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Mathematical communication competency in a setting with GeoGebra

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The core of mathematical communication competency is the ability to interpret others and the ability to express oneself mathematically. This paper examines the activation of this competency and its interplay with digital technologies in an 8th-grade classroom (age 14-16), focusing on two students' interaction with GeoGebra. The aim is to investigate and discuss the stated interplay from two different lenses: discourse and instrumental genesis. The lenses bring different perspectives. On the one hand, the results show that using GeoGebra increases the complexity of students' mathematical communication because the students need to switch between discourses. On the other hand, to support communication competency, GeoGebra must be an instrument for the students.

Introduction

In mathematics education, the focus on both digital technologies and mathematical competencies is increasing (Trouche, Drijvers, Geudet, & Sacristián, 2013; Niss & Højgaard, 2019). However, a significant problem arises when implementing these two paradigms, which seem to run separately. This paper focuses on the interplay between digital technologies and the mathematical communication competency (as described in Niss & Højgaard, 2019) in an 8^{th} grade classroom (students aged 14-16) in Denmark.

In Denmark, it is meaningful to investigate this interplay because of the implementation of both digital technologies and mathematical competencies within the national curriculum in both primary and secondary education (UVM, 2018; 2019). However, research on the mathematical communication competency is limited, and research on its interplay with digital technologies does not exist.

This paper aims to investigate and discuss the interplay between the use of digital technologies and mathematical communication competency using two theoretical dualities: visual mediators-routines (Sfard, 2008) and artefact-instrument (Guin & Trouche, 1998). This paper asks how the two theoretical dualities of visual mediators-routines and artefact-instrument contribute to understanding the activation of students' communication competency in a setting in which students work with GeoGebra.

This study investigates the interplay using an illustrative case of two 8^{th} grade students' problem solving as data. The mathematical problem, which the students worked with, was partly designed to activate communication competency and partly adjusted to the use of GeoGebra. To understand how the communication competency is activated and the use of GeoGebra, the competency description is not enough. To understand the mathematical communication aspect, Sfard's (2008) perspective on discourses is utilised as an analytic tool. Also, the theoretical concept of instrumental genesis is used as an analytical tool to examine the use of GeoGebra (Guin & Trouche, 1998). Two transcripts of the two students' work are presented, followed by an analysis from a discourse perspective and then, an analysis from the perspective of instrumental genesis. The discussion combines the analyses with the competency perspective.

Mathematical communication competency

In the Danish competencies framework (KOM), general mathematical competency is defined as "someone's insightful readiness to act appropriately in response to *a specific sort* of mathematical challenge in given situations" (Niss & Højgaard, 2019, p. 14). KOM consists of eight mathematical competencies, all of which are related yet different, and one of them is mathematical communication competency. Having mathematical communication competency means having

> […] ability to engage in written, oral, visual or gestural mathematical communication, in different genres, styles, and registers, and at different levels of conceptual, theoretical and technical precision, either as an interpreter of others' communication or as an active, constructive communicator (p. 17).

Frequently, mathematical communication includes one or more mathematical representations and generates the use of particular mathematical concepts, results, or theories (Niss & Højgaard, 2019).

Communication in mathematics

This section introduces theoretically to mathematical discourse, focusing on visual mediators and routines.

Sfard (2008) defines communication in mathematics as

> ...a rule-driven activity in that discursants' actions and re-actions are from certain well-established repertoires of options and are matched with one actioner in a nonaccidental patterned way. (p. 146)

This definition regards communication as a patterned activity between individuals who act on others' actions based on rules. According to Sfard (2008), mathematical sentences give an impression of handling material things, but mathematical nouns replace the names of the material objects. In colloquial discourses, the objects exist independently of the discourse involved in the communication, but this does not apply in mathematical discourse. Visual mediators might represent objects in mathematics, but the actors of the mathematical discourse never gain access to the actual objects (Sfard, 2008).

Sfard (2008) includes four elements in the definition of mathematical discourse: word use, visual mediators, narratives, and routines. First, an important characteristic is the use of words that are distinct for mathematics. These words could be ‘equation’, ‘slope’, ‘piecewise function’, and ‘graph’ when working with functions. Second, a visual mediator in a mathematics discourse is “a visual realization of the object of a discourse” (Sfard, 2008, p. 302). Particularly in mathematics, this involves mathematical symbols, but a realization could also include graphs, gestures, words, and drawings. Third, the narratives are descriptions and explanations of mathematical objects and activities. Forth, routines are “repetition-generated patterns of our actions” (Lavie, Steiner, & Sfard, 2019, p. 153).

Based on Sfard (2008), having mathematical communication competency (Niss & Højgaard, 2019) means that a student must be able to engage in a mathematical communication situation based on particular rules for the activity. The student must be able to use mathematical words; use and understand different visual mediators existing in the communication situation and understanding rules of engaging in a communication situation (Sfard, 2008). In a mathematical communication situation in which students work with GeoGebra, GeoGebra becomes a mediator of discourse. The students then both act and react on each other and the in- and outcomes provided by GeoGebra. (Antonini, Baccaglini-Frank, and Lisarelli, 2019).

Instrumental genesis

When students use digital technology within the classroom, the software influences the mathematics content that the students are learning, because new opportunities to interact with mathematical objects emerge (Guin & Trouche, 1998). The use of GeoGebra is no exception. Distinguishing between artefact and instrument is key when using instrumental genesis to look at students’ use of technology. An artefact is regarded as a material object; an instrument, on the other hand, does not exist in itself, but an artefact becomes an instrument for a person when she can use it in an activity (Verillon & Rabarbel, 1995). When an individual manipulates an artefact into an instrument, instrumental genesis happens (Guin & Trouche, 1998).

The complex process of instrumental genesis leads to the reorganization of activity – in this way, the student can manipulate the artefact, and it becomes an instrument. The student needs to acquire new knowledge to be able to make new procedures. At the same time, “the features of instrumented activity are specified” (Guin & Trouche, 1998, p. 201) concerning both the constraints and the possibilities inherent to the artefact. The constraints and the possibilities are subsequently connected to the new procedures of the artefact. Then, the student encounters the artefact, and the student can "identify, understand and mange in the course of this action" (Guin & Trouche, 1998, p. 201). Thereby, the instrumental genesis occurs when the student has new possibilities to use the artefact, and when a reorganization of the instrumented activity takes place. The artefact becomes an instrument for the student (Guin & Trouche, 1998). In an instrumented activity, it is essential to state the close relationship between the students' mathematical knowledge and knowledge about use of the instrumental (Lagrange, 2005)

The use of the digital tool offers different representations and words (Guin & Trouche, 1998), which is interesting when looking at mathematical communication (Niss & Højgaard, 2019).

Method

The aim of this study is reached by investigating the communication between two students that are solving a mathematical problem concerning piecewise functions, with the help of GeoGebra. The data consists of the students' speech, their actions, and the reactions from GeoGebra. Data is presented as transcripts of a discussion between the two students.

This section presents the educational context, the design of the mathematical problem, and the transcript used as data in the analyses.

Design of the mathematical problem

The students solve a task, based upon a released PISA task about newspaper sellers. Two job advertisements describing sellers' pay per week are presented from two different newspapers: Zedland Star and Zedland Daily. The Zedland Daily pays 60 Danish kroner every week and, additionally, 0.05 Danish kroner per sold newspaper. Zedland Star pays 0.2 Danish kroner per sold paper, and then 0.4 additional Danish kroner per newspaper sold after selling 240 newspapers in one week. (OECD, 2012, p. 70, task no. PM994Q).

The designed lesson lasted 90 minutes. The students worked together and shared one computer to promote communication. The task offered the use of different representations, essential for both mathematical discourse (Sfard, 2008) and the process of instrumental genesis (Guin, & Trouche, 1998). In PISA, the task was a multiple-choice question – the students had to choose between four pictures, which all contained both a graph for Zedland Star and Zedland Daily. In the present study, the students were guided by instructions about the newspapers' pay:

> Use GeoGebra to calculate the differences between how the newspapers pay the sellers.
>
> - Compare the graph and solutions for the two newspapers.
> - Choose the newspaper you would prefer to work for and explain thoroughly why.

- Prepare arguments for a discussion in plenum using your results, including different representations.

Zedland Daily is defined as $F_D(x) = 0.05x + 60$. Zedland Star is defined as $F_S(x) = 0.2x\ if\ 0 \leq x \leq 240$ and $F_S(x) = 0.6x - 96\ if\ x \leq 240$. (OECD, 2012, p. 70, task no. PM994Q).

Educational setting

Data collection took place in an 8th grade (students aged 14-16) classroom in Denmark. The students usually worked with GeoGebra in school, but they were not used to solve tasks concerning piecewise functions in GeoGebra. Learning about piecewise linear functions is mentioned in the Danish curriculum (Undervisningsministeriet, 2019). However, working with piecewise linear functions demands a more comprehensive understanding of functions than just working with linear function (Bayazit, 2010).

Transcripts of the students' dialogue

The data is presented as transcripts of dialogue between two students (S1 and S2). Two transcripts function as extracts and focus on the students' process of the mathematising of Zedland Star when using GeoGebra. Zedland star is the piecewise linear function.

The following is Transcript 1. S1 has, just before the transcript started, written $fx = 0{,}4x + 48$ in the algebra window in GeoGebra. The use of commas is essential here: In Denmark, the rule is to use decimal *commas* instead of decimal points in decimal numbers.

S1	This is what we agree on, right? For every x, we have 0,48? [*S1 looks at S2*] Then we are pressing 'enter'.
S2	Enter, enter, enter.
GeoGebra	Please check your input.
S1	What? No! Oh, it is because. I know what I did wrong [*S1 points with the mouse on fx*]. A parenthesis is missing there. There is a parenthesis missing there! [*S1 controls the computer and writes f(x) instead of fx.*]
S1	Otherwise, we will change it into y. [*S1 presses enter again].*
GeoGebra	Please check your input.
S1	What? [*S1 changes f(x) into y.*] How bad are we at this?
GeoGebra	Please check your input.
S1	Oh, it is because you need to use a period instead of a comma. It still needs to be y. [*S1 changes the comma into a decimal point.*]
S1	Wouldn't it [=GeoGebra] show it? [*S1 right-clicks with the mouse in the algebra window; she does not understand why it cannot be shown in the graphical window.*]

S1	Show object. Oh, it is because it starts at 48, we are so stupid. [*S1 zooms out.*]

Transcript 1: Decimal commas versus decimal points.

Transcript 2, below, is an extract of the dialogue when the students figure out that Zedland Star is a piecewise function.

S1	Ups. We did something wrong. This is not right, S2. [*S1 and S2 look at the face of GeoGebra.*] That one is not right. It says that when you have sold zero newspapers, you have 48 DKK (=danish valuta), but that is not true? When you sold that many when you have sold 240 newspaper, you have 48 DKK – then, you are over here, but I do not know how to do that. [*S1 points at (240, 48) in the coordinate system.*]
S2	Okay [*S2 takes on the control of the computer, deletes y=0.4x+48 in the algebraic window. Now it says, "y=".*]
S1	Okay. We will try to change it again.
S2	Okay. So, if you have sold 240, you get 48 DKK, okay?
	[…]
S1	This is really how you are supposed to write it, x… This is how it is until we reach 240, then we have to… [*S1 writes "y=0.2x", but does not press enter.*]
S2	Should we try something else? [*S2 adds "0.4" to the equation*].
S1	Okay 0,4x. I think that we are missing something on that one [*pointing at their present graph for the Zedland Star*]. There must be a limit, a parenthesis, or something else. Is there supposed to be something else, when it reaches 140 [meaning 240]?
S2	Yes.

Transcript 2: Understanding piecewise functions.

Analyses of the students' communication

This section presents two analyses of the presented transcripts. The first one focuses on the students' communication. The second one focuses on the use of GeoGebra.

Communication when using GeoGebra: Visual mediators and routines

This section analyse transcript 1 and 2 using Sfard's (2008) concepts *visual mediators* and *routines* to understand the mathematical communication between the students. The analysis focus on identifying different uses of *visual mediators,* which is uses of various mathematical realisations of mathematical objects, in this case piecewise functions, and students' different *routines*, that is, patterns of visible actions relating to the mathematical content.

In transcript 1, the students' communication involves different visual mediators such as "*0,48",* the equation written in GeoGebra, and the graphical representation of the function. Taking decimal numbers as visual mediators, a discrepancy between the discourse between the students and the discourse between the students and GeoGebra emerge. In the discourse between the students, decimal numbers are written using decimal comma (e.g., "0,48"). In the discourse between GeoGebra and the students, decimal numbers are written using decimal points (e.g., "0.48"). In the discourse between the students and GeoGebra, GeoGebra becomes a mediator of the mathematical discourse between the students. The students then need to switch between discourses and the different visual mediators used in the two discourses. Looking at transcript 2, a development of the students' use of decimal points and commas appears when the students *say* "comma", but uses decimal points when writing in GeoGebra. The students then know how to use the visual mediators of both discourses.

The students' use of a decimal comma when writing decimal numbers can also be described as a routine in their mathematical discourse between the students (i.e., when they write decimal commas in transcript 1 and say comma in transcript 1 and 2). In the discourse mediated by the students and GeoGebra, the use of decimals points when writing decimal numbers in GeoGebra is regarded as a meta-rule mediated by GeoGebra. The use of a decimal point is a pattern, repetitive in their action when working with decimal numbers. Mainly in transcript 2, this pattern of action appears when the students repeatedly keep on the writing the decimal points due to GeoGebra's demands.

Summing up, two different mathematical discourses exist when students use GeoGebra in class analysing the visual mediators and the routines in transcript 1 and 2.

The use of GeoGebra when communicating: artefact and instrument

This section analysis the students' use of GeoGebra in transcript 1 and 2 utilising two concepts from instrumental genesis: *artefact* and *instrument,* to understand how GeoGebra influences the mathematical communication between the students. The analysis focuses on identifying if GeoGebra is (still) an *artefact* for the students, which is when the program is just a material thing, or if GeoGebra has become an *instrument.* GeoGebra is regarded as an instrument if the students can use GeoGebra in an activity (Verillon & Rabardel, 1995). The process of *instrumental genesis* is identified by looking at reorganisations of the students' problem solving activity (Guin & Trouche, 1998).

At the beginning of transcript 1, GeoGebra is an artefact for the students since the students' ability to use GeoGebra is limited. The students keep using a comma instead of a point, which indicates that the students have not yet acquired the knowledge needed for the use of GeoGebra involving decimal numbers. Because of the feedback provided by GeoGebra, the students reorganize their activity to solve the task. Thereby, the feedback increases knowledge acquisition and the students slowly begin to understand the constraints of GeoGebra.

At the end of transcript 1, the students have learned how to use; that is, the need to use decimal points instead of decimal commas. The process of instrumental genesis has begun: at the very end of transcript 1, further possibilities for the use of GeoGebra arise, which is the zooming feature. This development indicates that the students' acquisition of knowledge is an ongoing process throughout the activity.

In the middle of transcript 2 show the reorganization of activity when writing decimal numbers within GeoGebra. GeoGebra has become an instrument for the students. Although the students have instrumented GeoGebra working with decimal numbers, the students experience new constraint when aiming at making a piecewise function at the end of transcript 1. Elements of GeoGebra remains artefacts.

The students' understanding of functions to include piecewise functions are lacking, which could be a reason why the students try to construct Zedland Star as one function for the whole interval. The mathematical knowledge concerning the object related to the artefact is essential for instrumental genesis. However, the students' mathematical understanding increases – or, their knowledge about representing piecewise functions in GeoGebra improves – as the students work in the teaching session.

Discussion: A Mathematical Communication Competency perspective

In this section, the activation of the mathematical communication competency from the two dualities, visual mediators-routines and artefact-instrument, is discussed.

As stated earlier, the communication competency consists of both the ability to express oneself and the ability to interpret others' communication (Niss & Højgaard, 2019). From a mathematical discourse perspective, the students need to master the different discourses that they participate in (Sfard, 2008). As seen in transcript 1 and 2, the students need to be able to switch between the two discourses. In a situation, in which the students activate their communication competency, they must understand how the routines and visual mediators of the particular discourses they participate in. The situation depends on who is engaged in the communication situation – a student taking the person that he communicates with into account master the communication competency better (Niss & Højgaard, 2019). The students must be able to switch between different discourses communities. For instance, the discourse changes occur between peers, in classroom discussions, and when working with software (Sfard, 2008; Antonini et al., 2019). Using the former analysis combined with the aspects of the communication competency, the students not only need to express themselves to other *people* without the use of software, but also to *software*, such as GeoGebra, and they need to express themselves through *software* to others (Niss & Højgaard, 2019). When students communicate to *software*, such as GeoGebra, they communicate based on GeoGebra's rules of discourse (transcript 1). The students express themselves through *software* when they use the results from the software in communication with others, depending on the current discourse (transcript 2). In the activation of the communication competency, the students express themselves to others based on rules of discourse outside the software, but the students must understand how to switch between discourses (Sfard, 2008; Antonini et al., 2019; Niss & Højgaard, 2019). If the students do not understand the different visual mediators and routines for each discourse, the communication is not as effective as it could be, and their communication competency would seem less developed (Niss & Højgaard, 2019).

Using instrumental genesis to analyse the students' activation of their communication competency, the students need to have GeoGebra as an instrument (Guin & Trouche, 1998). When GeoGebra functions as an instrument, the students understand and control the communication and the rules for the use of decimal points versus decimal commas. If students' attention is on how to write decimal numbers, their focus moves from the mathematical aspects of the activity (i.e. piecewise functions) to the features of GeoGebra, because GeoGebra then is yet an artefact (Verillon & Rabardel, 1995). At the end of transcript 1 and beginning of transcript 2, the students' have been through the process of instrumental genesis, which means that the students' problem solving activity has been reorganised (Guin & Trouche, 1998). After the reorganisation, the students can focus on representing Zedland Star using both equations and graphs. Knowledge of various representations makes the students more competent in mathematical communication when they express their mathematical ideas and solutions to the tasks (Niss & Højgaard, 2019).

The students' knowledge about the mathematical content (i.e., functions), seems to be very relevant not only for the use of GeoGebra but also to show mathematical communication competency. When the students do not know that Zedland Star must be formulated differently for the two intervals, the students' seems less competent when communicating because they express an incorrect understanding of piecewise functions (Niss & Højgaard, 2019; Bayazit, 2010).

Summing up, this study indicates that mathematical communication competency is more complex when students use GeoGebra to solve mathematical problems. Both perspectives indicate that the students must be familiar with the tool to support the competency. Using Sfard (2008), students must understand the different discourse they participate in and they must switch between them. Also, instrumental genesis must happen, involving a reorganisation of activities for the students (Guin & Trouche, 1998).

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