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Mathematics and physics at upper secondary school: an analysis of two lectures

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A physics lecture and a mathematics lecture, by the same teacher and partly the same students, were studied at upper secondary school. Both lectures covered ordinary differential equations. The main aim of the present paper was to investigate the teacher's different and similar ways to handle related mathematical content in the two school subjects. The findings show a structural use of mathematics with an analytical approach in mathematics and an applied approach in relation to formulas in physics. This study is part of a larger study about mathematics in physics education funded by the Swedish research council.

The theoretical framework used and some results

Mathematics in physics education has received international attention (Pospiech, Michelini, & Eylon, 2019), and an analytical model was developed to identify aspects of the classroom communication (Hansson et al., 2019). The model allows identification of relations made between Reality (R), Theoretical models (TM), and Mathematics (M). Reality refers to phenomena in the real world, divided in Reality (R) and a 'reduced reality' in Reality School (RS), e.g. frictionless movement. Theoretical models refers to theoretical models in physics and concepts related to them. We distinguish between an Instrumental approach (TMI) and a Relational approach (TMR) (Skemp, 1976). Mathematics in the model refers to concepts, theorems, representations, reasoning and methods. Technical use (MT) is distinguished from a Structural use of mathematics (MS), where the latter means that there is an emphasis on interpretations, consequences or logical reasoning (Karam, 2014). The model was used to analyse data from a study of upper secondary school teachers' use of mathematics in physics and mathematics classes. This presentation is of a teacher teaching a physics class and a mathematics class the year after at the natural science program (partly the same students). The lectures covered mainly the same mathematical content, as depicted in table 1.

	Physics lecture, year 2	Mathematics lecture, year 3
Topic	ODE. Induction	ODE. Logistic growth
Notations used	$\frac{d\Phi}{dt}$. Suitable for deriving Lenz's law	*y*' as a function of *y*. Suitable for the calculations with the chain rule
Mathematics in use	Functions, Formulas, Modelling, Derivatives, ODE, Limits, Graphs, Slope, Absolute values	Functions, Formulas, Modelling, Derivatives, Chain rule, ODE, Limits, Graphs, Extreme values

Introduction	Demonstration: Magnetic and non-magnetic objects through metallic slides [RS]	Growth of a population of bacteria. Discusses limitations and reasonability in models [R]
Main part in	[RS, TMR]	[MS, TMR]
Ending	Student asks where the new formula is in the formulary [TMI]	The teacher will hand out gathered formulas for the models [TMI]

Table 1: Content of the two lectures coded in the ternary model.

In the physics lecture the teacher derived a formula for voltage, Lenz's law, through formulas known to the students [RS, TMR, MS], writing $U = \frac{\Delta\Phi}{\Delta t}$ on the board. He told the students that the direction is opposite its origin and that the formula hence needs a minus sign. He wrote $U = -\frac{\Delta\Phi}{\Delta t}$ under the first formula and said that if the time intervals are much smaller we get the time derivative ($U = -\frac{\Delta\Phi}{\Delta t} = -\frac{d\Phi}{dt}$). After discussing a coil he wrote $= -N\frac{d\Phi}{dt}$. The board showed three different expressions for *U*. The teacher concluded that the minus sign is rarely used in this course, only for direction. The mathematics used was in a structural mode (Karam, 2014) in relation to theoretical models even though details were omitted, or imprecisely handled, leaving a focus on the physics formulas. In the mathematics lecture the teacher used an analytical approach with references to reality [MS, TMR, R]. He presented a formula for logistic growth from the students' prior experiences, *y*' = 2*y*, and changed it to provide for restrictions in the model, *y*' = *ky* (1- $\frac{y}{M}$). He tested chosen scenarios to justify the formula. The justifications included limits for testing the role of *y* in relation to *M* for values of *y*'. The lack of a given solution method drove the lecture and the teacher reasoned about limits in graphs representing solutions. He optimized the function *y*' using the chain rule in new ways for the students, i.e. implicitly in calculations, and in a graph with (*y*, *y*'). Compared to the physics lecture with an emphasis on deriving the formula from prior formulas, the mathematics lecture had an emphasis on analysing formulas. In both cases graphs were used and related to reality (school).

References

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