Pre-service teachers' explanations of division by zero and denseness of the number line

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Students' beliefs about division by zero and numbers on the number line were studied through explanations of the concepts in questionnaires and interviews during their teacher education to become primary school teachers in the years 4-6. The concepts were chosen for students' proven cognitive challenges in coping with them, with the aim to add to the existing knowledge in terms of specific and general explanation types. General and specific parts of the students' concept images were contradictory in several cases and the examples used for explaining were often based on other mathematical structures than the ones explained, e.g. $\frac{2}{1}$ instead of $\frac{2}{0}$ or a finite decimal recitation instead of an infinite one.

A brief description of the study and some results

A longitudinal study was initiated in 2016 to investigate three groups of pre-service teachers' (years 4-6) development to become mathematics teachers, with a focus on mathematics through their education. 78 students responded to an openly formulated questionnaire (Q1) about the mathematics teacher role and subject knowledge in mathematics. The questionnaire was distributed at the beginning of the students' first mathematics course to give a picture of the students' initial views on their pending profession, mathematics and goals. Eight students were interviewed (I1) two weeks after Q1. The interviews covered the same topics as Q1 and the students were asked to explain their answers from the questionnaire, e.g. answers to fictitious pupils' questions about division by zero and how many numbers are between 0,99 and 1. Six of the eight students were interviewed again in their third semester (I2), at the end of their second mathematics course. The remaining two students had left the program. The second interview included a questionnaire (Q2) about infinite sets, division by zero and denseness of the number line, e.g. items about whether or not 0,99… is equal to 1 (see Juter (2019) for details about Q2).

Tables 1 and 2 show a sample of four students' responses to the questionnaires and interviews, two female (F) and two male (M). The mathematical structures in their explanations are written within brackets. *Specific* (S) and *general* (G) explanations were categorized based on Mason and Pimm's (1984) definitions. A specific explanation may entail particular numbers and real situations s.a. dividing

2 apples, whereas a general explanation is based on general rules, e.g. two different numbers have other numbers between them. The tasks to respond to were mostly specifically formulated and it is hence natural that the explanations were as well.

	Q1, 2 divided by 0	I1, 2 divided by 0		I2, 2 divided by 0
1(F)	S: No answer $\left(\frac{0}{2}\right)$	S: Incorrect $\left(\frac{0}{2}\right)$	G: Incorrect $\left(\frac{0}{x}\right)$	S: Incorrect $\left(\frac{2}{0}\right)$
2(M)	G: Incorrect $\left(\frac{x}{0}\right)$	S: Incorrect $\left(\frac{0}{2}\right)$	S: Incorrect $\left(\frac{2}{1}\right)$	S: Partly correct $\left(\frac{2}{0}\right)$
3(M)	S: No answer $\left(\frac{2}{1}\right)$	S: Incorrect $\left(\frac{2}{0}\right)$	S: Incorrect (2-2)	S: Correct $\left(\frac{2}{0}\right)$
4(F)	S: No answer $\left(\frac{2}{0} \& \frac{2}{1}\right)$	S: Incorrect $\left(\frac{2}{1}\right)$		S: Correct $\left(\frac{2}{0}\right)$

Table 1: General rules (G) and specific examples (S) about division by zero.

	Q1, 0,99 and 1	I1, 0,99 and 1	Q2, 0,99 … and 1	I2, 0,99 … and 1
1(F)	G: No answer (SP)	G: No answer (D)	G: Correct S: Incorrect	S: Incorrect (NB, R)
2(M)	S: No answer (D)	S: No answer (D)	G: Correct S: Incorrect	S: Incorrect (R)
3(M)	S: Incorrect (CN)	S: No answer (CN)	G: Partly correct S: Contradictory	G: Incorrect (NB) S: Incorrect (R,NB)
4(F)	S: No answer (D)	S: Correct (D)	G: Correct S: Incorrect	S: Incorrect (R)

Table 2: General rules (G) and specific examples (S) about the number line.

The students mainly used specific examples to explain their standpoints, albeit incorrectly to a high extent. Regarding division by zero , the examples were often based on different types of misconceptions of what 2 divided by zero means, e.g. confusing it with zero divided by 2, 2 divided by 1 or even 2 minus 2. The confusion with division by 1 often stemmed from the impossibility to distribute two objects in zero piles leaving the objects in the hand. In the explanations used to answer how many numbers there are between 0,99 and 1, many added decimals (D) to explain. Some looked at smaller parts (SP) or chose to only regard specific classes of numbers (CN), e.g. hundredths. When it came to 0,99… and 1, a majority knew applicable general rules, but relied more on specific explanations (S) incorrectly opposing the general rule. The two explanations used here was that there are numbers between 0,99… and 1 (NB) and adding another nine after the last nine in the recitation of nines (R), even though it is an infinite recitation.

References

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Mason, J. & Pimm, D. (1984). Generic examples seeing the general in the particular. Educational Studies in Mathematics, 15 (3), 277–289.