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On the notion of 'background and foreground' in networking of theories

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In this paper, we report on a finding in an ongoing literature review on Networking of Theories. As theories are the focus of networking practices, discussion of what is meant by theory is an ongoing debate. In our reading of these discussions, we experience a discrepancy in the use of the notion of background theories and foreground theories, which can be related to an absolute or a relative understanding of these notions. We account for this discrepancy and discuss potential consequences of each perspective to argue that a new notion ' framing theories' or a distinction between 'background theory inside mathematics education' and 'background theory outside mathematics education' may accommodate these consequences.

Introduction

The term 'networking of theories' stems from the thematic working group (TWG) on theoretical perspectives and approaches in mathematics education research (MER) at the *Congress of European Research in Mathematics Education* (Kidron, Bosch, Monaghan & Palmér, 2018). The group confronts the issue of the diversity of theories in mathematics education, and claims that "theoretical approaches can *only* become fruitful *if* connections between them are actively established" (Bikner-Ahsbahs et al., 2014, p. 8). Taking this stance, the group has embarked on the challenge of how to establish connections between theories by developing 'networking of theories' as a research practice. Several important questions and issues have been discussed over the years. Kidron and colleagues (2018) state the following examples: "What are the aims of connecting theories? […] To what extent does the networking depend on the theories that are considered?" (p. 257); "To what extent do we share the same notion of theory? (p. 257); "What are the different aims of networking?" (p. 258); "What do researchers do when they use more than one theory? Do the different approaches use the same words with different meanings?" (p. 258). Such questions have been addressed in the literature on networking of theories, e.g. Bikner-Ahsbahs and Prediger (2006) the ZDM article "Diversity of theories in mathematics education—How can we deal with it?", the ZDM issue "Comparing, Combining, Coordinating – Networking Strategies for Connecting Theoretical Approaches" edited by Prediger, Bikner-

Ahsbahs and Arzarello (2008), and not least in the recent book "Networking of Theories as a Research Practice in Mathematics Education" edited by Bikner-Ahsbahs and Prediger (2014). Surely, the potential answers must to some extent draw on a common notion of 'what theory is' – we return to this below. For now, we draw the attention to the observation that in the available literature on networking of theories, there are often references to the notion of background theories and foreground theories (to be explained in more depth below) – this often occurs with specific reference to Mason and Waywood (1996), who initially introduced the terms into MER. Our ongoing review, which so far encompasses 96 publications on networking of theories, reveals the observation that the use of these two terms in more recent literature do not necessarily align with the original description by Mason and Waywood. More precisely, although some theoretical perspectives are attributed the role of background theories; these are not necessarily used in the sense of Mason and Waywood. Hence, there is a discrepancy between the descriptions and the actual use. In this paper, we ask the question: *How are the notions background theories and foreground theories used in the literature on networking of theories?*

We do not provide a full account of the 96 publications due to the space limitations of this paper. Instead, we present and discuss our finding through two carefully selected illustrative cases, showing the discrepancy in the use of background theory. Before we get to these cases, we briefly discuss the notion of theory itself and explicate the original notion of background and foreground theories as defined by Mason and Waywood (1996).

What is 'theory' in mathematics education research?

In networking of theories, a minimum requirement must be that we can agree on what is and what is not *a theory*. The literature – not only in mathematics education – is rich on various attempts of coining what theory is. For the reader who is unfamiliar with this discussion, we provide a brief account in this section. The reason we do this is not to apply this in our further analyses, but rather as a general comment to the ongoing discussion on what a theory actually is, and not least what a theory must be described by in order to be networked with other theories. We shall consider *a theory* from the perspective of networking theories, not least, with reference to what has taken place in this literature.

Kidron et al. (2018) state that the questions of what a theory is and how theoretical frameworks shape MER "came into play when comparing or just talking about theories is the heterogeneity of what is considered as a theoretical framework in MER and the consequent possible incommensurability of the investigations that are carried out in different theories" (p. 261). Radford (2008) suggested that a theory is a way of producing understanding and ways of action based on a triplet PMQ:

> A system, P, of basic principles, which includes implicit views and explicit statements that delineate the frontier of what will be the universe of discourse and the adopted research perspective.
>
> A methodology, M, which includes techniques of data collection and data-interpretation as supported by P.
>
> A set, Q, of paradigmatic research questions (templates or schemas that generate specific questions as new interpretations arise or as the principles are deepened, expanded or modified). (p. 320).

Around the same time, Prediger, Bikner-Ahsbahs and Arzarello (2008) surveyed different notions of theory found in the literature. This led them to distinguish between static and dynamic notions of theory, eventually pleading for a dynamic understanding: "theories or theoretical approaches are constructions in the state of flux" and they "consist of a core, of empirical components, and its application area. The core includes basic foundations, assumptions and norms, which are taken for granted." (p. 169). Niss (2019), however, notes: "The fact that theories or theoretical approaches are in a state flux doesn't mean that the definitions of the concepts are as well." We agree with Niss (2019) that: "Anything called a theory (or theoretical framework, construct etc.) is a *theory of something!* I.e. it deals with certain sorts of *objects* and *phenomena*, as well as *terms* for these." Mason and Waywood (1996) define such *objects* as the "sorts of things that are studied, even if they are not perceived as 'things' in any material way" (p. 1058). From Radford's (2008) account, it is unclear where these objects reside, although several researchers in networking of theories seem to consider them as part of the principles (P).

Foreground and background theories

As mentioned in the introduction, Mason and Waywood's (1996) distinction between foreground and background theories is often referred to in the discussion of the concept of theory. In this section, we outline our interpretation of the distinction as a basis for further discussion. Mason and Waywood (1996) present theory as a "hypothesis, or possibility such as a concept that is not yet verified but that if true would explain certain facts or phenomena." (p. 1055). They define foreground theory as *explicit* hypothesising based on the process of asking and answering questions within mathematics education, because:

> …the foreground aim of most mathematics education research is to locate, precise and refine theories *in* mathematics education about what does and can happen within and without educational institutions. (Mason & Waywood, 1996, p. 1056).

Thus, from the process of questioning 'things' within a local or specific area of mathematics education research gives rise to new theories in forms of explicit hypotheses about what is happening, or what can happen under certain circumstances. The foreground theories are generated *within* mathematics education and can have one or more of four different functions: descriptive; explanatory; predictive and informing practice. Conversely to foreground theory, Mason and Waywood define background theory as *implicit* hypothesising or as a belief that guides behaviour. They consider that "every act of teaching and of research can be seen as based on a theory *of* or *about* mathematics education" with reference to Thom (1976), who puts it as "all mathematical pedagogy, even if scarcely coherent, rest on a philosophy of mathematics." (quoted in Mason & Waywood, 1996, p. 1056). In this sense, the theory remains in the background and implies an implicit way of action or behaviour of the teacher or researcher, but is not used with an explicit aim. It is important to notice that a background theory does not become a foreground theory, just because the hypothesis becomes explicit. Mason and Waywood (1996) emphasise that as a researcher, it is important to be aware and explicit about one's own background theories and their implicit assumptions and hypotheses. They explain:

> Background theories encompass an object (aims and goals of the research, including what constitutes a researchable question [...]), objects (what sorts of things are studied, [...]), methods (how research is carried out, validated and applied), and situation (as perceived by the researcher), and provide a language for discussing these. The situation necessarily assumes, manifests, encompasses, and is constituted through a philosophic stance manifested in the discourse and in other practices (p. 1058)

This implies that the activities of research, such as framing researchable questions, using an appropriate method, collecting data, using analytical tools and looking at results as well as the validation hereof, are all determined and constructed by the background theory. This is elaborated with examples of how theoretical positions such as post-modernism, phenomenology and different directions within constructivism stress different ways and methods to investigate sociological and psychological dimensions and phenomena in educational research. Hence, we understand background theory as the theory that affords the conditions for the structure of the research, but it is not a theory generated within mathematics education research (MER). In addition, MER draws on theories from domains such as psychology and sociology, and their philosophical positions as well as their methods (Mason & Waywood, 1996). Accordingly, we understand Mason's and Waywood's (1996) explanation of background theories as theories establishing the view by which we look at mathematics education, for example critical theory, constructivism, social-constructivism, phenomenology or ethnology. It also follows that we understand their term of foreground theory as the theoretical constructs generated and developed by research in mathematics education that have explicit aims in forms of describing, explaining, predicting and/or informing specific situations, concepts and practices happening or possible to happen in the teaching and learning of mathematics.

As an example of the differences between foreground and background theories, we use Vergnaud's (2009) Theory of Conceptual Fields (TCF). As TCF is a theory developed in MER, specifically concerned with mathematical learning, it is a foreground theory. To consider the background theories of TCF, we must understand what theories precede TCF. As Vergnaud (2009) argues for his perception of schemes, he draws on Vygotsky's (1962) as well as Piaget's (1977) understanding of schemes. These two constructivist perspectives both have a broader scope on learning as they are developed outside of MER. Hence, we position them as the background theories of TCF.

A hermeneutic literature review

The following is a brief overview of our initial literature review on networking of theories. This review was conducted as a hermeneutic literature review (Boell & Cecez-Kecmanovic, 2014). As a part of a hermeneutic process, the understanding of the literature is never final; a constant re-interpretation is taking place. Our literature review began by scanning CERME proceedings, relevant ZDM issues and books. We also scanned the reference lists for the relevant literature to expand our literature base. Furthermore, we did literature searches in MathEduc and ERIC, although this did not reveal many relevant sources. Only literature describing the practice of networking of theories in mathematics education were included in the final cohort. Describing each relevant piece of literature in the following categories made our findings about background theories more explicit: 1) actual results; 2) how is networking of theories used and discussed; 3) what theories are being networked; 4) what strategies and methods are applied; and 5) perspectives with particular relevance to our overall project.

In our efforts to grasp the discussions of category 2, we compared the use of the notion of foreground and background theories in the literature on networking of theories to the original reference by Mason and Waywood (1996). Our two cases are carefully chosen to illustrate the result of this comparison: Each case utilises background theory explicitly, yet differently. But first, a further elaboration on the different uses of background theory in networking of theories.

Foreground and background theories in networking of theories

In relevant literature, the use of Mason and Waywood (1996) is widespread, both in paragraphs concerning theory and in discussions thereof. At CERME5, a communication problem within the field of MER was noticed:

> Researchers from different theoretical frameworks sometimes have difficulties to understand each other in depth because of their different backgrounds, languages and implicit assumptions (Arzarello, Bosch, Lenfant, & Prediger, 2007, p. 1618)

This quotation emphasises the need to understand the origin and background of theories as well as their implicit assumptions and hypotheses. According to Bikner-Ahsbahs and Prediger (2006), the distinction between background and foreground theories seems applicable when analysing theories and their functions in different phases of research. This could be the characterisation of foreground theories and their respective background theories. An example is:

> The theory of interest-dense situations is a foreground theory with a middle range scope (Mason and Waywood, 1996), situated in the background theoretical framework of interpretative research on teaching and learning (Bikner-Ahsbahs & Halverscheid, 2014, p. 99)

According to Bikner-Ahsbahs and colleges (2014), the underlying theoretical assumptions must be explicit when networking theories. Bikner-Ahsbahs and Prediger (2006) point out that "the background theory and its philosophical base are deeply interwoven" (p. 53). For instance, when taking a constructivist perspective, mathematics has a philosophical view on the construction of knowledge. Nevertheless, the use of foreground and background theories is regarded neither as a definite definition of theories, nor as an absolute categorisation of theories. This leads to a more relative use of background and foreground theories, than originally intended by Mason and Waywood (1996), e.g.: "In contrast [to the absolute definition], the status of some parts of the theory can change from foreground to background theory or vice versa within the research process" (Bikner-Ahsbahs & Prediger, 2006, p. 54). We interpret this statement to mean that a theory is not only *of/about* MER or only *in* MER, but that a theory can act as either, depending on the situation. Bikner-Ahsbahs and colleges (2014) contribute to this meaning by referring to foreground and background theories as *relative distinctions*. Still, and despite the discussions of making background theories explicit, authors reporting on networking processes and results seldom explicate the distinction. Hence, the way these terms are used within research practices are less apparent that one might initially anticipate.

Examples on the different use of background theory

Our first case is an example of the relativism of the notions as presented in Bikner-Ahsbahs and Prediger (2006). Koichu (2013) describes the work of a colleague in which a selected framework is contrasted with another. The insights obtained in the contrasting process are used in a following process of unpacking a selected construct in the selected framework:

> To this end and consistently with the Bikner-Ahsbahs and Prediger's (2006) terminology, the former theory can be seen as a foreground one, and the latter—as a background one. On the other hand, they use the Hershkowitz et al.'s (2001) work as a background theory or as an overarching framework, in which their own foreground theory is embedded (Koichu, 2013, p. 2841)

The relativism of the status of a given framework thus becomes apparent as something that emerges in particular situations in research activities expressing the relation between frameworks in use.

Our second case is an example of another use of the notion of background theory. First, Fetzer (2013) addresses a specific perspective, namely Latour's Actor Network Theory (ANT) as a background theory to understand objects in mathematics education:

> *Latour's approach is fascinating and irritating and provokes the research question, if and respectively how actor network theory can be a fruitful background theory to get a better understanding about the role objects play in mathematical learning processes* (Fetzer, 2013, p. 2800, italics in original)

Using Latour's ANT, Fetzer (2013) presents an example in line with Mason and Waywood's (1996) distinction between foreground and background theories. Latour's ANT, as a theory outside of mathematics education research, is used as a background theory determining the researchers' definition of an object, the researchable objects, methods and situations. Similar utilisations are found in Bikner-Ahsbahs and Prediger (2014) and Bikner-Ahsbahs and Halverscheid (2014). This way of using the notion of background theory implies that it is a perspective outside of MER, which allows the researcher to understand mathematics education through a particular philosophical or epistemological stance.

To sum up, our literature review on networking of theories indicates that the original terms, as defined by Mason and Waywood (1996), have undergone further development. The use of the notion of background and foreground theories in the networking of theories literature now also encompasses a more relative definition of background theory, i.e. one focusing on the relations of theories and frameworks within MER.

Discussion of the coexistence of two notions of background theory

In the discussion of theories related to networking of theories, Bikner-Ahsbahs and Prediger (2014) suggest to take "the notions of foreground and background theory as offering relative distinctions rather than an absolute classification, they can help to distinguish different views on theories (p. 6). This quotation clearly describes the development of the definitions of foreground and background theories. Hence, in line with the findings of our literature review, and as showcased by the two illustrative cases above (Fetzer, 2013 and Koichu, 2013), the relative and the absolute distinction of the foreground and background theories coexist in literature on networking of theories. Schoenfeld (2007) emphasizes a need for specificity of concepts in research, as loosely defined terms can produce variation in results. Looking at the absolute distinction of background theory, this satisfies Schoenfeld's criteria for specificity. However, what are the potential consequences of an absolute distinction of background and foreground theory? One consequence is that it causes a large number of foreground theories, because all theoretical developments and contributions generated inside MER are considered as such. Another consequence of the absolute distinction is an untended need for a notion that denotes the experienced distinctions between theories *inside* MER. Using Koichu (2013) as an example of Bikner-Ahsbah and colleges' (2014) relative use of the notions, theory *in* mathematics education has a similar role as a background theory. Hence, the use of foreground theory as a background theory seems to confuse the use of background theory, since *background theories inside mathematics education* and *background theories outside mathematics education* then coexist.

Moving to the relative distinction of background and foreground theories, also this might not withstand Schoenfeld's (2007) criteria for specificity. A first consequence of a relative distinction is a less clear definition of foreground and background theories. A second consequence is the existence of different utilizations of the notion of background theory. When different utilizations of background theory exist, a third consequence occurs: The importance of the background theories *outside* MER and its philosophical base might be indistinct. If researchers do not take their background theories *outside* MER into account, the implicit assumptions and hypotheses continue to be tacit.

Conclusion

Our study shows that both a relative and an absolute distinct of foreground and background theories exist in the literature of networking of theories. Koichu's (2013) uses the relative distinction when denoting the relation between theories or frameworks in use. Fetzer (2013) uses the absolute distinction when she considers the underlying beliefs or epistemological position that determines the researches' goals, aim, questions and objects. Considering both the absolute and the relative distinctions, the following consequences appear:

- Adhere to the *absolute distinction*: a need for a new notion distinguishing background theories emerges when networking.
- Adhere to the *relative distinction*: different utilizations of background theories appear.

The consequences of both reveal the need for distinguishing between foreground theories, background theories *inside* MER and background theories *outside* MER. In networking of theories, the relative distinction also builds on the changing relationship between the theories used in a research practice (Bikner-Ahsbahs & Prediger, 2006). This means that one theory may act as both foreground and background *inside* MER.

Looking at the consequences of an absolute and a relative distinction between foreground and background theories, these indicate the need for a new distinction/notion. We suggest that the *background theories inside mathematics education research* are referred to as *framing theories*. Looking at Koichu (2013), the new distinction informs and describes the different roles of foreground and background theories in networking. If the notion *framing theories* is applied, the importance of background theories *outside* MER arises and the implicit assumptions and hypotheses in background theories *outside* MER thus becomes clearer. The new notion is not needed to characterise Fetzer's (2013) networking practice and the distinction between foreground and background. However, given the use of background theory outside MER and foreground theory inside MER, the theories involved do not change between the two types in a networking practice. This means that the dynamic relationship between theories only exist between *framing theories* and foreground theories *inside* MER.

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