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Investigating grade 6 students' mathematical problem solving in small group interaction

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This paper presents an investigation of mathematical problem solving in two small groups of students in grade 6. The interaction within the groups was analysed with proof schemes as analytical tool, in combination with principles and procedures of Ethnomethodological Conversation Analysis (EMCA). In addition, the groups' orientation to social and sociomathematical norms was analysed. The analysis showed that one group oriented to social and sociomathematical norms that gave rise to a potentially positive learning opportunity, whereas the other group primarily oriented to a social norm of equality that overshadowed the mathematical discussion. This study serves as a pre-study for the analysis of larger material, where EMCA appears as a promising methodological contribution.

Introduction

Within the field of mathematics education, there is a consensus that collaboration is beneficial for students' mathematical development (e.g. Wood & Kalinec, 2012). However, letting students take part in group discussions and collaborative tasks does not automatically lead to productive mathematical work; sometimes participation as such is favoured, prior to discussions regarding the mathematical content of the activity (e.g. Kilhamn, Hillman and Säljö, 2019).

This paper presents an investigation of grade 6 students' mathematical problem solving in small group interaction. The study is based on the *ethnomethodological approach*, which is infrequently applied to previous research within the field of mathematics education (Ingram, 2018). According to ethnomethodology, social interaction is a process where participants orient to shared norms of conduct, and where actions are organised in recognisable patterns (Heritage, 1984). By organising actions in patterns, the participants contribute to the establishment of norms (Ingram, 2018), which in the mathematics classroom consist of both general *social norms* and *sociomathematical norms* that are specific to mathematical activities (Yackel & Cobb, 1996). According to Yackel and Cobb (ibid.), the development of sociomathematical norms creates a "taken-as-shared" sense of when and how to contribute to mathematical discussions, as the norms concern both the content and the process of participating in a mathematical activity. For the same reason, McClain and Cobb (2001) argue that the development of sociomathematical norms is closely related to the students' mathematical development. Kazemi and Stipek (2008), who compare learning opportunities in varying classroom practices, confirm McClain and Cobb's statement. However, further analysis of the role of social interaction in the mathematics classroom is valuable, particularly regarding how teachers' and students' turns build upon each other, and contribute to patterns that organise mathematical classroom communication (Drageset, 2015). Wood and Kalinec (2012) also call for the analysis of the relation between academic and social aspects of students' collaboration in the mathematics classroom. The aim of this study is thus to analyse both mathematical and social aspects, and their relationship, of grade 6 students' mathematical problem solving in small group interaction. Our research questions are as follows:

1. What characterizes explanations and solutions that students consider mathematically acceptable?
2. What social and sociomathematical norms do students orient to, when engaging in mathematical problem solving?

As only two groups were observed, this study serves as a pre-study for the analysis of larger material.

Theoretical framework

This study is based on a combination of *ethnomethodology* and the *emergent perspective*. The aim of ethnomethodology is to analyse what people, who participate in various kinds of everyday activities, do in order to make those activities meaningful (Heritage, 1984). The emergent perspective shares this aim, but focus specifically on mathematical activities (Cobb & Yackel, 1998).

According to the emergent perspective, the relation between the individual student and the social context of the mathematics classroom is central. As the constitution of norms is an important factor of the classroom culture, Yackel and Cobb (1996) developed the concept of *sociomathematical norms*, in connection to the formulation of the emergent perspective. Sociomathematical norms are normative aspects that specifically concern mathematical activities, whereas general social norms apply to relations between participants. One example of a social norm that is applicable in all classrooms, regardless of subject matter, is that students are expected to explain and account for their solutions to a problem. The corresponding sociomathematical norm is to expect that an explanation of a solution to a *mathematical* problem is *mathematically acceptable*. Similarly, in any discussion, a new suggestion of a solution to a problem should be different from what has already been suggested, but in a discussion of a *mathematical* problem, the suggestions have to be *mathematically different*. Students should also be able to recognize and appreciate the variety in *sophistication* of different mathematical solutions (McClain & Cobb, 2001).

What counts as mathematically acceptable, different and sophisticated solutions varies between classrooms, as sociomathematical norms emerge through interactive processes between the teacher and the students (Yackel & Cobb, 1996). For instance, the teacher contributes to the development of sociomathematical norms by explicitly asking her students to present different solutions to a given task, as shown by McClain and Cobb's (2001) analysis of discussions in primary school classrooms. The teacher's way of asking questions or giving attention to certain explanations also implicitly contributes to the development of normative patterns in the social interaction, which Partanen and Kaasila (2015) reveal in their investigation of the development of norms during upper secondary school students' collaboration in small groups.

McClain and Cobb (2001), as well as Kazemi and Stipek (2008), investigated norms of entire classrooms, whereas we, like Partanen and Kaasila (2015), focus on small group work. However, we do not analyse the actual establishment of norms over time; instead, we focus on how participants orient to norms during the course of group interaction. Sociomathematical norms were part of the theoretical framework in Levenson, Tirosh and Tsamir's (2009) investigation of the discrepancy between the norms teachers endorse and students perceive, as well as in Wester's (2015) analysis of the tension between the teacher's intentions and students' perception of norms. The difference between these two studies, and the study reported in this paper, is that our aim is to analyse students' interaction, rather than the interaction between students and teachers. Students' interaction was also the focus in Tatsis and Koleza's (2008) identification of norms in students' problem solving in pairs. However, Tatsis och Koleza performed their study as an experiment, whereas ethnomethodological investigations concern naturally occurring activities (Heritage, 1984).

Empirical material and method for analysis

The empirical material of this study consists of video recordings of two heterogeneous groups of students in grade 6, solving a mathematical problem. The observed group work represents a natural and authentic classroom situation, as the teacher of the class often let her students collaborate in small groups. The two groups (Group A and Group B) were observed during one lesson where they solved a combinatorial problem that was formulated as follows:

> There is a line-up at the bus stop. In how many different ways can:
>
> a) 2 persons stand in line?
>
> b) 3 persons stand in line?
>
> c) 4 persons stand in line?
>
> Try to find a rule for calculating the number of line-ups.

The students have been given fictitious names: Alice, Anna, Alan and Andy in Group A, and Bea, Bibi, Benny and Billy in Group B. At the beginning of the lesson, they were instructed by their teacher to start out by working individually with the problem. After a few minutes, they were asked to turn to the classmate sitting next to them, and agree on a "pair-solution". Thereafter, the dyads formed groups of four, and were told to agree on a "group-solution".

In order to characterize the explanations and solutions that the students considered mathematically acceptable (RQ1) we used the concept of *proof schemes* as analytical tool. According to Sowder and Harel (1998), proof schemes are what makes people convinced that an assertion is true. In mathematics, the three main classes of proof schemes are *analytic, empirical* and *externally based.* Students who are able to reason in a logical and general manner demonstrate *analytic proof schemes*, whereas *empirical proof schemes* consist of explanations that rely on the perception of examples or concrete objects. The *externally based proof schemes* describe situations where the convincing factors are located outside of the student, such as what an authority (for instance the teacher) has stated. The initial stage of the analysis was thus to code the students' utterances with regard to demonstrated proof schemes.

To further analyse the social interaction in the groups, we used principles and procedures of *Ethnomethodological Conversation Analysis (EMCA)*. One principle of EMCA is to analyse naturally occurring data. Another principle is to treat talk-in-interaction as contextually embedded, in the sense that participants' utterances and actions only can be understood in relation to what other participants say and do (Heritage, 1984). One major finding in EMCA research is that *repair practices* (that is, participants' handling of various kind of trouble during talk, such as problems of understanding) is interactionally organized in at least three parts: *trouble source*, *initiation of repair* and *repair* (Sidnell, 2010). Each spoken turn in the analysed sequences was therefore related to the preceding and the following turns, and their contributions were interpreted with regard to how the participants displayed an understanding of the turn (or not). As norms can be identified as patterns in social interaction (Ingram, 2018; Yackel & Cobb, 1996), we also analysed sequences of turns that demonstrated the groups' orientation to social and sociomathematical norms (RQ2).

Although the analytical process is described above as consisting of three distinct stages, the analytical phenomena in these stages were intertwined in the interaction. For instance, students sometimes demonstrated a certain proof scheme when engaging in repair, and the repair practices were, in turn, contributing to the group's orientation to norms.

Below, in the analysis of the interaction in the two groups, dialogue excerpts are organised in turns at talk, including relevant embodied actions and handling of artefacts. The spoken turns are unmarked for intonation. Descriptions by the transcriber are marked with ((double parenthesis)) and speaker's emphasis with underlining.

Analysis of the interaction in Group A

During individual work, Andy initially wrote a table of the possible combinations of line-ups for three persons, where the digits 1, 2 and 3 represented the persons in line. Thereafter he wrote two sets of six combinations, all beginning with number 1 and 2 respectively, representing four persons in line, as shown in figure 1:

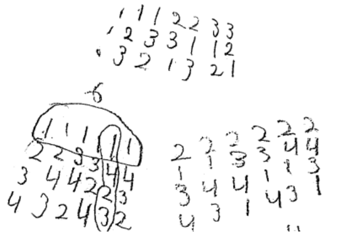

Fig. 1: Excerpt from Andy's notes.

Using the two tables with six combinations, Andy was able to calculate the total number of possible line-ups for four persons. He explained his calculation to his "pair-partner" Alan in the following way:

> if the same person is in the front, then there are six different ways for the ones behind to stand, and then there are four persons who can be in the front, so I just figured six times four is twenty four

Andy reduced the problem from four to three persons, by fixating one person in the first place, and was then able to calculate the number of line-ups: "six times four". As Andy formulated a solution based on logical reasoning about a general character of the problem, he demonstrated an *analytic proof scheme*. However, at first, the other group members could not quite understand Andy's explanation. In the following sequence, Andy initially repeats his analytic explanation, which Anna and Alice respond to:

41	Andy:	if the 1 is in the first place all the time then there are six different ways if there are three persons behind
42	Anna:	yeah but what if everyone else is in the first place
43	Andy:	((writes combinations with number 4 in the first position)) as there's a 4 you have to take 4123 4132 42, well you get it
44	Anna:	yes but all the others can be in the front
45	Andy:	but check this out, if there's only the 4 in the front
46	Alice:	yeah
47	Anna:	yeah
48	Andy:	with these three persons behind, they can move about so there are six different ways

49	Alice:	and then we can to the same thing with the others
50	Andy:	yes and then y'know there are four persons, six different ways, six times four is twenty four
51	Alice:	mm ((nods)) good let's do this as our group's

Anna's "yeah" (42) indicated that she accepted Andy's solution (41), but her following question also revealed that she did not understand his explanation. Andy treated Anna's question (42) as a *trouble source* when he switched to an *empirical proof scheme* and, provided a concrete example (43) as an attempt to *repair* her difficulties to understand. However, Anna showed that she still did not understand Andy's general explanation, and extended the repair sequence, when once again stating that everybody can be in the first position (44). Andy then continued to explain, by stressing the importance of keeping the same number in the first place: "only the 4" (45). Anna and Alice showed that they listened, by saying "yeah" (46, 47). Alice also showed that she now could follow Andy's reasoning (49), which Andy confirmed by repeating his initial way of explaining his solution (50), which closed the repair sequence. Alice then made it public that the group had agreed on Andy's suggestion as the group's joint solution (51).

Andy's group mates did not simply accept his solution, but strived to understand his general explanation, as shown by Anna, raising the same objection twice (42, 44). Together with Andy's thorough explanation of his solution (43, 45, 48), the repair sequence displayed an orientation to a social norm to *strive for joint understanding*. Andy also seemed to contribute to the others' ability to reason in a logical manner as Alan, soon after the sequence presented above, intuitively formulated the factorial function when telling how the number of line-ups for three and four persons can be calculated:

> one times two is two times three is six, one times two is two times three is six
> six times four is twenty four

The group assessed Alan's formulation as "cool", "smart" and "magic", and also told the teacher that their solution was "great" and "awesome". Together with the effort to understand Andy's logical reasoning, the group's response to Alan's formulation of the solution displayed an orientation to a norm that *solutions to mathematical problems should be formulated on a general level*. This corresponds to the primal sociomathematical norm of recognizing the sophistication of a specific solution.

Analysis of the interaction in Group B

In Group B, Bibi initially wrote all possible combinations of three persons in line. Thereafter, she let the digits 1, 2, 3 and 4 represent four persons in line, and presented her solution as a list of six combinations, all beginning with 1, as shown in figure 2:

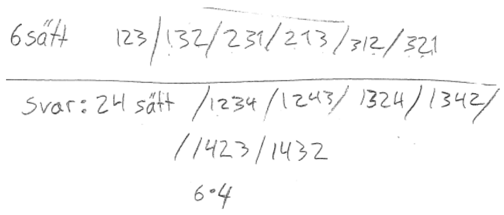

Fig. 2: Excerpt from Bibi's notes.

Just like Andy, Bibi formulated a solution based on logical and general reasoning when explaining her calculation to her "pair-partner" Bea:

> here are all the ways with a 1 in the front, then all the ways with a 2 in the front that should get exactly the same number as there are just as many figures [so] with four figures I just did six times four and that's twenty four

Bibi demonstrated an *analytic proof scheme* in formulating a general character in her solution: "with a 2 in the front, that should get exactly the same number". Benny and Billy, on the other hand, assumed that everyone had to change places to form a new combination. They presented their solution as tables of letters and dots, as shown in figure 3, thus demonstrating an *empirical proof scheme*:

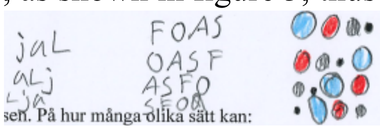

Fig. 3: Excerpt from Billy's notes.

When the dyads came together as a group, they instantly realised that they had interpreted the problem in completely different ways. However, the students did not discuss the mathematical content of their solutions. Instead, Billy asked the teacher to join them in order to assess Bibi's solution of the number of line-ups of three persons:

43	Billy:	((points at Bibi's solution)) can you, can you do it this way, what's it now, eh, 123, 132, 231, 213
44	Teacher:	((interrupts Billy)) you have kind of drawn how these different people stand
45-46		((omitted talk about who wrote the solution))
47	Teacher:	yes you could do it that way
48	Bea:	yes
49	Benny:	okay

Billy's request for the teacher's assessment (43) demonstrated an *externally based proof scheme*, which also Bea and Benny accepted in their responses to the teacher's positive answer (48, 49). The possible problem of having two solutions was, however, not resolved. Shortly after the sequence presented above, Benny turned to the teacher, again demonstrating an externally based proof scheme, in asking which of the dyads' solutions the group should choose:

71	Benny:	which one should we have as the group's, eh, joint

72	Teacher:	((to Benny and Billy)) well since you thought about it in a different way ((omitted talk about different parts of the task)) maybe you could present a and b in your two different ways of thinking 'cause you just thought about it in a different way, you haven't, like, done anything wrong
73-76		((omitted talk about which worksheet to write on))
77	Benny:	we write both of them
78	Bibi:	we, like, write both of the ways
79	Teacher:	you could do that
80	Bea:	mm
81	Teacher:	it can be quite interesting, there could be others in the class who thought about it in this way

The teacher's response (72, 81) resembles the sociomathematical norm that suggested solutions to a mathematical problem have to be mathematically different. However, the teacher also oriented to a social norm of equality, in telling Benny and Billy that they "haven't, like, done anything wrong" (72). Together with the statement that they had "just thought about it in a different way" (72), the teacher's stance towards the dyads' differing solutions became guiding for the group's subsequent assessment:

98	Bibi:	but your idea was also very well thought out
99	Benny:	right
100	Bibi:	because it is, it depends, both of them can be the correct answer y'know
101	Billy:	well both, none of them is actually the correct one
102	Bibi:	no exactly
103	Benny:	it all just depends on how you think about it
104	Bibi:	exactly
105	Billy:	both are just as correct
106	Benny:	both were just as good

Initially, Bibi assessed Benny and Billy's solution positively (98), which Benny agreed with (99). The group also agreed that both solutions were equally correct (100, 101, 105, 106) and Benny repeated that the only difference between the two solutions was "how you think about it" (103). Although the students talked about their mathematical solutions, this sequence demonstrates an orientation to a social norm that *both parties are equal.*

Conclusions and discussion

Many studies regarding classroom norms (e.g. Kazemi & Stipek, 2008; Levenson, Tirosh and Tsamir, 2009; McClain & Cobb, 2001; Yackel & Cobb, 1996; Wester,

2015) analyse whole class interaction, and interaction between teachers and students. The study reported in this paper adds to previous research in that we focus mainly on social interaction within small groups of students, engaging in mathematical problem solving. The students in the observed groups had belonged to the same class, with the same teacher, for two years. It is therefore reasonable to think that they had taken part in the same mutual processes of developing classroom norms. Nevertheless, our results show that there were significant differences regarding which social and sociomathematical norms the groups oriented to. As the development of sociomathematical norms “gives rise to learning opportunities” (Yackel & Cobb, 1996, p. 466), our results imply that students’ in the very same classroom create and experience a variety of learning opportunities, within different groups.

The solution that Group A considered to be mathematically acceptable was characterized by an analytic proof scheme, as Andy’s explanation was based on logical reasoning about general features of the problem. By engaging in a collaborative repair sequence, the students oriented to a social norm to strive for joint understanding. In positively assessing Alan’s formulation of how to perform calculations, the students also displayed an orientation to a sociomathematical norm that solutions should be formulated on a general level. The group therefore (intuitively) created a potentially positive learning opportunity, characterized by what Kazemi and Stipek (2008) denote *press for conceptual learning*.

In Group B, the dyads presented two solutions, characterized by analytic and empirical proof schemes respectively, but instead of discussing each other’s solutions, the students invited the teacher’s authority in choosing which one to present as the group’s joint solution. The teacher conveyed that it is interesting to present different solutions, which is in line with the primal sociomathematical norm that various suggestions to a mathematical solution have to be mathematically different (Yackel & Cobb, 1996). However, as McClain and Cobb (2001) argue, if there is no assessment of the sophistication of the different solutions, the discussion does not contribute to the students’ mathematical development. As neither the group nor the teacher discussed the actual content of the two solutions, they oriented primarily to a social norm of equality that overshadowed the mathematical discussion.

Wood and Kalinec (2012) suggest that researchers should focus on both the mathematical activities and the social talk, in order to better understand how teachers’ arrangements of groups, and the design of tasks, might support students’ group work. Kilhamn, Hillman and Säljö’s (2019) finding that the mathematical concepts and ideas of collaborative tasks do not always get adequate attention underlines the importance of investigating academic as well as social aspects of students’ collaboration. Kilhamn et al. also encourage continued investigation of how sociomathematical norms can be made explicit to students, to support their mathematical development. The analysis on a turn-by-turn basis (as called for by Drageset, 2015) of students' demonstrations of proof schemes, and orientation to social and sociomathematical norms, showed how differing conversational patterns may shape mathematical content as well as social practices in students' group work. EMCA therefore appears as a promising methodological contribution to the analysis of collaborative problem solving.

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