqvist et al., 2019). This analysis will hopefully reveal if the students fall into behavioural patterns when reading the tasks and how quick this behaviour might occur. Hypothetically, students that are presented with solution templates starts to disregard other information bearing representations (e.g., illustrations) since the task can be solved with the given solution template. Another hypothesis is that students that practice by creative tasks will initially use the illustration to solve the task but leave it in later tasks when the constructed solution has been consolidated.

In the short communication I will present the previous eye-tracking study (Norqvist et al., 2019) and its results, and also discuss the ongoing analysis, which by then will have come further.

References

Brousseau, G. (1997). *Theory of didactical situations in mathematics*. Dordrecht ; Boston: Kluwer Academic Publishers.

Granberg, C., & Olsson, J. (2015). ICT-supported problem solving and collaborative creative reasoning: Exploring linear functions using dynamic mathematics software. *Journal of Mathematical Behavior, 37*, 48-62. https://doi.org/10.1016/j.jmathb.2014.11.001

Jonsson, B., Norqvist, M., Liljekvist, Y., & Lithner, J. (2014). Learning mathematics through algorithmic and creative reasoning. *The Journal of Mathematical Behavior, 36*, 20-32. https://doi.org/10.1016/j.jmathb.2014.08.003

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press.

Lithner, J. (2008). A Research Framework for Creative and Imitative Reasoning. *Educational Studies in Mathematics, 67*(3), 255-276. https://doi.org/10.1007/s10649-007-9104-2

Norqvist, M., Jonsson, B., Lithner, J., Qwillbard, T., & Holm, L. (2019). Investigating algorithmic and creative reasoning strategies by eye tracking. *The Journal of Mathematical Behavior*. https://doi.org/10.1016/j.jmathb.2019.03.008